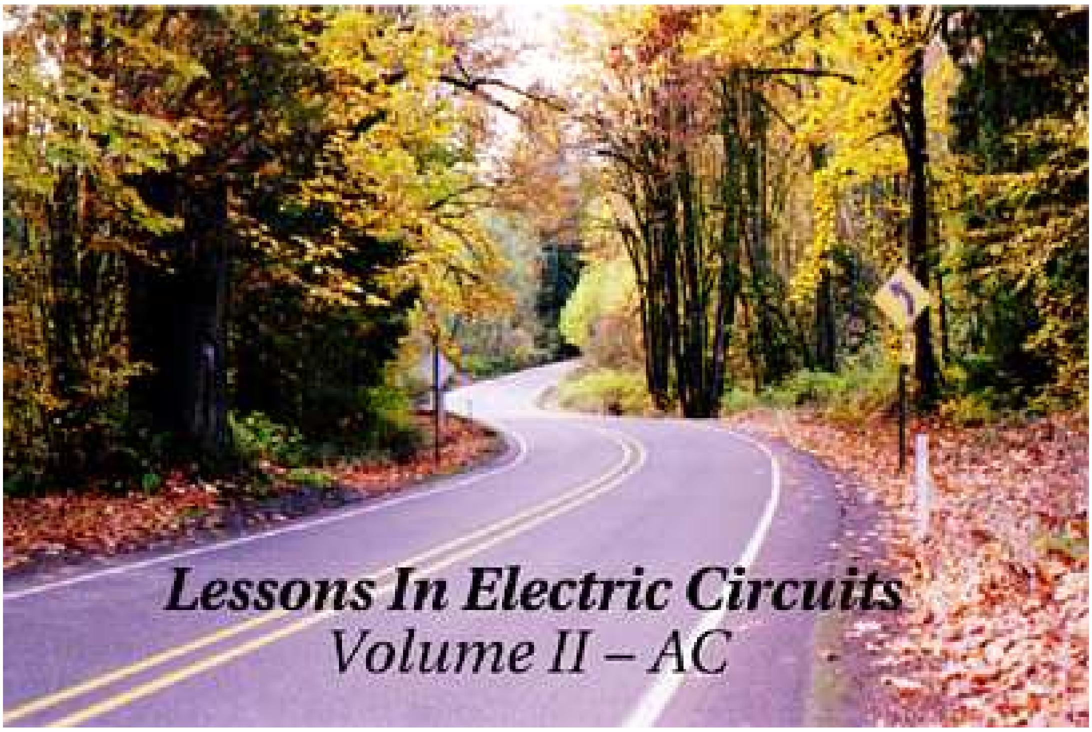

Cinquième Édition, dernière mise à jour le 10 Janvier 2004

Leçon sur les Circuits Électriques, Volume II – AC

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Cinquième Édition, dernière mise à jour le 10 Janvier 2004

Available in its entirety as part of the Open Book Project collection at `http://www.ibiblio.org/obp`

PRINTING HISTORY

- First Edition: Printed in June of 2000. Plain-ASCII illustrations for universal computer readability.
- Second Edition: Printed in September of 2000. Illustrations reworked in standard graphic (eps and jpeg) format. Source files translated to *Texinfo* format for easy online and printed publication.
- Third Edition: Equations and tables reworked as graphic images rather than plain-ASCII text.
- Fourth Edition: Printed in November 2001. Source files translated to *SubML* format. SubML is a simple markup language designed to easily convert to other markups like LaTeX, HTML, or DocBook using nothing but search-and-replace substitutions.
- Fifth Edition: Printed in November 2002. New sections added, and error corrections made, since the fourth edition.

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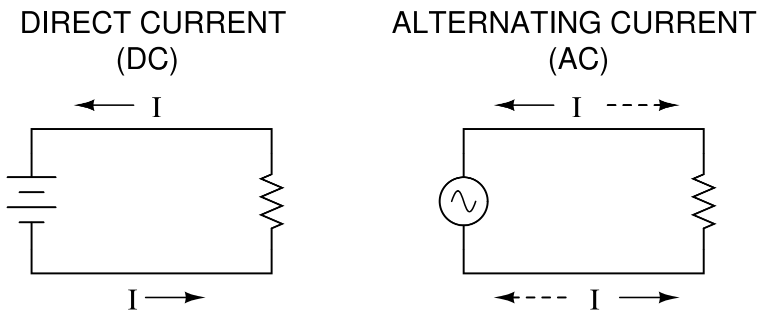
Chapter 1

THÉORIE AC DE BASE

1.1 Qu'est ce que le courant alternatif (AC)?

La plupart des étudiants en électricité commencent leur études avec ce qui est connu comme le *courant direct* (DC), qui est le flux électrique dans une direction constante et/ou possédant une tension avec une polarité constante. DC est le type d'électricité fabriqué par une batterie (avec des terminaux définitivement positifs et négatifs) ou le type de charge généré par frottement d'un certain type de matériaux avec un autre.

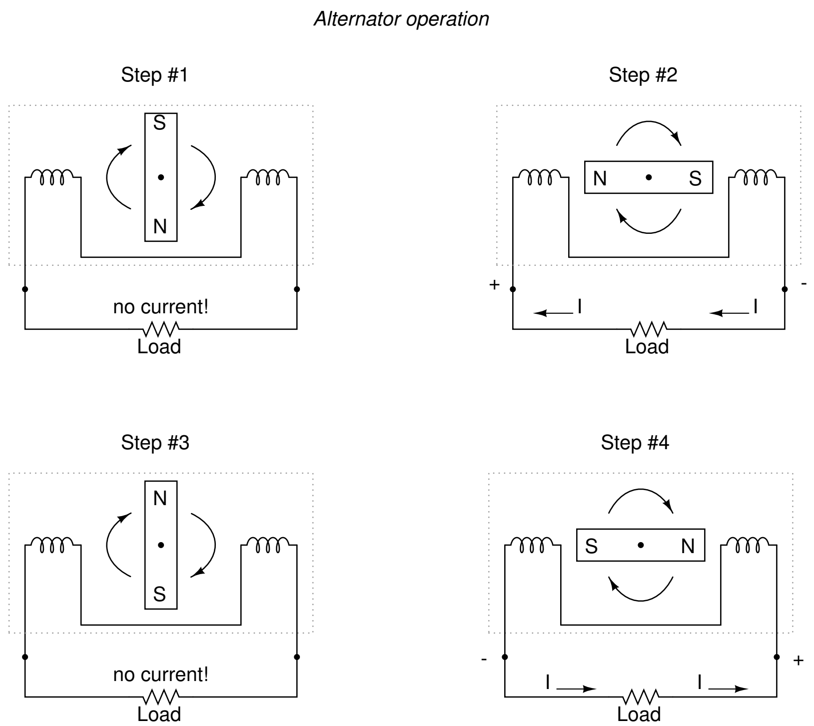
Si utile et facile à comprendre que l'est le courant continu (DC), ce n'est pas le seul "type" d'électricité utilisé. Certaines sources d'électricité (en grande majorité les générateurs rotatifs électro-mécaniques) produisent naturellement des tensions alternant leur polarité, changeant du positif au négatif au cours du temps. Que ce soit un changement de la polarité de la tension ou un changement de direction du courant d'avant en arrière, ce "type" d'électricité est connu comme Courant Alternatif (AC) :



Alors que le symbole familier de la batterie est utilisé comme symbole générique pour les sources de tensions DC, le cercle incorporant la ligne ondulée est le symbole générique pour toutes les sources de tensions AC.

On pourrait se demander pourquoi quiconque voudrait s'ennuyer avec des choses comme l'AC. Il est vrai que dans certains cas, l'AC n'a pas d'avantages par rapport au DC. Dans les applications où l'électricité est utilisée pour dissiper de l'énergie sous la forme de chaleur, la polarité ou la direction du courant est sans importance, tant qu'il y a suffisament de tension et de courant pour la charge afin de produire la chaleur souhaitée (dissipation de puissance). Avec l'AC, il est est néanmoins possible de construire des générateurs électriques, des moteurs et des systèmes de distribution de puissance qui sont beaucoup plus efficaces que pour le DC et nous trouvons donc de l'AC utilisé majoritairement de part le monde dans les applications à haute puissance. Pour expliquer les détails de l'état des lieux, un peu de culture générale à propos de l'AC est nécessaire.

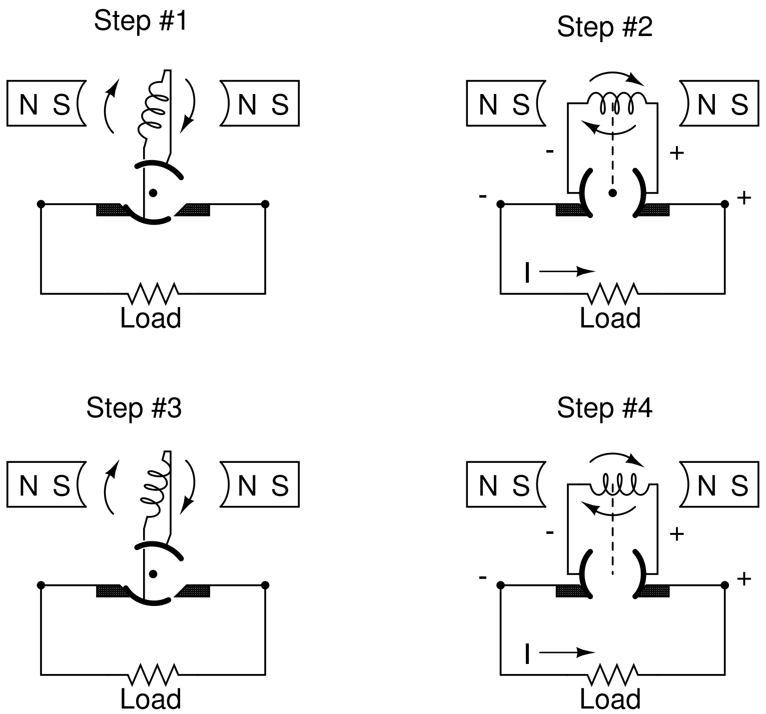
Si une machine est construite pour faire tourner un champ magnétique autour d'un jeu de bobines de fils stationnaires avec la rotation d'un axe, une tension AC sera produite dans les fils de la bobine lorsque l'axe est tourné, en accord avec la Loi de Faraday sur l'induction électromagnetique. C'est le principe opérationnel de base d'un générateur AC, aussi connu comme un *alternateur* :



Notez comme la polarité de la tension dans les bobines de fils s'inversent quand les poles opposés des aimants rotatifs passent dessus. Connectés à une charge, cette inversion de polarité de tension créera une inversion de la polarité du courant dans le circuit. Plus l'arbre de l'alternateur va vite, plus vite tourne les aimants, résultant dans une tension et un courant alternatif qui changent de direction plus souvent en un temps donné.

Alors que les générateurs DC fonctionnent sur le même principe général d'induction électromagnétique, leur construction n'est pas aussi simple que leur homologue AC. Avec un générateur DC, la bobine de fil est montée sur l'arbre alors que c'est l'aimant qui y est sur l'alternateur AC et les connexions électriques se font de ces bobines tournantes par le biais de "balais" carbones stationnaires faisant contact avec des plots de cuivre sur l'axe tournant. Tout ceci est nécessaire pour commuter le changement de la polarité de sortie vers un circuit extérieur de telle manière qu'il voit une polarité constante :

(DC) Generator operation

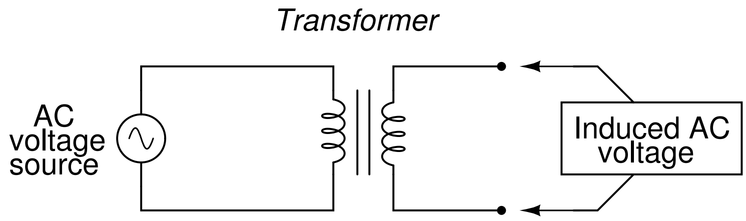


Le générateur montré plus haut produira deux pulsations de tension par révolution de l'arbre, chaque impulsion dans la même direction (polarité). De manière à produire une tension *constante* pour un générateur DC, plutôt que d'avoir des impulsions à chaque 1/2 révolution, il existe plusieurs jeux de bobines réalisant un contact intermittent avec les balais.Le graphique montré ci-dessus est un peu plus simplifié que ce qu'il serait dans les vie réelle.

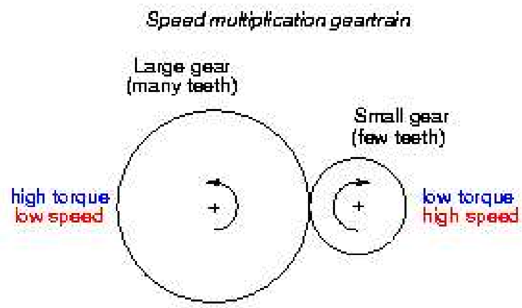
Les problèmes impliqués avec l'amorçage et l'arrêt des contacts électriques avec une bobine en mouvement devraient être évidents (étincelles et échauffements), spécialement si le balais du génrateur se déplace rapidement. Si l'athmosphère entourant la machine contient des vapeurs inflammables ou explosives, les problèmes pratiques de la production d'étincelles par les balais en encore plus importants. Un générateur AC (alternateur) ne nécessite pas de balais et de commutateurs pour fonctionner et est donc immununisé contre ce type de problèmes expérimentés par les générateurs DC.

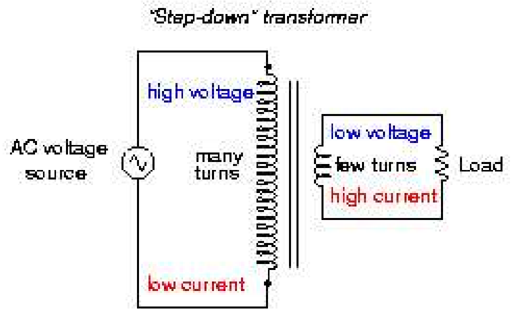
Les bénéfices de l'AC sur le DC avec du point de vue de la conception du générateur sont aussi reflétés dans les moteurs électriques. Alors que les moteurs DC nécessitent l'utilisation de balais pour réaliser un contact électrique avec les bobines de fils mobiles, les moteurs AC n'en ont pas besoin. En fait, les conceptions des moteurs AC et DC sont assez similaires quant à leur partie générateur (identique pour ce tutoriel), le moteur AC étant dépendant du changement de champ magnétique produit en alternant le courant passant par ces bobines de fils stationnaires pour faire tourner le champ magnétique autour de son axe et le moteur DC dépendant des contacts de balais créant et coupant les connexions pour inverser le courant dans les bobines rotatives chaque demi rotation (180 degrés).

Nous savons donc maintenant que les générateurs et les moteurs AC sont plus simples que ceux en DC. Cette simplicité relative se traduit en une plus grande fiabilité et des coûts de fabrication moindres. Mais çà quoi d'autre l'AC est-il bon? Il doit sûrement y avoir plus que des détails de conception de générateurs et de moteurs! C'est effectivement le cas. C'est un effet de l'électromagnétisme connu comme *l'induction mutuelle*, où au moins deux bobines de fils sont placées de telle manière que le changement de champ magnétique créé par l'un induise une tension dans l'autre. Si nous avons deux bobines à inductance mutuelle et que nous énergisions une bobine avec du courant AC, nous créerons une tension AC dans l'autre bobine. Lorsqu'il est utilisé de cette manière, ce composant est connu comme un *transformateur* :

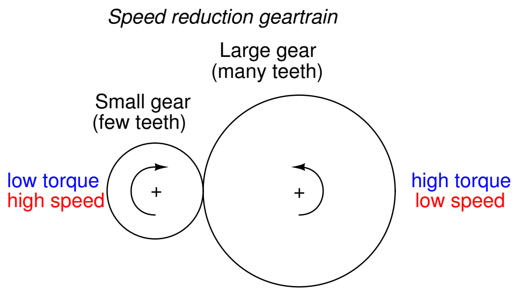


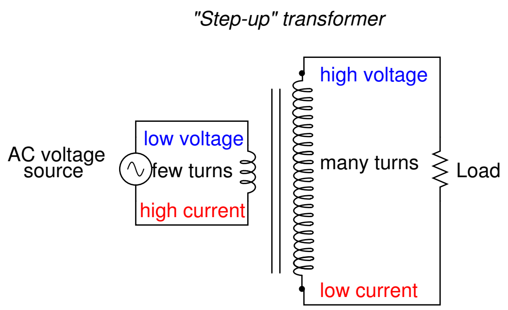
La signification fondamentale d'un transformateur est son apttiude d'augmenter ou de diminuer la tension depuis la bobine alimentée vers l'autre. La tension AC induite dans la bobine non alimentée ("secondaire") est égale à la tension AC aux bornes de la bobine alimentée ("primaire") multipliée par le ratio des tours de la bobine secondaire sur ceux de la primaire. Si la bobine secondaire alimente une charge, le courant dans la bobine secondaire est juste l'opposé : le courant de la bobine primaire est multiplé par le ratio des tours du primaire sur le secondaire. Cette relation est très proche de l'analogie mécanique, en utilisant le couple et la vitesse pour représenter la tension et le courant, respectivement :



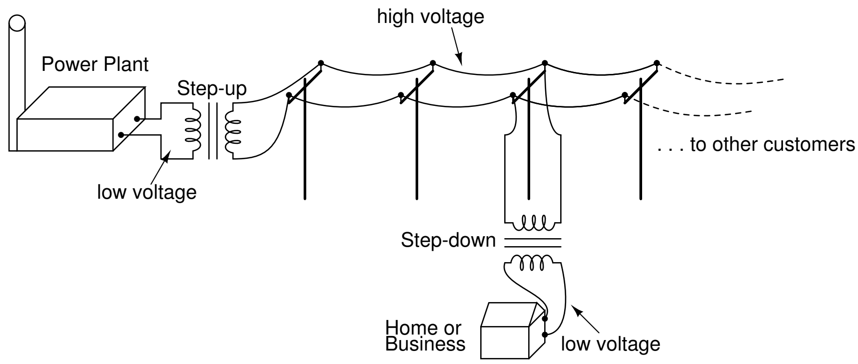


Si le ratio de bobinage est inversé, de telle manière que la bobine primaire a moins de tours que la bobine secondaire, le transformateur "monte" la tension depuis le niveau de la source vers un plus haut niveau sur la charge :





L'aptitude du transformateur à augmenter ou descendre la tension AC facilement lui donne un avantage inatteignable au DC dans le royaume de la distribution de puissance. Lors de la transmission de puissance électrique sur de longues distances, il est beaucoup plus efficace de le faire avec des tensions augmentées et des courants réduits (des fils avec un diamètre plus petits avec des pertes de puissances réduites), puis abaisser la tension et augmenter le courant pour l'utilisation dans l'industrie, le business ou chez les consommateurs.



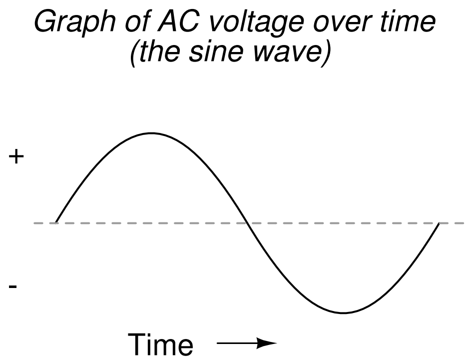
La technologie des transformateurs est issue d'une longue pratique de distribution de la puissance électrique. Sans la possibilité d'augmenter ou de diminuer la tension, il serait hors de prix de construire des systèmes de puissance polyvalents mais proches (dans un rayon de quelques kilomètres au mieux).

Aussi utiles que soient les transformateurs, ils ne fonctionnent qu'avec de l'AC, pas le DC. Comme le phénomène d'inductance mutuelle repose sur le *changement* des champs magnétiques et que le courant continu (DC) peut seulement produire des champs magnétiques statiques, les transformateurs ne fonctionnent tout simplement pas avec le courant continu. Bien sûr, le courant continu peut être interrompu (pulsé) dans le bobinage primaire d'un transformateur pour créer un champ magnétique variable (comme cela est réalisé dans les systèmes d'allumage automobile pour produire de la haute tension afin d'alimenter les bougies depuis une batterie DC basse tension) mais le DC pulsé n'est pas si différent de l'AC. Peut être bien plus que toute autre raison, c'est pourquoi l'AC a trouvé des applications étendues dans les systèmes de puissance.

- **RÉSUMÉ :**
- DC signifie "Direct Current ou Courant Continu" signifiant que la tension ou le courant garde une polarité ou une direction constante, respectivement, au cours du temps.
- AC signifie "Alternating Current ou Courant Alternatif" signifiant que la tension ou le courant change de polarité ou de direction, respectivement, au cours du temps.
- Les générateurs électromécaniques AC, aussi connus comme *alternateurs*, sont plus simples de fabrication que les générateurs électromécaniques DC.
- Les conceptions de moteurs AC et DC suivent des principes de création respectifs très semblables.
- Un *transformateur* est une paire de bobines induites mutuellement, utilisées pour transporter de la puissance AC d'une bobine vers une autre. Le nombre de tours dans chaque bobine est destiné à créer une augmentation ou une diminution de tension depuis la bobine alimentée (primaire) vers la seconde bobine (secondaire).
- Tension secondaire = Tension primaire (tours secondaires / tours primaires)
- Courant secondaire = Courant primaire (tours primaires / tours secondaires)

1.2 Ondes AC

Lorsqu'un alternateur produit une tension AC, la tension change de polarité au cours du temps mais il le fait d'une manière très particulière. Lorsqu'elle est tracé au fil du temps, "l'onde" de cette tension à la polarité alternative, provenant d'un alternateur prend une forme distinctive, connue comme une *onde sinus* :



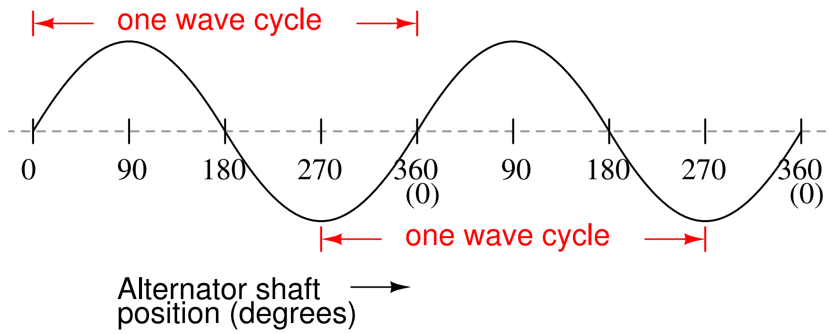
Avec le tracé de tension d'un alternateur électromécanique, le changement d'une polarité vers l'autre est atténué, le niveau de tension changeant plus rapidement au point zéro ("crossover") et plus lentement à son sommet. Si nous devions tracer le graphe de la fonction trigonométrique de la fonction "sinus" sur un éventail horizontal de 0 à 360 degrés, nous trouverions exactement le même motif :

```
Angle            Sinus(angle)
en degrés
0   ..............  0.0000  -- zéro
15  ..............  0.2588
30  ..............  0.5000
45  ..............  0.7071
60  ..............  0.8660
75  ..............  0.9659
90  ..............  1.0000  -- pic positif
105 ..............  0.9659
120 ..............  0.8660
135 ..............  0.7071
150 ..............  0.5000
165 ..............  0.2588
180 ..............  0.0000  -- zéro
195 .............. -0.2588
210 .............. -0.5000
225 .............. -0.7071
240 .............. -0.8660
255 .............. -0.9659
270 .............. -1.0000  -- pic négatif
285 .............. -0.9659
```

```
300 .............. -0.8660
315 .............. -0.7071
330 .............. -0.5000
345 .............. -0.2588
360 ..............  0.0000  -- zéro
```

C'est à cause de son organisation physique qu'un alternateur électromécanique sort une onde sinus AC. La tension produite par les bobines stationnaires en fonction du déplacement rotatif des aimants est proportionnel au taux auquel le flux magnétique y change perpenduculairement (Loi de Faraday sur l'induction Électromagnétique). Ce taux est plus grand lorsque les poles magnétiques sont proches des bobines et moindre lorsqu'il s'en éloignent. Mathématiquement, le taux de flux magnétique change à cause des aimants en suivant une fonction sinus, la tension produit par les bobines suit donc cette même fonction.

Si nous devions suivre le changement de tension produite par une bobine dans un alternateur, sur un graphique, depuis tout point de l'onde sinus jusqu'au point où la forme de l'onde se répète, nous aurions alors parcouru un *cycle* de cette onde. Ceci est plus facilement montré par l'intervale de distance entre des pics identiques mais peut être mesure entre deux point correspondants quelconques du graphique. La marque de degré sur l'axe horizontal du graphique représente le domaine de la fonction trigonométrique sinus et aussi la position angulaire de notre alternateur à axe à deux pôles lors de sa rotation :



Comme l'axe horizontal de ce grapique peut autant montrer l'évolution au cours du temps que la position de l'axe en degrés, la dimension indiquée pour un cycle est souvent mesurée par unité de temps, le plus souvent en secondes ou fractions de secondes. Lorque c'est exprimé en mesure, c'est souvent appelé la *période* d'une onde. La période d'une onde en degrés est *toujours* 360 mais la somme de temps qu'une période occupe dépend du taux des oscillations de tension avant et arrière.

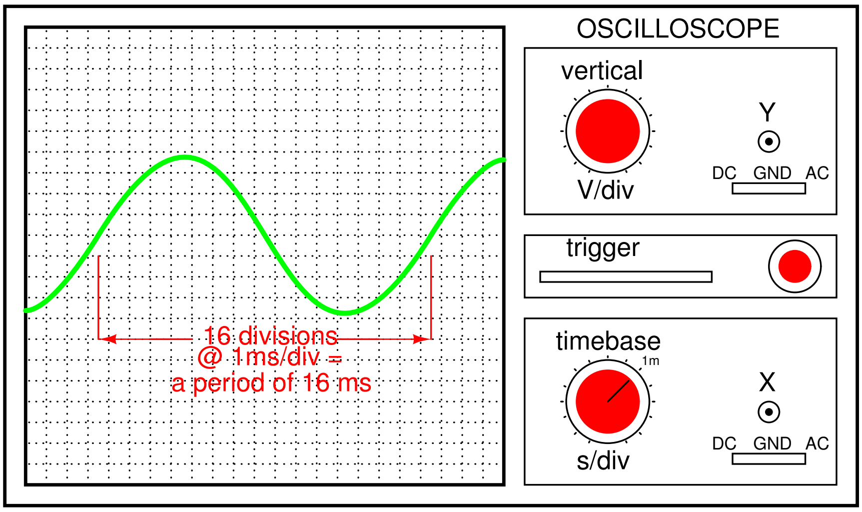
Un mesure plus populaire que la description de la vélocité d'une onde de tension ou de courant AC par la *période* est la taux d'oscillation avant-arrière. C'est appelé la *fréquence*. L'unité moderne pour la fréquence est le Hertz (abbrévié en Hz), qui représente le nombre de cycles d'ondes complétés pendant une seconde. Aux États-Unis, la fréquence des lignes de puissance est de 60 Hz, ce qui signifie que la tension AC oscille à une vitesses de 60 cycles d'allers-retours complets chaque seconde. En Europe, où la fréquence du secteur est à 50 Hz, la tension AC n'effectue que 50 cycles chaque secondes. Un transmetteur de station radio diffuse à une fréquence de 100 MHz, générant une tension AC oscillant à une vitesse de 100 *million* de cycles par seconde.

Avant la normalisation de l'unité des Hertz, la fréquence était exprimée simplement comme des "cycles par seconde". Les vieux multimètres et équipements électroniques portent souvent les unités de fréquence "CPS" (Cycles Par Seconde) au lieu de Hz. Plusieurs personnes croient que le changement depuis des unités qui s'expliquent d'elles-même comme les CPS vers les Hertz constituent un retour en arrière de la clarté. Un changement similaire s'est produit lorsque l'unité des "Celsius" a remplacé celle des "Centigrades" pour une mesure de température métrique. Le nom de Centigrade était basé sur une échelle ("-grade") à 100 pas ("Centi-") représentant les points de fusion et d'ébulition de H_2O, respectivement. Le nom Celsius, d'un autre coté, ne donne aucun élément sur l'origine de l'unité ou sa signification.

La période et la fréquence sont les réciproques mathématiques l'une de l'autre. Cela signifie que si une onde a une période de 10 secondes, sa fréquencesera de 0.1 Hz ou 1/10 de cycle par seconde :

$$\text{Frequency in Hertz} = \frac{1}{\text{Period in seconds}}$$

Un instrument appelé un *oscilloscope* est utilisé pour afficher un changement de tension au cours du temps sur un écran. Il est possible que vous soyez familier avec l'apparence d'un *ECG* ou *EKG* (électrocardiographe), utilisé par les physiciens pour tracer les oscillations d'un coeur de patient au cours du temps. L'ECG est un oscilloscope spécialisé, expressement conçu pour l'usage médical. Les oscilloscopes ont la possibilité d'afficher des tensions depuis virtuellement toute source de tension, tracé comme un graphe avec le temps comme variable indépendante. La relation entre la période et la fréquence est très utile pour savoir lorsqu'il y a un affichage en tension AC ou en courant sur un écran d'oscilloscope. En mesurant la période de l'onde sur l'axe horizontal de l'écran d'oscilloscope et en calculant la réciproque de cette valeur de temps (en secondes), vous pouvez déterminer la fréquence en Hertz.



$$\text{Frequency} = \frac{1}{\text{period}} = \frac{1}{16\text{ ms}} = 62.5\text{ Hz}$$

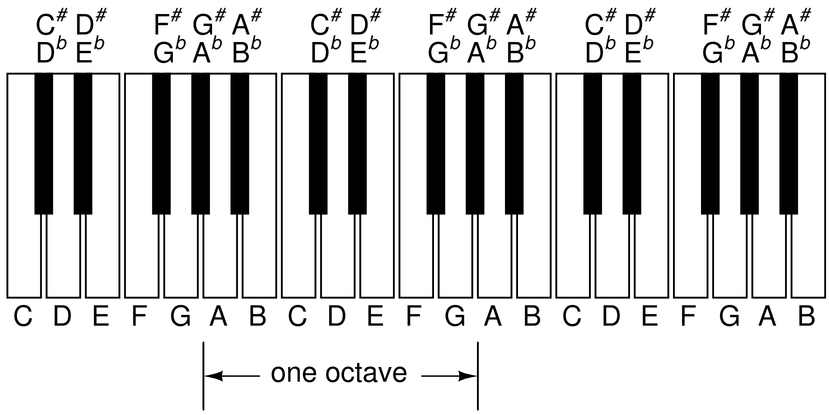
La tension et le courant sont les seules variables physiques sujettes à des variations au cours du temps. Beaucoup plus proche de notre expérience quotidienne, le *son* n'est rien de plus que l'alternance de la compression et de la décompression (ondes de pression) des molécules d'air, interprétée par nos oreilles comme sensation physique. Comme le courant alternatif est un phénomène d'onde, il partage beaucoup des propriétés des autres phénomènes d'ondes, comme le son. Pour cette raison, le son (spécialement la musique structurée) fournit une analogie excellente pour en liaison avec les concepts AC.

En termes musicaux, la fréquence est équivalente au *pitch*. Les notes low-pitch tels que celles produites par un tuba ou basson consistent en des vibrations de molécules d'air qui sont relativement lentes (basse fréquence). Les notes high-pitch telles que celles produites par une flute ou un sifflet consistent au même type de vibrations dans l'air, vibrant seulement à une vitesse beaucoup plus rapide (haute fréquence). Vous avez ici une table montrant les fréquences réelles du champ des notes de musiques habituelles :

Note	**Musical designation**	**Frequency (in hertz)**
A	A_1	220.00
A sharp (or B flat)	$A^{\#}$ or B^{b}	233.08
B	B_1	246.94
C (middle)	C	261.63
C sharp (or D flat)	$C^{\#}$ or D^{b}	277.18
D	D	293.66
D sharp (or E flat)	$D^{\#}$ or E^{b}	311.13
E	E	329.63
F	F	349.23
F sharp (or G flat)	$F^{\#}$ or G^{b}	369.99
G	G	392.00
G sharp (or A flat)	$G^{\#}$ or A^{b}	415.30
A	A	440.00
A sharp (or B flat)	$A^{\#}$ or B^{b}	466.16
B	B	493.88
C	C^1	523.25

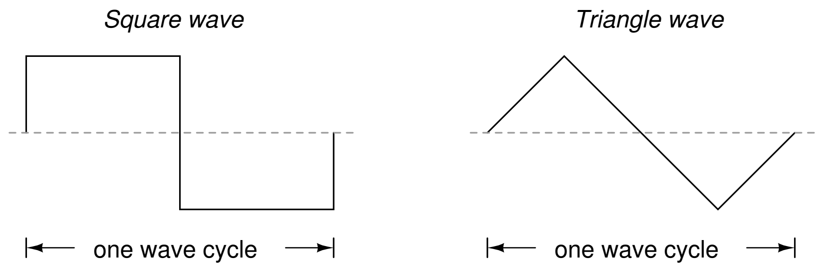
Les observateurs malins noteront que toutes les notes sur la table portant les même désignations de lettre sont liées par un ration de fréquence de 2 :1. Par exemple, la première fréquence montrée (désignée avec la lettre "A") est de 220 Hz. La note suivante "A" un peu plus haute possède une fréquence de 440 Hz – exactement le double de cycles d'ondes de son par seconde. Le même ratio 2 :1 reste vrai pour la première A sharp (233.08 Hz) et la A sharp suivante (466.16 Hz) et pour toute paire de note trouvée dans la table.

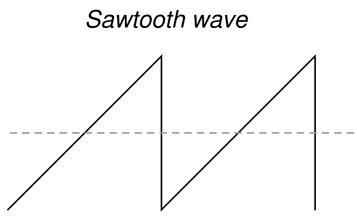
Auditivement, deux notes dont les fréquences sont exactement le double l'une de l'autre semblent remarquablement similaires. Cette similarité du son est musicallement reconnue, l'étendue la plus courte d'une échelle musicale séparant de telles paires de notes est appelée une *octave*. En suivant cette règle, la note suivante "A" plus haute (une octave au-dessus de 440 Hz) sera 880 Hz, la "A" suivante plus basse (une octave en-dessous de 220 Hz) sera 110 Hz. Une vue d'un clavier de piano aide à mettre cette échelle en perspective :



Comme vous pouvez le voir, une octave est égale à une distance de *huit* touches blanches sur le clavier d'un piano. Le mnémonique musical habituel (doe-ray-mee-fah-so-lah-tee-doe) – oui, le même motif immortalisé dans la chanson de whimsical Rodgers et Hammerstein chanté dans The Sound of Music – couvre une octave de C à C.

Alors que les alternateurs électromécaniques et plusieurs autres phénomènes physques produisent naturellement des ondes sinus, ce n'est pas le seul type d'ondes alternées dans l'existance. D'autres "ondes" AC sont habituellement produites dans des circuits électroniques. Vous avez ici quelques exemples de formes d'ondes et leur désignation habituelle :





Ces formes d'ondes ne sont aucunement les seuls types de forme d'onde dans l'existence. Elles font simplement partie des quelque unes qui sont suffisamment communes pour obtenir des noms distincs. Même dans les circuits qui sont supposés émettre des formes d'onde en tension/courant sinus, carrés, triangles ou dent-de-scie "purs", le résultat dans la vie réelle est souvent une version distordue des formes d'ondes désirées. Quelques formes d'ondes sont si complexes qu'elles défient les classifications comme "type" particulier (incluant les formes d'ondes associées avec plusieurs types d'instruments de musique). En généralisant, toute forme d'onde présentant une ressemblance avec une onde sinus parfaite est appelée *sinusoïdale*, alors que tout ce qui est différent est considéré comme *non-sinusoïdal*. Étant donné que la forme d'onde d'une tension ou un courant AC est cruciale pour son impact dans un circuit, nous avons besoin de faire attention au fait que les ondes AC possèdent plusieur types de formes.

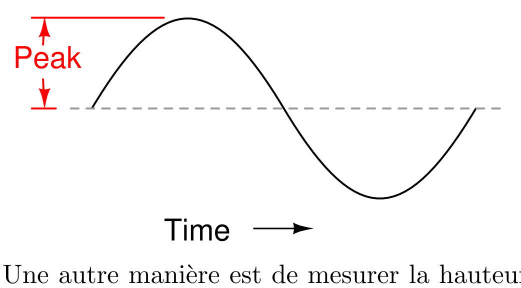
- **RÉSUMÉ :**
- Les ondes AC produites par un alternateur électromécanique suivent la forme graphique d'une onde sinus.
- Un *cycle* d'onde est évolution complète de sa forme jusqu'au point où elle est prête à se reproduire.
- La *période* d'une onde est le temps qu'il faut pour compléter un cycle.
- La *fréquence* est le nombre de cycles complets qu'une onde complète pendant une période de temps. Habituellement mesuré en Hertz (Hz), 1 Hz étant égal à un cycle d'onde complet par seconde.
- Fréquence = 1/(période en secondes)

1.3 Mesures de l'amplitude AC

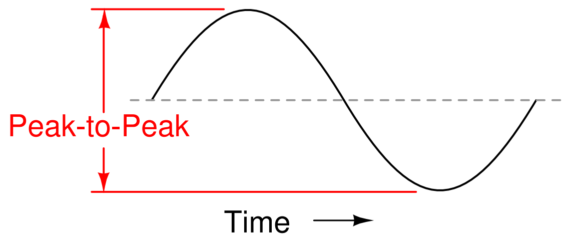
De ce qu'on en sait, la tension AC alternent en polarité et le courant AC alternent en direction. Nous savons aussi que l'AC peut alterner de diverses manières et en traçant l'alternance au cours du temps, on peut la tracer comme une "forme d'onde". Nous pouvons mesurer la vitesse des alternances en mesurant le temps qu'il faut pour faire évoluer une onde avant qu'elle ne se reproduise (la "période") et l'exprimer comme cycles par unité de temps ou "fréquence". En musique, la fréquence est la même qu'un *pitch*, qui est la propriété essentielle distinguant une note d'une autre.

Nous rencontrons néanmoins un problème de mesure si nous tentons d'exprimer quelle est l'amplitude d'un quantité AC. Avec le DC, où les quantités de tension et de courant sont généralement stables, nous avons peu de problèmes pour trouver combien de tension ou de courant nous avons dans toutes les zones du circuit. Mais comment pouvons-nous garantir une simple mesure de l'amplitude de quelque chose qui change constamment?

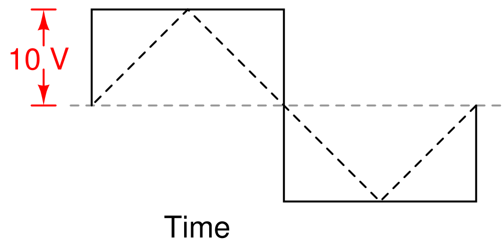
Une manière pour exprimer l'intensité ou la magnitude (aussi appelé l'*amplitude*) d'une quantité AC est de mesurer sa hauteur de pic dans le graphe d'une forme d'onde. C'est connu comme une valeur *pic* ou *crête* d'une forme d'onde AC :

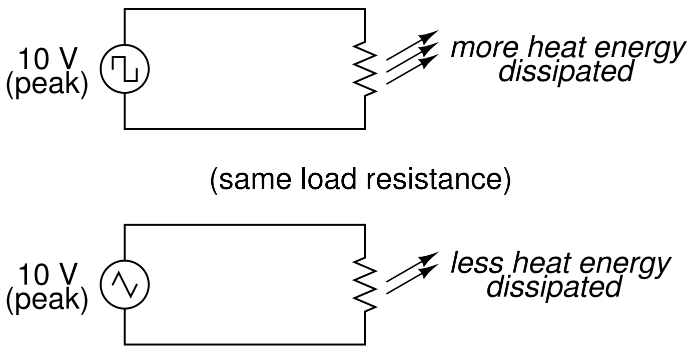


Une autre manière est de mesurer la hauteur totale entre les pics opposés. C'est connu comme la valeur *pic-à-pic* (P-P) d'une forme d'onde AC :

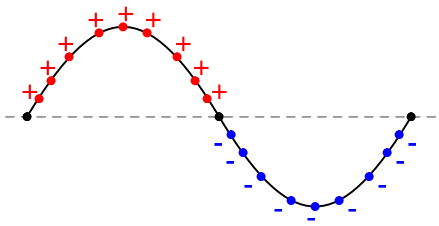


Malheureusement, l'expressions de forme d'onde peut amener des malentendus qui peuvent comparer deux types d'ondes différents. Par exemple, une onde carrée avec un pic à 10 volts possède une valeur de tension plus grande par unité de temps qu'une onde triangle avec un pic à 10 volts. L'effet de ces deux tensions AC alimentant une charge sera assez différent :



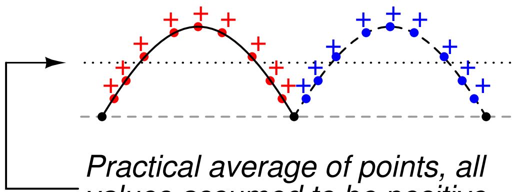


Une manière d'exprimer l'amplitude des différentes formes d'ondes d'une manière plus équivalente est de moyenner mathématiquement les valeurs de tous les points du graphe de la forme d'onde en un seul nombre global. Cette mesure d'amplitudeest simplement connue comme la valeur *moyenne* de la forme d'onde. Si nous moyennons tous les points sur la forme d'onde algébriquement (ceci étant, en considérant leur *signe*, soit positif, soit négatif), la valeur moyenne pour la plupart des formes d'ondes est techniquement zéro car tous les points positifs annulent tous les points négatifs lors d'un cycle complet :



True average value of all points (considering their signs) is ***zero****!*

Ceci, bien sûr, sera vrai pour toutes les formes d'ondes ayant une proportion égale de zones au-dessous et au-dessus de la ligne "zéro" du tracé. Néanmois, comme une mesure *pratique* de la forme d'onde accumule la valeur, la "moyenne" est habituellement définie comme une moyenne mathématique de tous les points en *valeur absolue* lors d'un cycle. En d'autres mots, nous calculons la valeur moyenne pratique de la forme d'onde en considérant tous les points de l'onde comme des quantités positives, comme si la forme d'onde ressemblait à ceci :

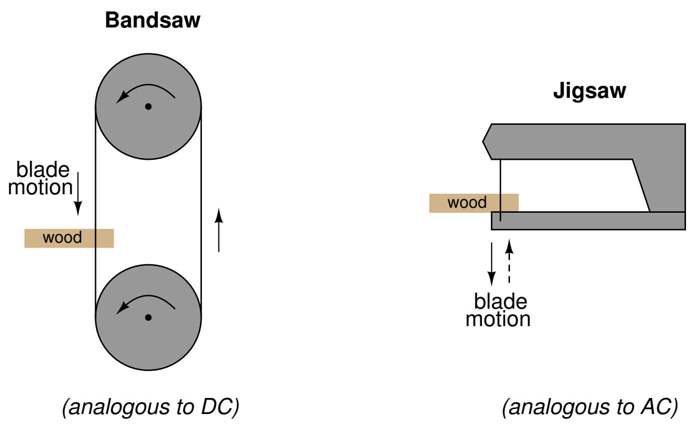


Practical average of points, all values assumed to be positive.

Les mouvements de l'indicateur mécanique insensible à la polarité (l'indicateur doit être conçu pour permettre une réponse égale aux demi-cycles positifs et négatifs d'une tension ou d'un courant alternatif) s'enregistrent en proportion de la valeur moyenne de la forme d'onde (pratique) car l'inertie de l'aiguille par rapport à la tension du ressort moyenne naturellement la force produite par la variation de tension/courant au cours du temps. Inversement, les mouvements de l'indicateur mécanique sensible à la polarité vibrent inutilement si exposés à une tension ou un courant AC, leurs aiguilles oscillent rapidement autour du zéro, indiquant la vraie valeur (algébrique) moyenne du zéro pour la forme d'onde symétrique. Lorsque la valeur "moyenne" d'une forme d'onde est indiquée dans ce texte, il sera supposé que la définition "pratique" de la moyenne sera utilisée à moins que cela ne soit précisé.

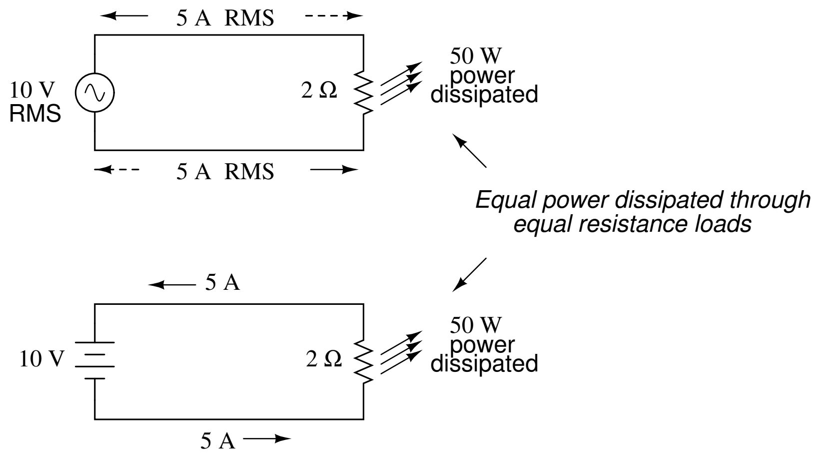
Une autre méthode dérivée pour avoir une valeur globale de l'amplitude d'une forme d'onde est basée sur l'aptitude de la forme d'onde à effectuer un travail utile lorsqu'elle est appliquée à une résistance de charge. Malheureusement, une mesure AC basée sur le travail effectué par une forme d'onde n'est pas la même que la valeur "moyenne" de cette forme d'onde car la *puissance* dissipée par une charge donnée (travail effectué par unité de temps) n'est pas directement proportionnel à la magnitude de la tension ou le courant qui y passe. La puissance est plutôt proportionnelle au *carré* de la tension ou du courant appliqué à une résistance ($P = E^2/R$ et $P = I^2R$). Bien que les mathématiques d'une telle mesure d'amplitude puissent ne pas être directes, leur utilité l'est.

Considérons une scie à bande et une scie sauteuse, deux outils d'un équipement moderne pour le travail du bois. Les deux types de scies découpent le bois avec une fine lame de métal en dent de scie motorisée. Mais alors que la scie à bande utilise un déplacement continu de la lame pour couper, la scie sauteuse utilise un déplacement de va-et-vient. La comparaison des courants alternatifs (AC) avec le courant direct (DC) peut être approché de la comparaison de ces deux types de scie :



Le problème pour tenter de décrire les changements de tension ou de courant AC en une seule mesure globale est aussi présent dans l'analogie des dents : comment pouvons-nous exprimer la vitesse d'une lame à scie sauteuse? Une lame à bande se déplace avec une vitesse constante, identique à la manière qu'une tension DC est fournie et que le courant DC se déplace avec une amplitude constante. Une lame de scie sauteuse, d'un autre côté, se déplace de haut en bas, avec une vitesse en constant changement. En plus, le déplacement de haut en bas de deux scies sauteuses peut ne pas être du même type, en fonction du dessin mécanique des dents. Une scie sauteuse peut déplacer sa lame avec un déplacement en forme d'onde sinus, alors qu'une autre peut avoir un déplacement avec une onde en triangle. Évaluer la vitesse d'une lame de scie sauteuse basée sur les *pics* serait trompeur lors de la comparaison d'une scie sauteuse avec une autre (ou d'une scie sauteuse avec une scie à bande!). Malgré le fait que ces scies déplacent leurs lames de manière différente, elles sont égales à certains égards : elles coupent toutes le bois et une comparaison quantitative de cette fonction commune peut servir de base commune pour évaluer la vitesse de la lame.

En dessinant une scie à bande et scie sauteuse l'une à côté de l'autre, équipées avec des lames identiques (même écartement de dents, angle, etc.), capable également de couper la même épaisseur du même type de boisà la même vitesse. Nous pourrions dire ques les deux scies sont équivalentes ou égales dans leur capacité de découpe. Cette comparaison peut-elle être utilisée pour assigner une vitesse de lame "équivalente d'une scie à bande" au déplacement en va-et-vient d'une lame de scie sauteuse; pour relier l'efficacité d'une découpe de bois de l'une par rapport à l'autre? C'est l'idée générale utilisée pour assigner une mesure "équivalente DC" à toute tension ou courant AC : quelque soit l'amplitude de la tension ou du courant DC, il y aura production de la même dissipation d'énergie thermique au travers de résistances égales :



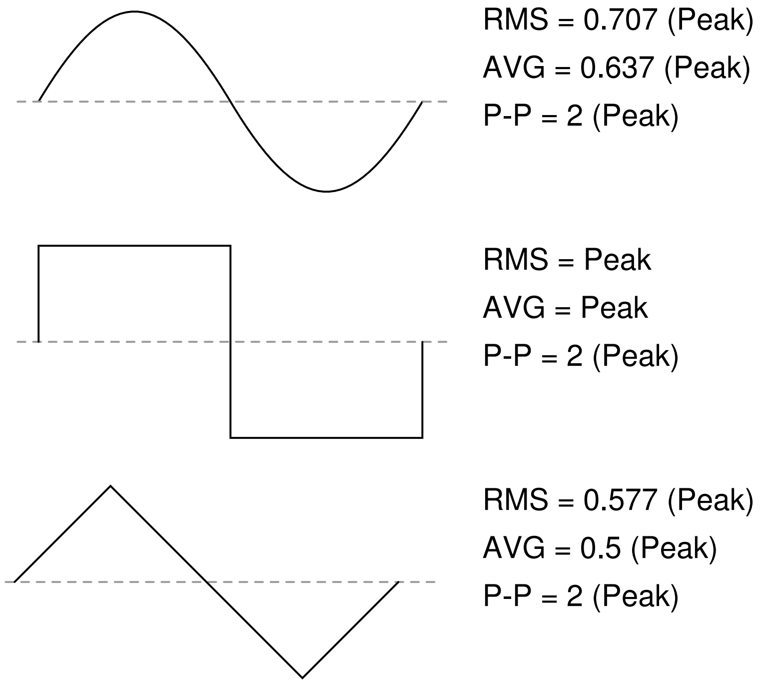
Dans les deux circuits ci-dessus, nous avons la même résistance de charge (2 Ω) dissipant le même niveau de puissance sous la forme de chaleur (50 watts), l'une alimentée en AC et l'autre en DC. Comme la source de tension AC dessinée ci-dessus est équivalente (en terme de puissance délivrée à une charge) à une batterie DC de 10 volts, nous l'appelerons une source AC de "10 volt". Plus spécifiquement, nous allons noter sa valeur de tension comme étant à 10 volts *RMS*. Le qualificatif "RMS" signifie *Root Mean Square*, l'algorithme utilisé pour obtenir la valeur DC équivalente depuis les points d'un graphe (la procédure consiste essentiellement à prendre le carré des points positifs et négatifs sur le graphe d'une forme d'onde, en moyennant ces valeurs élévées au carré puis en prennant la racine carrée de la moyenne pour obtenir la réponse finale). Quelques fois, des termes alternatifs *équivalent* ou *équivalent DC* sont utilisés au lieu de "RMS" mais les valeurs et le principe sont les mêmes.

La mesure de l'amplitude RMS est la meilleure manière de relier les valeurs AC à celles en DC ou d'autres valeurs AC de formes d'ondes différentes, lors du traitement avec des mesures de puissances électriques. Pour les autres considérations, les mesures pic ou pic à pic peuvent être peuvent être les meilleures à employer. Par exemple, lorsqu'il faut déterminer la bonne taille d'un fil (ampérage) pour conduire une puissance électrique depuis une source vers une charge, la mesure du courant RMS est la plus adaptée car l'intérêt principal avec le courant est la surchauffe du fil, qui est en fonction de la dissipation de puissance créée par le courant dans la résistance du fil. Néanmois, lorsqu'il faut déterminer l'isolation nécessaire dans des applications AC à haute tension, la valeur pic de la tension est plus appropriée car l'intérêt principal, ici, est le perçage d'isolateurs "(flashover)" causé par de brefs pics de tension, indépendamment du temps.

Les mesures pic et pic à pic sont facilement faites avec un oscilloscope qui peut capturer les crêtes des formes d'ondes avec un haut niveau de précision grâce au tube qui réagit très rapidement aux changements de tension. Pour les mesures RMS, les mesures analogiques de mouvements (D'Arsonval, Weston, iron vane, mesure électrodynamique) fonctionneront s'ils ont été calibrés pour le RMS. Comme l'inertie mécanique et les effets d'amortissement de la mesure électromécanique de mouvements rend la déviation de l'aiguille naturellement proportionelle à la valeur *moyenne* de l'AC, pas la vraie valeur RMS, les mesures analogiques doivent être spécifiquement calibrés (ou mal-calibré, selon la manière dont vous le calibrez) pour indiquer la tension ou le courant en unités RMS. La précision de cette calibration dépend d'une forme d'onde supposée, habituellement une onde sinus.

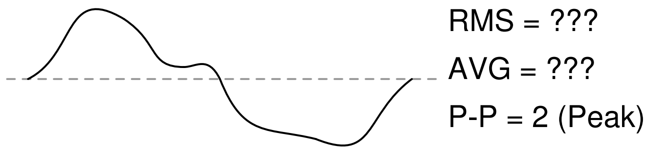
Les instruments électroniques spécifiquement conçus pour la mesure RMS sont les plus adaptés pour cette tâche. Quelques fabricants d'instruments ont conçu des méthodes ingénieuses pour déterminer la valeur RMS de toute forme d'onde. Un de ces fabricants produit des instruments "RMS-vrai" avec une petite résistance de chauffe alimentée par une tension proportionelle à ce qui est mesuré. L'effet de chauffe de cet élément de résistance est mesuré thermiquement pour donner une vrai valeur RMS sans calculs mathématiques, simplement les lois de la physique en action en accord parfait dans la définition du RMS. La précision de ce type de mesure RMS est indépendante de la forme de l'onde.

Pour une forme d'onde "pure", de simples coefficients de conversion existent pour comparer les mesures pic, pic à pic, moyenne (pratique, pas algébrique) et RMS les unes avec les autres :



En plus des mesures RMS, moyennes, pic (crête) et pic à pic d'une forme d'onde AC, il existe des ratios exprimant la proportionalité entre certaines de ces mesures fondamentales. Le *facteur de crête* d'une forme d'onde AC, par exemple, est le ratio entre cette valeur pic (crête) divisée par sa valeur RMS. Le *facteur de forme* d'une forme d'onde AC est le ratio de sa valeur pic divisé par sa valeur moyenne. Les formes d'ondes carrées ont toujours des facteurs de forme et de crête égaux à 1, comme la valeur pic est la même que les valeurs RMS et moyennes. Les formes d'ondes sinusoïdales ont des facteurs de crête de 1.414 (la racine carrée de 2) et des facteurs de forme de 1.571 ($\pi/2$). Les formes d'ondes triangle et dent de scie ont des valeurs de crête de 1.732 (la racine carrée de 3) et des facteurs de forme de 2.

Gardez en mémoire que les constantes de conversion montrées ici pour les amplitudes d'ondes sinus, triangle et carrée, pic, RMS et moyennes restent vraies seulement pour des formes *pure* de ces formes d'ondes. Les valeurs RMS et moyennes de formes d'ondes distordues ne sont pas reliées par les même ratios :



C'est un concept très intéressant à comprendre lors de l'utilisation des mouvements d'un vu-mètre analogique pour mesurer la tension ou le courant AC. Un vu-mètre analogique, calibré pour indiquer une amplitutde sinus RMS, ne sera précis que pour la mesure d'ondes sinus pures. Si la forme d'onde de la tension ou du courant mesuré est autre chose qu'une onde sinus pure, l'indication donnée par par le vu-mètre ne sera pas une valeur RMS vraie de la forme d'onde car le degré de la variation de l'aiguille d'un vu-mètre analogique est proportionnel à la valeur *moyenne* de la forme d'onde, pas la RMS. La calibration pour le RMS est obtenue en "biaisant" le débatement du vu-mètre afin qu'il affiche un petit multiple de la valeur moyenne, qui sera égale à la valeur RMS pour une forme d'onde particulièren et *une forme d'onde particulière seulement*.

Comme la forme de l'onde sinus est la plus courante dans les mesures électriques, c'est la forme d'onde qui est supposée pour la calibration des vu-mètres analogiques et le petit multiple utilisé dans la calibration du vu-mètre est 1.1107 (le facteur de forme $\pi/2$ divisé par le facteur de crête 1.414 : le ratio du RMS divisé par ma moyenne pour une forme d'onde sinusoïdale). Toute forme d'onde autre qu'une pure onde sinus aura un ratio différent des valeurs RMS et moyenne et un vu-mètre calibré pour les tensions et courants à ondes sinus n'indiquera pas le RMS-vrai lors de la lecture d'une onde non-sinusoïdale. Gardez en mémoire que cette limitation s'applique seulement aux vu-mètres analogiques simples qui n'emploient pas la technologie "RMS-vrai".

- **RÉSUMÉ :**

- *L'amplitude* d'une forme d'onde AC est son sommet comme décrit sur un graphe au cours du temps. Une mesure d'amplitude peut prendre la forme de quantités pic, pic à pic, moyenne ou RMS.

- L'amplitude *pic* est le sommet d'une forme d'onde AC, mesurée depuis le zéro jusqu'au point du graphe positif le plus haut ou négatif le plus bas. C'est aussi connu comme l'amplitude *crête* d'une onde.

- L'amplitude *pic à pic* est la hauteur totale d'une forme d'onde AC mesurée depuis le pic maximum du positif au pic maximum négatif du tracé. C'est souvent abbrévié en "P-P".

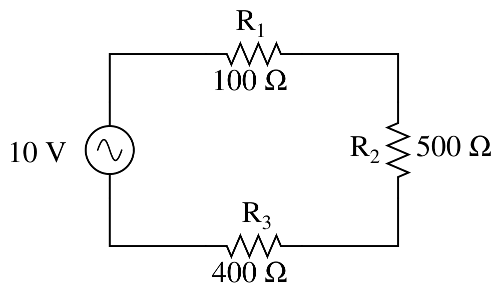
- L'amplitude *moyenne* est la "moyenne" mathématique de tous les points de la forme d'onde lors d'un cycle. Techniquement, l'amplitude moyenne de toute forme d'onde avec des portions égales au-dessus et au-dessous de la ligne du "zéro" sur un tracé est zéro. Néanmoins, une mesure pratique de l'amplitude, une valeur moyenne de la forme d'onde est souvent calculée comme la moyenne mathématique de tous les points en *valeur absolue* (en prenant toutes les valeurs négatives et en les considérant comme positives). Pour une onde sinus, la valeur moyenne calculée est approximativement 0.637 de sa valeur pic.

- "RMS" signifie *Root Mean Square* et est une manière d'exprimer une valeur de tension ou de courant AC en termes fonctionnels équivalents au DC. Par exemple, 10 volts AC RMS est la valeur de tension qui produira la même valeur de dissipation de chaleur au travers d'une résistance d'une valeur donnée qu'une alimentation de 10 volt DC. C'est aussi connu comme la valeur "équivalente" ou "équivalente DC" d'une tension ou courant AC. Pour une onde sinus, la valeur RMS est approximativement 0.707 de sa valeur pic.

- Le *facteur de crête* d'une forme d'onde AC est le ratio de son pic (crête) sur sa valeur RMS.
- Le *facteur de forme* d'une forme d'onde AC est le ratio de sa valeur pic (crête) sur sa valeur moyenne.
- Les mouvements d'un vu-mètre électromécanique analogique répondent proportionellement à la valeur *moyenne* d'une tension ou d'un courant AC. Lorsque l'indication RMS est souhaitée, la calibration du vu-mètre doit être "biaisée" en conséquence. Ceci signifie que la précision de l'indication d'un vu-mètre RMS électromécanique dépend de la pureté de la forme de l'onde : si c'est exactement la même forme d'onde que celle utilisée pour la calibration.

1.4 Calculs de circuits AC simples

Au cours des quelques chapitres, vous apprendrez que la mesure de cuircuit AC et les calculs peuvent être très compliqués à cause de la nature complexe des courants alternatifs dans des circuits avec inductances et capacitances. Néanmoins, avec de simples circuits n'impliquant pas plus qu'une source d'alimentation AC et des résistances, les même lois et règles du DC s'appliquent simplement et directement.



$R_{total}=R_1 + R_2 + R_3$
$R_{total}=1k\Omega$

$$I_{total} = \frac{E_{total}}{R_{total}}$$

$$I_{total} = \frac{10\ V}{1\ k\Omega}$$

$$I_{total} = 10\ mA$$

$E_{R1}=I_{total}R_1$ ——— $E_{R2}=I_{total}R_2$ ——— $E_{R3}=I_{total}R_3$
$E_{R1}=1V$ ——— $E_{R2}=5V$ ——— $E_{R3}=4V$

Les résistances en série continuent de s'ajouter, celle en parallèle de se diviser et les Lois de Kirchhoff et d'Ohm restent vraies. Comme nous allons le découvrir plus loin, ces règles et lois restent réellement *toujours* vraies, c'est simplement que nous devons exprimer les quantités de tension, courant et d'opposition au courant sous des formes mathématiques plus avancées. Avec des circuits purement résistifs, néanmoins, cette complexité de l'AC n'a pas de conséquence pratique et nous pouvons donc traiter les nombres comme si vous avions seulement des quantités DC.

Comme toutes ces relations mathématiques restent encore vraies, nous pouvons utiliser notre méthode de "table" familière pour l'organisation de valeurs de circuits tel qu'avec le DC :

	R_1	R_2	R_3	Total	
E	1	5	4	10	Volts
I	10m	10m	10m	10m	Amps
R	100	500	400	1k	Ohms

Un avertissement important doit être donné ici : toutes les mesures de tension et de courant AC doivent être exprimées dans les mêmes termes (pic, pic-à-pic, moyen ou RMS). Si la source de tension est donné en tension AC pic alors tous les courants et toutes les tensions par conséquent calculés le sont dans les termes d'unités pic. Si la source de tension est donnée en volts AC RMS alors tous les courants et les tensions calculés le sont aussi dans l'unité AC RMS. Cela reste vrai pour *tous* les calculs basés sur les Lois d'Ohm, les Lois de Kirchhoff, etc. A moins que cela ne soit spécifié; toutes les valeurs de tension et de courant dans les circuits AC sont généralement supposés comme étant RMS plutôt que pics, moyen ou pic-à-pic. Dans quelques zones de l'électronique, les mesures pic sont supposées mais dans la plupart des applications (spécialement l'inductrie électronique) l'hypothèse est du RMS.

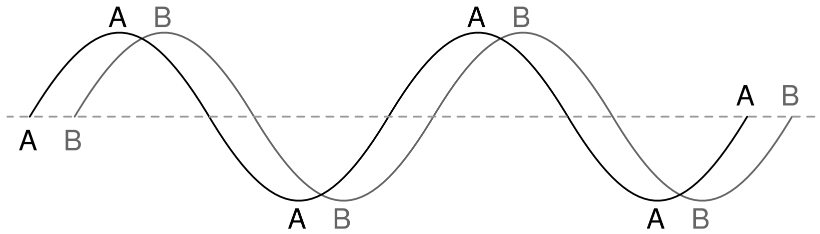
- **RÉSUMÉ :**

- Toutes les anciennes règles et loi du DC (Tension de Kirchhoff et de Courant, Loi d'Ohm) reste encore vrai pour AC. Néanmoins, avec des circuits plus complexes, nous pouvons avoir besoin de représenter les quantités AC dans une forme plus complexe. Vous en saurez plus dans peu de temps, je vous le promet!

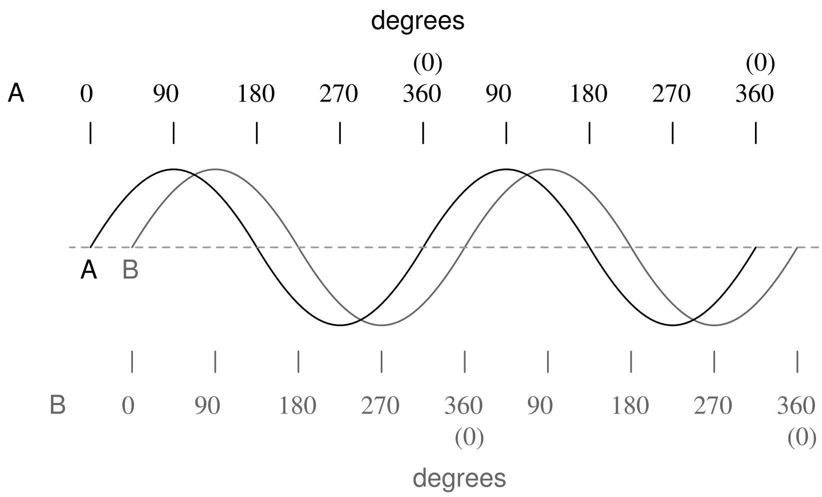
- La méthode de la "table" pour l'organisation des valeurs du circuit reste encore un outils d'analyse valide pour les circuits AC.

1.5 Phase AC

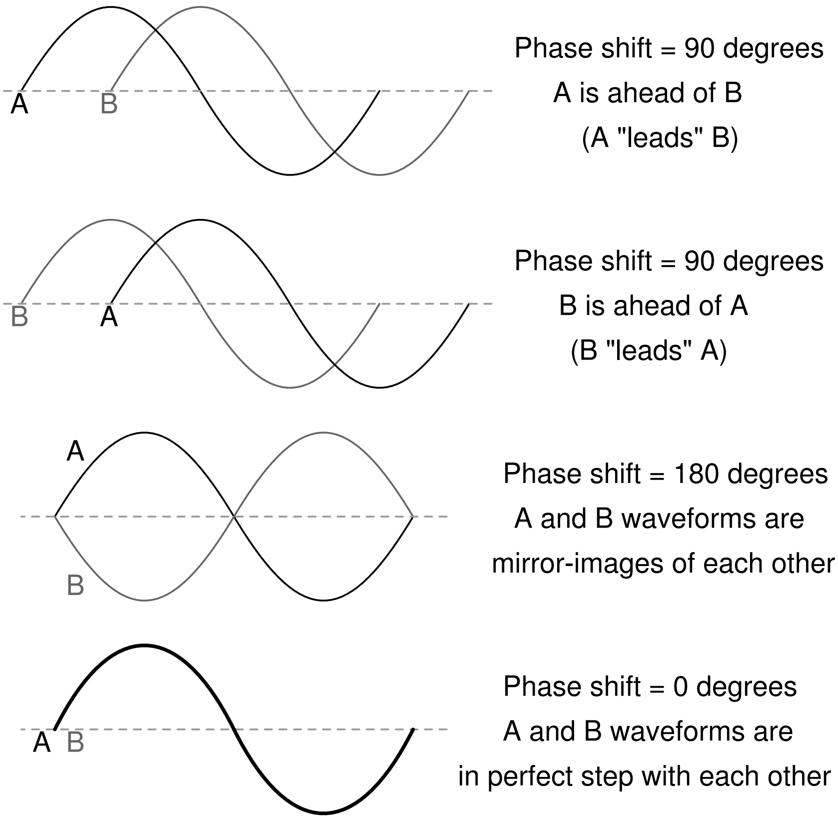
Les choses commencent à devenir compliquées lorsque nous avons besoin d'associer deux ou plusieurs tensions ou courants AC qui sont ne sont pas au même pas l'une avec l'autre. Par "au même pas", je veux dire que les deux formes d'onde ne sont pas synchronisées : que leur pic et leur point de zéro ne correspondent pas aux mêmes points dans le temps. Les graphes suivants illustrent un de ces exemples :



Les deux ondes montrées au-dessus (A versus B) ont la même amplitude et fréquence mais il sont décalés l'un avec l'autre. En termes techniques, c'est appelé un *décalage de phase*. Nous avons vu plus tôt comment nous pouvons tracer une "onde sinus" en calculant la fonction sinus trigonométrique pour des angles allant de 0 à 360 degrés, un cercle complet. Le point de départ d'une onde sinus est à zéro en amplitude et à zéro degrés, progressant vers une amplitude positive complète de 90 degrés, zéro à 180 degrés, complète négative à 270 degrés et retour au point de départ de zéro à 360 degrés. Nous pouvons utiliser cette échelle d'angle le long de l'axe horizontal de notre tracé de forme d'onde pour exprimer simplement de combien est le décalage d'une forme d'onde par rapport à une autre :



Le décalage entre ces deux formes d'ondes est de 45 degrés, l'onde "A" est avant l'onde "B". Un exemple des différents décalages de phase est donné dans les graphes suivants pour mieux illustrer ce concept :

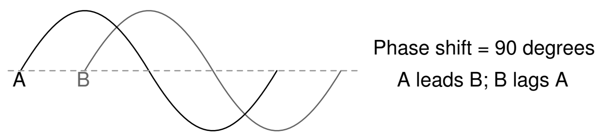


Comme les formes d'ondes dans les exemples ci-dessus sont à la même fréquence, ils seront décalés de la même valeur angulaire pour chaque pour point au cours du temps. Pour cette raison, nous pouvons exprimer le décalage de phase pour deux ou plusieurs formes d'ondes de la même fréquence comme une constante pour l'onde complète et pas seulement l'expression du décaleage entre deux points particuliers parmi les ondes. Ceci étant, on peut dire quelque chose comme, "la tension 'A' est décalé de 45 degrés avec la tension 'B'". N'importe laquelle des formes d'onde qui est en avant dans son évolution est dite être *en avance* et celle qui est en retrait est dite *en retard*.

Le décalage de phase, comme la tension, est toujours une mesure relative entre deux choses. Il n'existe pas de forme d'onde avec une mesure de phase *absolue* car il n'existe pas de référence universelle pour la phase. Typiquement, dans l'analyse de circuits AC, la forme d'onde de la tension de l'alimentation est utilisée comme phase de référence, cettte tension étant considérée comme ayant "xxx volts à 0 degrés". Toute autre tension ou courant AC dans ce circuit aura son décalage de phase exprimé en termes relatifs à cette source de tension.

C'est ce qui rend les calculs sur circuits AC plus compliqués que pour le DC. Lors de l'application des Loi d'Ohm et de Kirchhoff, les quantités de tension et de courant AC doivent refléter les déphasages et les amplitutdes. Les opérations mathématiques d'addition, soustraction, multiplication et division doivent opérer sur ces valeurs de déphasage ainsi que d'amplitude. Heureusement, il existe un système mathématique pour ces valeurs appelé *nombres complexes* idéalement adapté pour cette tâche de représentation d'amplitude et de phase.

Comme ce sujet sur les nombres complexes est si essentiel à la compréhension des circuits AC, le chapitre suivant sera dédié à ce seul sujet.

- **RÉSUMÉ :**
- Le *déphasage* est lorsque deux ou plusieurs formes d'ondes sont décalées l'une par rapport à l'autre.
- La valeur de déphasage entre deux ondes peut être exprimée en termes de degrés, comme défini par l'unité des degrés sur l'axe horizontal du graphe de la forme d'onde utilisée dans le tracé de la fonction trigonométrique sinus.
- Une forme d'onde en *avance* est définie comme une forme d'onde qui est avant l'autre dans son évolution. Une forme d'onde en *retard* est derrière l'autre. Exemple :
- 

- Les calculs pour l'analyse de circuit AC doivent prendre en considération et l'amplitude et le déphase des formes d'ondes de la tension et du courant pour être totalement précis. Ceci nécessite l'utilisation de systèmes mathématiques appelés *nombres complexes*.

1.6 Principes de radio

Une des applications les plus fascinantes de l'électricité est dans la génération de vagues d'énergie appelée *ondes radio*. Le champ limité de cette leçon sur le courant alternatif ne permet pas la complète exploration du concept, seuls certains des principes de base seront couverts.

Avec la découverte accidentelle de l'électromagnétisme d'Oersted, il a été réalisé que l'électricité et le magnétisme étaient reliés l'un à l'autre. Lorsqu'un courant électrique passe dans un conducteur, un champ magnétique est généré, perpendiculaire à l'axe du flux. De la même manière, si un conducteur a été exposé à un changement de flux magnétique perpendiculaire au conducteur, une tension a été produite le long de ce conducteur. Ainsi, les scientifiques savaient que l'électricité et le magnétisme semblent toujours s'affecter l'un l'autre aux angles corrects. Une découverte majeure, néanmoins, reste cachée juste derrière ce concept qui semble simple, de perpendicularité reliée et dévoilement fut un des moments charnière dans la science moderne.

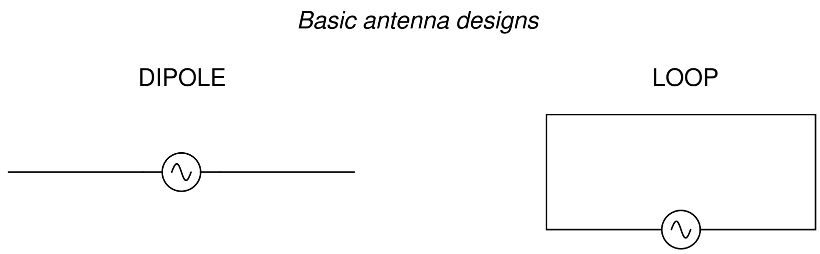
Cette brêche dans la physique est difficile à décrire. L'homme responsable de cette révolution conceptuelle état le physicien écossais James Clerk Maxwell (1831-1879), qui a "unifié" l'étude de l'électricité et du magnétisme en quatre équations relativement ordonnées. En essence, ce qu'il a découvert était que les *champs* magnétiques et électriques étaient intrinsèquement liés l'un à l'autre, avec ou sans la présence d'un chemin conducteur pour le flux des électrons. En l'établissant plus formellement, la découverte de Maxwell était ceci :

> **Un champ électrique changeant produit un champ magnétique perpendiculaire**, and **Un champ magnétique changeant produit un champ électrique perpendiculaire**.

Tout ceci peut prendre place en espace ouvert, les champs électriques et magnétiques se supportant l'un l'autre lorsqu'ils traversent l'espace à la vitesse de la lumière. Cette structure dynamique de champs électriques et magnétiques se propageant à travers l'espace est mieux connue comme une *onde électromagnétique.*

Il existe plusieurs types d'énergie radiative naturelle composée d'ondes électromagnétiques. Même la lumière est électromagnétique par nature. De même que les radiations des rayons X et les rayons "gamma". La seule différence entre ces types de radiation électromagnétique est la fréquence de leurs oscillations (l'alternance de champs électriques et magnétiques dont la polarité va et vient). En utilisant une source de tension AC et un instrument spécial appelé une *antenne*, nous pouvons créer des ondes électromagnétiques (à une beaucoup plus basse fréquence que la lumière) facilement.

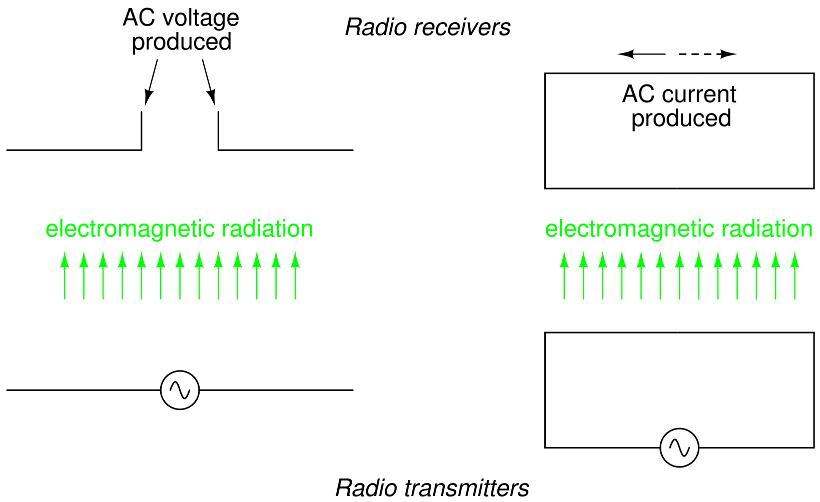
Une antenne n'est rien de plus qu'un instrument construit pour produire un champ électrique ou magnétique dispersé. Les deux types fondamentaux d'antennes sont les *dipoles* et les *boucles* :



Alors que le dipole ne ressemble à rien de plus qu'un circuit ouvert et que la boucle à un court circuit, ces morceaux de fils radient effectivement des champs électromagnétiques lorqu'ils sont connectés à une source AC à la bonne fréquence. Les deux fils ouverts du dipole agissent comme une sorte de condensateur (deux conducteurs séparés par un diélectrique), avec un champ électrique ouvert pour disperser au lieu d'être concentré entre deux plaques à l'espacement rapproché. Le chemin des fils fermés de l'antenne boucle agissent comme un inducteur avec un grand coeur à air, fournissant encore une grande opportunité pour le champ à se disperser de l'antenne au lieu d'être concentré et contenu comme dans une inductance normale.

Lorsque le dipole alimenté radie sont champ électrique changeant dans l'espace, un champ magnétique changeant est produit avec un bon angle pour soutenir le champ électrique plus loin dans l'espace et ainsi de suite alors que l'onde se propage à la vitesse de la lumière. Comme l'antenne boucle alimentée radie son champ magnétique dans l'espace, un champ magnétique changeant est produit au bon angle, avec le même résultat final d'une onde électromagnétique continue expulsée de l'antenne. Les deux antennes réalisent la même tâche basique : la production contrôlée d'un champ électromagnétique.

Lorsqu'elle est attachée à une source AC à haute fréquence, une antenne agit comme un instrument de *transmission*, convertissant une tension et un courant AC en une onde magnétique énergétique. Les antennes ont aussi la capacité d'intercepter des ondes électromagnétiques et de convertir leur énergie en une tension et un courant AC. Dans ce mode, une antenne agit comme un instrument de *réception* :



Alors qu'il y a *beaucoup* plus à dire à propos de la technologie des antennes, cette brève introduction est suffisante pour vous donner une idée générale sur ce qui se passe (et peut être suffisamment d'informations pour vous permettre quelques expériences).

- **RÉSUMÉ :**
- James Maxwell a découvert que changer des champs électroniques produisait des champs magnétiques perpendiculaires et inversement, même dans le vide.
- Un jeu jumeau de champs magnétiques et électriques, oscillant avec un angle correct l'un par rapport à l'autre et se déplaçant à la vitesse de la lumière, constitue une *onde électromagnétique*.
- Une *antenne* est un composant constitué de fils, conçu pour irradier un champ électrique ou un champ magnétique changeant lorsqu'il est alimenté par une source AC haute fréquence ou pour intercepter un champ électromagnétique et le convertissant en une tension ou un courant AC.
- Une antenne *dipole* consiste en deux pièces de fils (ne se touchant pas), générant d'abord un champ électrique lorsqu'elle est alimentée puis générant ensuite un champ magnétique dans l'espace.
- L'antenne *en boucle* consiste en une boucle de fil, générant d'abord un champ magnétique lorsqu'elle est alimentée et générant ensuite un champ électrique dans l'espace.

1.7 Contributeurs

Les contributeurs à ce chapitre sont listés dans l'ordre chronologique de leur contribution, depuis le plus récent jusqu'au premier. Voyez l'annexe 2 (Liste des contributeurs) pour les dates et les informations de contact.

Harvey Lew (February 7, 2004) : Corrected typographical error : "circuit" should have been "circle".

Duane Damiano (February 25, 2003) : Pointed out magnetic polarity error in DC generator illustration.

Mark D. Zarella (April 28, 2002) : Suggestion for improving explanation of "average" waveform amplitude.

John Symonds (March 28, 2002) : Suggestion for improving explanation of the unit "Hertz."

Jason Starck (June 2000) : HTML document formatting, which led to a much better-looking second edition.

Chapter 2

NOMBRES COMPLEXES

2.1 Introduction

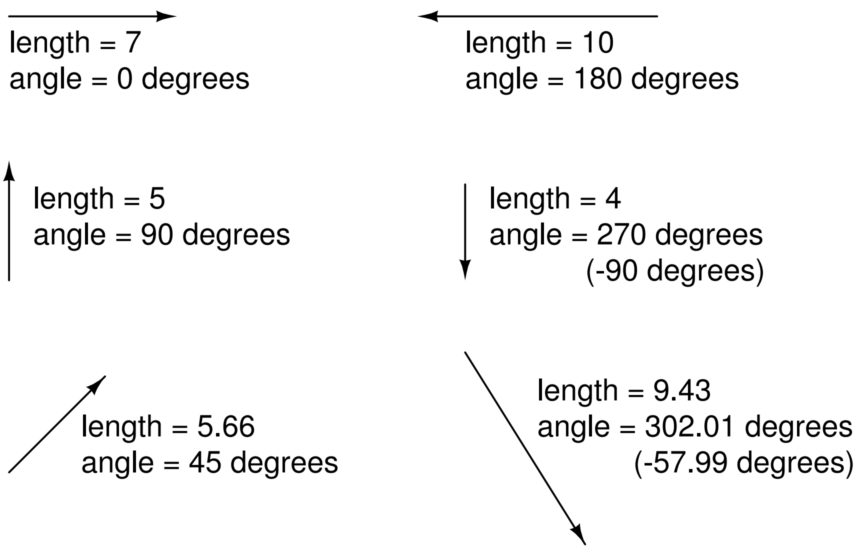
Si j'ai besoin de décrire la distance entre deux villes, je peux fournir une réponse constituée d'un simple nombre en kilomètres, en miles ou toute autre unité de mesure linéaire. Néanmoins, si je devais décrire comment voyager d'une ville à une autre, je devrais fournir plus d'informations que simplement la distance entre ces deux villes; Je devrais aussi fournir des informations sur la *direction* à suivre.

Le type d'information qui exprime une seule dimension, telle qu'une distance linéaire, est appelée une quantité *scalaire* en mathématiques. Les nombres scalaires sont les types de nombres que vous avez utilisé jusqu'ici dans la plupart des applications mathématiques. La tension produite par une batterie, par exemple, est une quantité scalaire. De même pour la résistance d'un bout de cable (ohms) ou le courant qui le traverse (amps).

Néanmoins, lorsque nous commençons à analiser les circuits à courants alternatifs, nous trouvons des valeurs de tension, de courant et même de résistance (appelée *impédance* en AC) qui ne sont par les quantités familières à une dimension que nous avons mesuré dans les circuits DC. Au contraire, ces quantités car elles sont dynamiques (direction et amplitude alternatives), possèdent d'autres dimensions qui doivent être prises en compte. La fréquence et le décalage de phase sont deux de ces dimensions qui rentrent en jeu. Même avec des circuits AC relativement simples, où nous avons une fréquence simple, nous avons encore la dimension du décalage de phase en plus de l'amplitude.

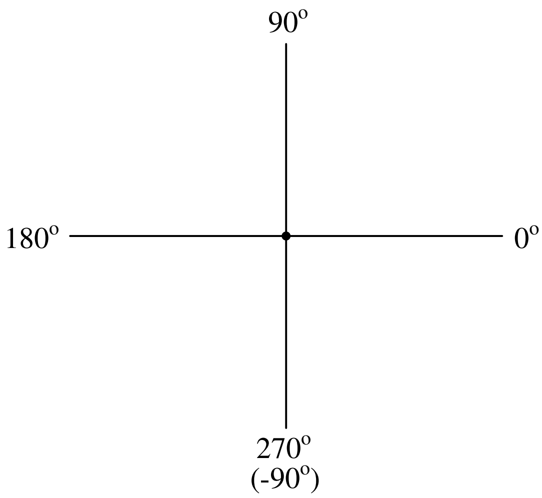
De manière à analyser avec succès les circuits AC, nous devons travailler avec les objets mathématiques et les techniques capables de représenter ces quantités multi-dimensionnelles. C'est ici que nous abandonnons les nombres scalaires pour quelque chose de plus adapté : les *nombres complexes*. Juste comme l'exemple de donner les directions d'une ville à une autre, les quantités AC dans un circuit à une fréquence unique ont à la fois une amplitude (analogie : distance) et un décalage de phase (analogie : direction). Un nombre complexe est une quantité mathématique simple permettant d'exprimer ces deux dimensions d'amplitude et de décalage de phase en une fois.

Les nombres complexes sont sont plus facile à appréhender lorsqu'ils sont représentés graphiquement. Si je dessine une ligne avec une certaine longueur (magnitude) et un angle (direction), j'ai une représentation graphique d'un nombre complexe qui sont habituellement connus en physique comme des *vecteurs* :

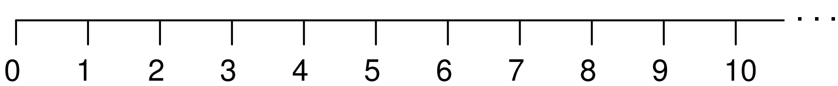


Comme les distances et les directions sur une carte, il doit y avoir une même structure de référence pour que les expressions d'angles aient une signification précise. Dans ce case, la droite est considérée comme étant le 0^o et les angles sont comptés d'une manière positive en allant dans le sens des aiguilles d'une montre :

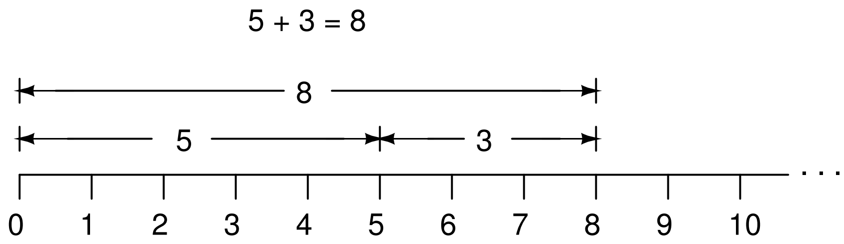
The vector "compass"



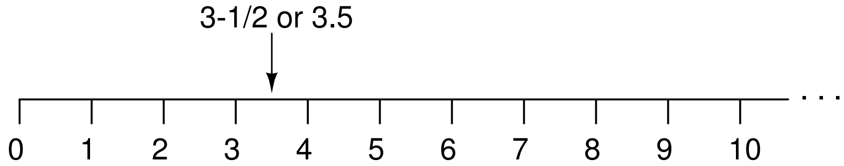
L'idée de représenter un nombre sous une forme graphique n'est pas nouvelle. Nous l'avons tous appris au primaire avec la "ligne des nombres" :



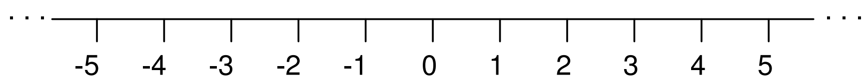
Nous avons même apris comment les additions et les soustractions fonctionnent en voyant comment les longueurs (magnitudes) sont accumulées pour donner la réponse finale :



Nous avons appris, plus tard, qu'il y avait des méthodes pour désigner les valeurs *entre* les nombre entiers marqués sur la ligne. Ce sont les quantités fractionnaires ou décimales :



Nous avons appris plus tard que les nombres de la ligne pouvaient aussi s'étendre à gauche du zéro :

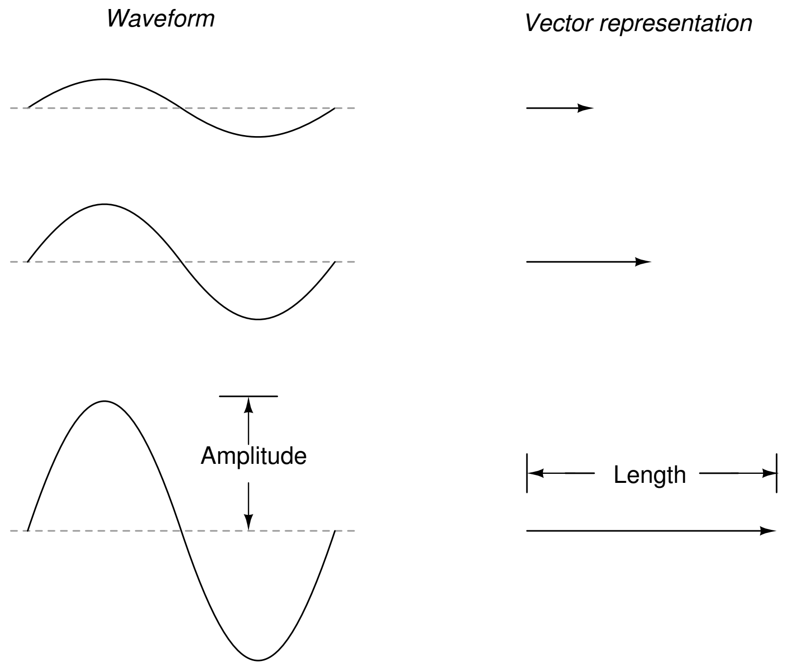


Ces champs de nombres (entiers, rationel, irrationel, réel, etc.) apris au primaire partagent une caractéristique commune : ils sont tous à *une dimension.* La droiture de la ligne des nombres le montre graphiquement. Vous pouvez aller à gauche ou à droite de la ligne mais tous les "déplacements" sur cette ligne sont restreints à un simple axe (horizontal). Les nombres à une dimension, scalaires sont parfaitement adaptés pour compter les perles, représenter des poids ou mesurer la tension DC d'une batterie mais ils sont trop limités pour représenter quelque chose de plus complexe comme la distance *et* la direction entre deux ville ou l'amplitude *et* la phase d'un signal AC. Pour représenter ces types de quantités, nous avons besoin de représentation multidimensionnelles. En d'autres mots, nous avons besoin d'une ligne numérotée qui peut pointer différentes directions et c'est exactement ce que permettent les vecteurs.

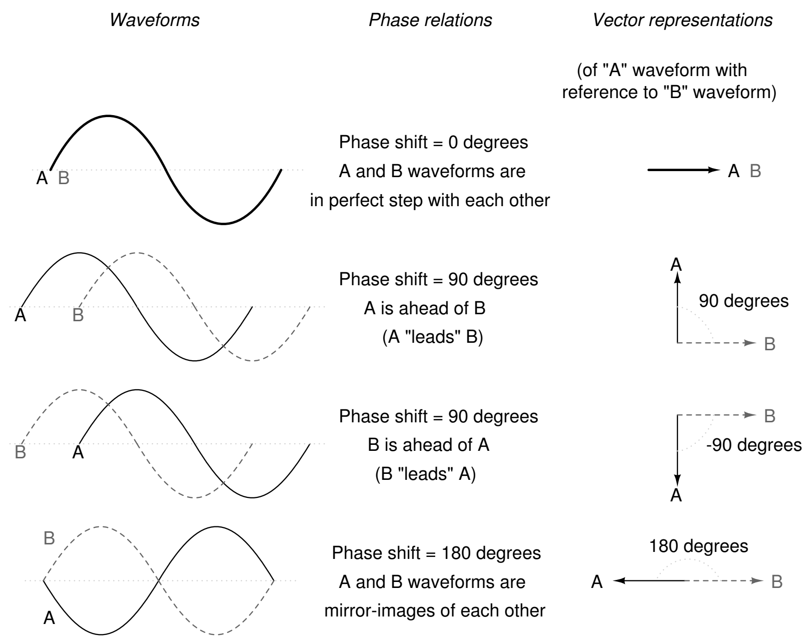
- **RÉSUMÉ :**
- Un nombre *scalaire* est le type d'objet mathématique que les personnes sont habituées à utiliser dans la vie de tous les jours : une quantité à une dimension comme la température, la longueur, le poids, etc.
- Un *nombre complexe* est une quantité mathématique représentatnt deux dimensions de magnitude et de direction.
- Un *vecteur* est une représentation graphique d'un nombre complexe. Il ressemble à une flèche, avec un point de départ, une pointe, une longueur et une direction définie. Le mot *phasor* est quelque fois utilisé dans des applications électriques où l'angle du vecteur représente le décalage de phase entre les formes d'ondes.

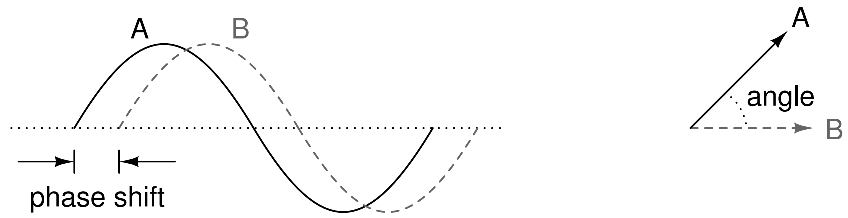
2.2 Vecteurs et signaux AC

Ok, donc comment pouvons-nous exactement représenter les quantités de tension ou de courant AC sous la forme d'un vecteur? La longueur du vecteur représente la magnitude (ou l'amplitude) de l'onde, comme ceci :



Plus grande est l'amplitude de l'onde, plus grande est la longueur du vecteur correspondant. L'angle du vecteur, néanmoins, représente le décalage de phase en degrés entre l'onde en question et une autre onde agissant comme "réference" de temps. Habituellement, lorsque la phase d'une onde dans un circuit est exprimée, elle est référencée à la forme d'onde de la tension d'alimentation (arbitrairement déclaré comme étant "à" 0^o). Rappelez-vous que cette phase est toujours une mesure *relative* entre deux ondes plutôt qu'une propriété absolue.





Plus grand est le décalage de phase en degrés, plus grande est la différence d'angle entre les vecteurs correspondants. Étant une mesure relative, comme la tension, le décalage de phase (angle de vecteur) n'a seulement de signification qu'en référence à une onde standard. Généralement, cette onde de "référence" est la tension principale de puissance AC du circuit. S'il y a plus d'une source de tension AC, alors une de ces sources est arbitrairement choisie pour être la référence de phase pour toutes les autres mesures du circuit.

Ce concept de point de référence n'est pas contraire au point "ground" d'un circuit au bénéfice d'une référence de tension. Avec un point clairement identifié comme "ground," dans le circuit, il devient possible de parler de tensions "sur" ou "à" des simples points dans un circuit, en comprenant que ces tensions (toujours relatives entre *deux* points) sont référencées au "ground." De même, avec un point de référence clairement défini pour la phase, il devient possible de parler de tensions et de courant dans un circuit AC ayant des angles de phases définis. Par exemple, si le courant dans un circuit AC est décrit comme "24.3 milliampères à -64 degrés", cela signifie que l'onde de courant possède une amplitude de 24.3 mA et il a un retard de 64^o après l'onde de référence, habituellement supposée être l'onde de tension de l'alimentation principale.

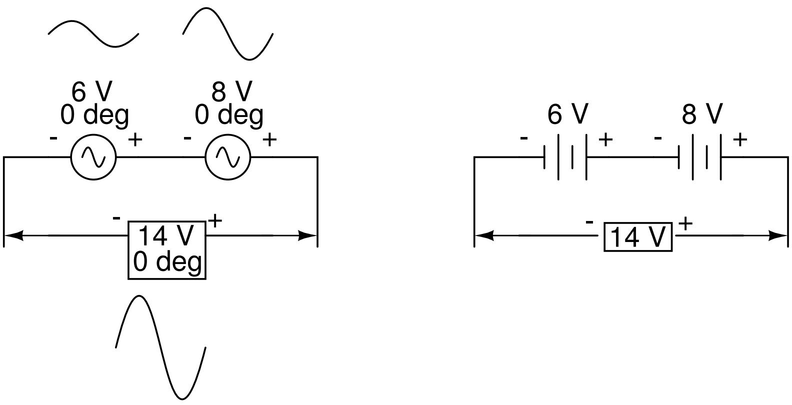
- **RÉSUMÉ :**

- Lorsqu'il est utilisé pour décrire une quantité AC, la longueur du vecteur représente l'amplitude de l'onde alors que l'angle du vecteur représente l'angle de phase de l'onde, relativement à une autre onde (de référence).

2.3 Addition simple de vecteur

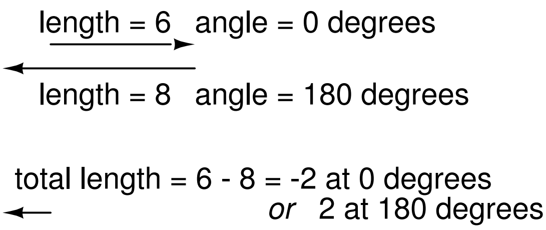
Rappelez-vous que les vecteurs sont des objets mathématiques comme les chiffres sur la ligne des nombres : ils peuvent être ajoutés, soustraits, multipliés et divisés. L'addition est peut être l'opération sur les vecteurs la plus facile à visualiser, nous allons donc débuter avec elle. Si des vecteurs avec des angles simples sont ajoutés, leurs magnitudes (longueurs) sont ajoutées exactement comme des quantités scalaires courantes :

De même, si des sources de tension AC avec le même angle de phase sont connectées ensemble en série, leurs tensions s'ajoutent de la même manière que pour les batteries DC :

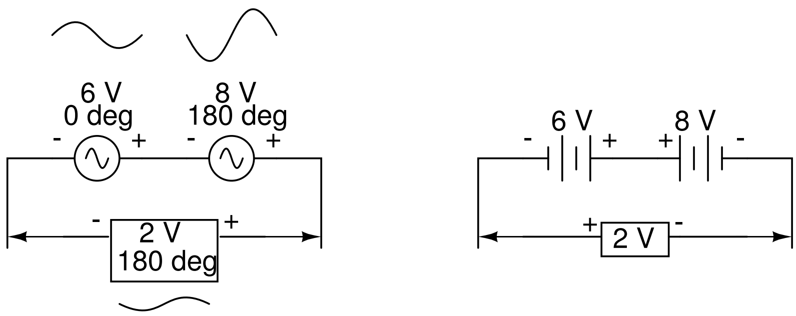


Veuillez noter que les polarités (+) et (-) marquées à coté des broches des deux sources AC. Même si nous savons que le courant alternatif n'a pas de "polarité" dans le même sens que peut en avoir le courant continu, ces marques sont essentielles pour savoir comme référencer les angles de phases des tensions données. Ce la deviendra plus évident dans l'exemple suivant.

Si les vecteurs directement opposés l'un à l'autre (180^o de décalage de phase) sont ajoutés ensemble, leurs magnitudes (longueurs) se soustraient comme les quantités scalaires positives et négative le font lorsqu'elles sont ajoutées :

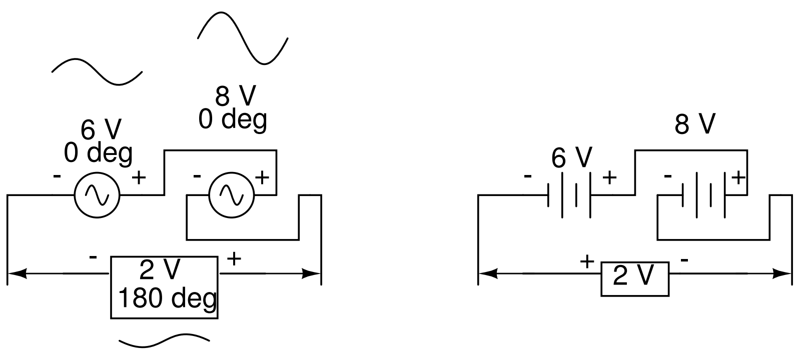


De la même manière, si des sources de tension alternative opposées sont connectées en série, leurs tensions se soustraient comme vous pouvez vous y attendre avec des batteries DC connectées d'une manière opposée :

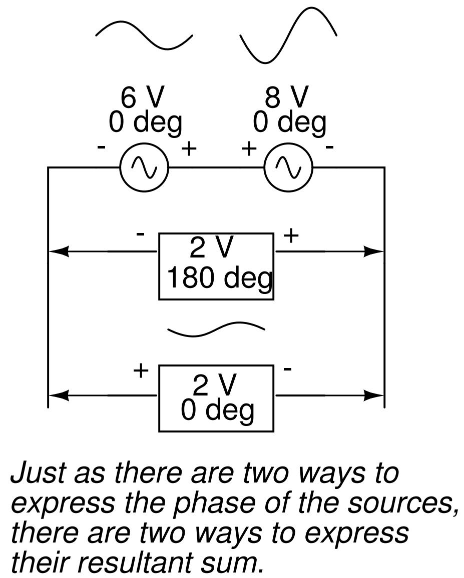


Pour déterminer si ces sources de tension sont opposées l'une à l'autre, il faut examiner leur polarités *et* leurs angles de phase. Notez comment les marques de polarités dans le diagramme ci-dessus semblent indiquer des tensions additives (de la gauche vers la droite, nous voyons - et + sur la source 6 volts, - et + sur la source 8 volts). Bien qu'aux bornes, nous aurions un effet *additif* d'indiqué, dans un circuit DC (les tensions fonctionnant ensemble pour pour produire une tension totale plus grande), dans ce circuit AC, elles poussent réellement dans des directions opposées car une de ces tensions a un angle de phase de 0^o et l'autre, un angle de phase de 180^o. Le résultat, bien sur, est une tension globale de 2 volts.

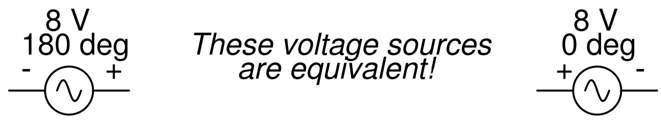
Nous aurions aussi bien pu montrer les tensions opposées soustraites en série comme ceci :



Notez comment les polarités apparaissent être opposées l'une à l'autre maintenant, avec l'inversion des connections des fils sur la source de 8 volts. Comme les deux sources sont décrites comme ayant des angles de phases égaux (0^o), elle sont vraiment opposées l'une à l'autre et l'effet global est le même qu'avec le scénario précédent avec des polarités "additionnées" et des angles de phase différents : une tension totale de seulement 2 volts.

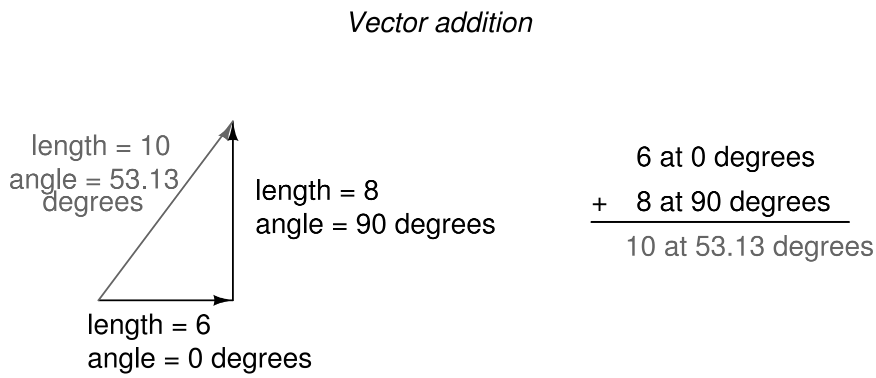


La tension résultante peut être exprimée de deux différentes manières : 2 volts à 180^o avec le symbole (-) sur la gauche et le symbole (+) sur la droite ou 2 volts à 0^o avec le symbole (+) sur la gauche et le symbole (-) sur la droite. Une inversion des fils de la sourrce de tension AC est la même chose que le décalage de phase de la source de 180^o.

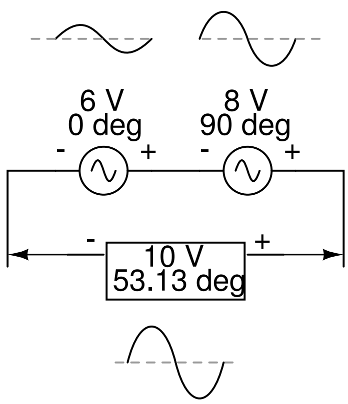


2.4 Addition complexe de vecteurs

Si des vecteurs avec des angles différents sont ajoutés, leurs magnitudes (longueurs) s'ajoutent differement que les magnitudes scalaires :



Si deux tensions AC – déphasées à 90^o – sont ajoutées ensemble en étant connectées en série, leurs magnitudes de tension ne s'ajoutent ou ne se soustraient pas directement comme pour les tensions scalaires en DC. Au contraire, ces quantités de tension sont des quantités complexes et comme les vecteurs ci-dessus, qui s'ajoutent à la manière trigonométrique, une source de 6 volts 0^o ajoutée à une source de 8 volts à 90^o donne une tension de 10 volts avec un angle de phase de 53.13^o :



Comparé à l'analyse de circuits DC, cela est, en effet, très étrange. Notez qu'il est possible d'obtenir des indications sur le voltmètre de 6 et 8 volts, respectivement, aux bornes des deux sources de tensions, et de ne lire que 10 volts pour la tension globale!

Il n'existe pas d'analogie en DC avec ce qui se produit avec deux tensions AC déphasées. Les tensions DC peuvent seulement s'ajouter ou s'opposer directement, sans rien entre. En AC, deux tensions peuvent être ajoutées ou opposées *quelque soit les degrés* en y incluant l'addition ou l'opposition totale. Sans l'utilisation de la notation vectorielle (nombre complexe) pour décrire les quantités AC, il serait *très* difficile d'effectuer des calculs mathématiques pour les analyses de circuit AC.

Dans la section suivante, nous apprendrons comment représenter symboliquement les quantités de vecteur plutôt que dans leur forme graphique. Les diagrammes de vecteurs et de triangle suffisent pour illustrer le concept général mais les méthodes symboliques plus présises doivent être utilisées si des calculs plus sérieux doivent être effectués sur ces quantités.

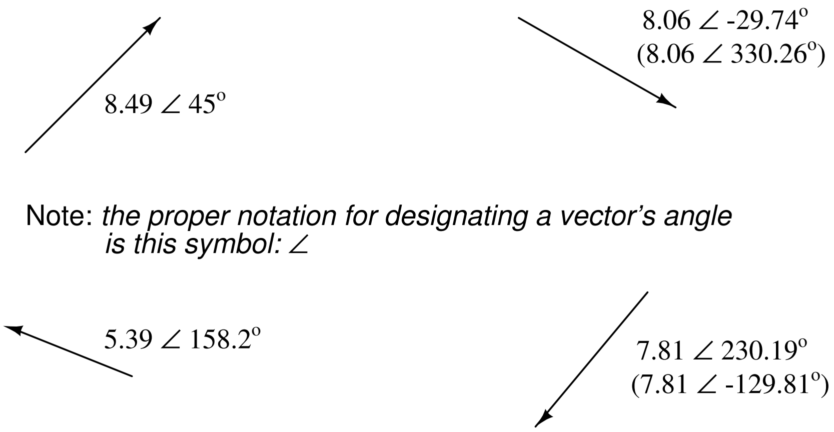
- **RÉSUMÉ :**

- Les tensions DC peuvent soit s'ajouter soit s'opposer l'une à l'autre lorsqu'elles sont connectées en série. Les tensions AC peuvent s'ajouter ou s'opposer *queque soit le degré* en fonction du décalage de phase entre eux.

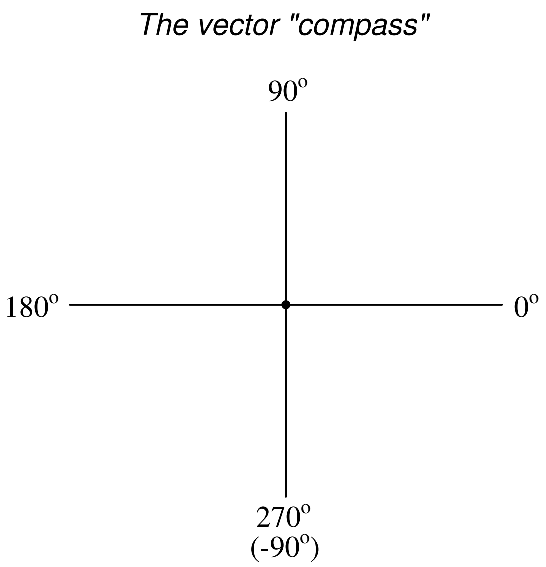
2.5 Notation polaire et rectangulaire

De manière à travailler avec ces nombres complexes sans dessiner de vecteurs, nous avons d'abord besoin d'une notation mathématique standard. Il y a deux formes de base pour la notation des nombres complexes : *polaire* et *rectangulaire*.

La forme polaire est lors de la notation d'un nombre complexe par la *longueur* (aussi connu comme la *magnitude*, la *valeur absolue* ou le *module*) et l'*angle* de son vecteur (habituellement noté par un symbole d'angle qui ressemble à ceci : $\angle$). Pour utiliser une analogie avec la carte, la notation polaire du vecteur de New York City à San Diego serait quelque chose comme "2400 miles, southwest".Vous avezici deux exemples de vecteurs et leurs notations polaire :

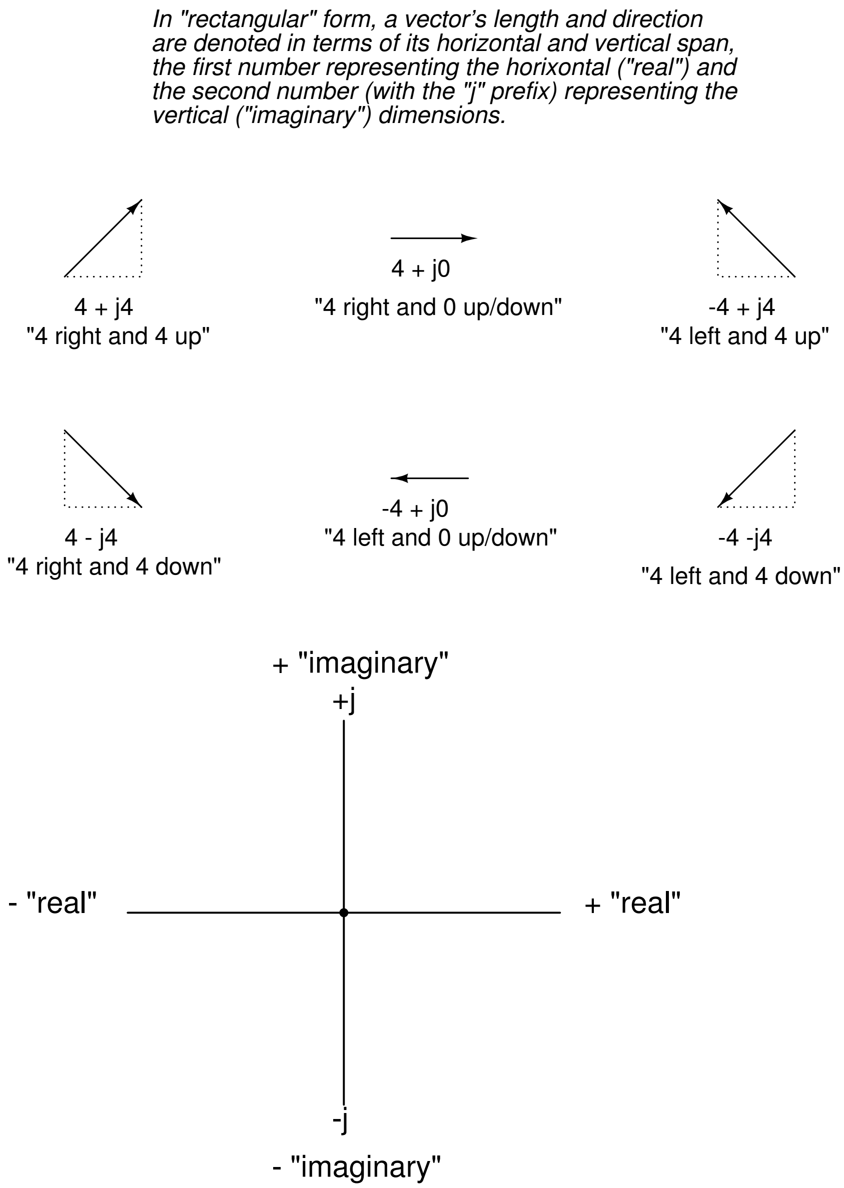


L'orientation standard pour les calculs des angles de vecteurs dans un circuit AC définit 0^o comme étant à la droite (horizontale), 90^o vers le haut, 180^o vers la gauche et 270^o vers le bas. Veuillez noter que les angles de vecteurs orientés vers le "bas" peuvent avoir des angles représentés en forme polaire comme des nombres positifs au-delà de 180 ou de nombres négatifs à moins de 180. Par exemple, un vecteur dont l'angle est $\angle\ 270^o$ (vers le bas) peut aussi être considéré comme ayant un angle de -90^o. Le vecteur ci-dessus, sur la droite, ($5.4 \angle 326^o$) peut aussi être noté comme $5.4 \angle -34^o$.



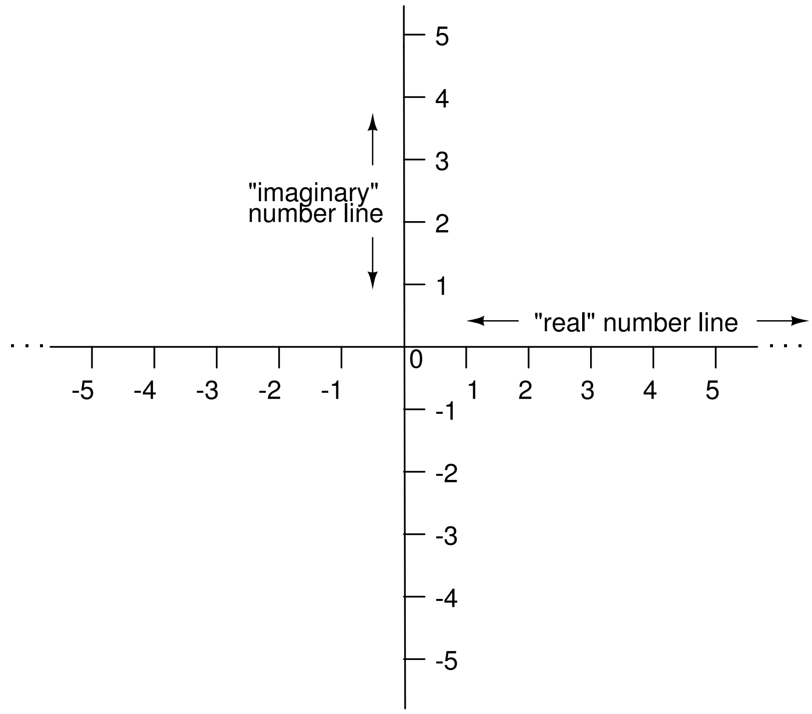
La forme rectangulaire, d'un autre côté, est lorsqu'un nombre complexe est noté par ses composantes horizontales et verticales respectives. Au fond, le vecteur angle est pris comme étant l'hypothénuse d'un triangle droit, décrit par les longueurs des cotés adjacents et opposés. Plutôt que décrire une longueur de vecteur et une direction en notant la magnitude et l'angle, il est décrit dans les termes de ”à quelle distance de droite/gauche” et ”à quelle distance en bas/haut”.

Ces formes à deux dimensions (horizontales et verticales) sont symbolisées par deux formes numériques. De manière à distinguer les dimensions hozizontales et verticales l’une de l’autre, la verticale est préfixée avec un ”i” minuscule (en mathématiques pures) ou ”j” (en électronique). Ces lettres en minuscule ne représentent pas une variable physique (tel qu’un courant instantané, lui aussi symbolisé par la lettre ”i” en minuscule) mais sont plutôt des *opérateurs* mathématiques utilisés pour distinguer la composante verticale du vecteur de sa composante horizontale. Comme nombre complexe complet, les quantités horizontales et verticales sont écrites comme une somme :



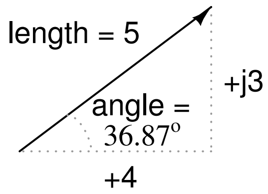
La composante horizontale est indiquée comme étant *réelle* car cette dimension est compatible avec des nombres normaux, scalaires ("réels"). La composante verticale est indiquée comme étant *imaginaire* car cette dimension est dans une direction différente, totallement étranger à l'échelle des nombres réels.

L'axe "réel" du graphe correspond à la ligne familiere des nombres que nous avons vus plus tôt : ceux qui ont des valeurs positives et négatives. L'axe "imaginaire" du graphe correspond à une autre ligne de nombres située à 90^o de celle "réel". Les vecteurs ayant deux dimensions, nous devons avoir une "carte" à deux dimensions sur laquelle les appliquer, donc les deux lignes de nombres perpendiculaires l'une par rapport à l'autre :



Chaque méthode de notation est valide pour les nombres complexes. La raison première d'avoir deux méthodes de notation est de faciliter les calculs courant, la forme rectanguaire réservée pour l'addition et la soustration et la forme polaire à la multiplication et la division.

La conversion entre les deux formes de notation n'impliquent que de la simple trigonométrie. Pour effectuer la conversion du polaire vers le rectangulaire, trouvez la composante réelle en multipliant la magnitude polaire par le cosinus de l'angle et la composante imaginaire en multipliant la magnitude polaire par le sinus de l'angle. Ceci peut être compris plus facilement en dessinant les valeurs comme de côté des triangles droits, l'hypothénuse du triangle représentant le vecteur lui-même (sa longueur et son angle en fonction de l'horizontale constituant la forme polaire), les côtés horizontaux et verticaux représentent les composantes rectangulaires "réelles" et "imaginaires", respectivement :



$5 \angle 36.87^o$ (forme polaire)
$(5)(\cos 36.87^o)=4$ (composante réelle)
$(5)(\sin 36.87^o)=3$ (composante imaginaire)
4+j3 (forme rectangulaire)

Pour faire une conversion depuis la forme rectangulaire vers la polaire, trouvez la magnitude polaire avec l'utilisation du Théorème de Pythagore (la magnitude polaire est l'hypothénuse du triangle droit et les composantes réelles et imaginaires sont les côtés adjacent et opposés, respectivement) et l'angle en prennant l'arctangente de la composante imaginaire divisée par la composante réelle :

$4 + j3$ **(rectangular form)**

$c = \sqrt{a^2 + b^2}$ (pythagorean theorem)

$$\text{polar magnitude} = \sqrt{4^2 + 3^2}$$

$$\text{polar magnitude} = 5$$

$$\text{polar angle} = \arctan \frac{3}{4}$$

$$\text{polar angle} = 36.87^o$$

$5 \angle 36.87^o$ **(polar form)**

- **RÉSUMÉ :**
- La notation *polaire* indique un nombre complexe en terme de longueur de vecteur et de direction angulaire depuis le point de départ. Exemple : volez pendant 45 miles $\angle$ 203^o (West by Southwest).
- La notation *rectangulaire* indique un nombre complexe en terme de dimensions horizontales et verticales. Exemple : conduisez 41 miles West puis tournez et 18 miles South.
- En notation rectangulaire, la première quantité est la composante "réelle" (dimension horizontale du vecteur) et la seconde quantité est la composante "imaginaire" (dimension verticale du

vecteur). La composante imaginaire est précédée par une minuscule "j," quelque fois appelée *l'opérateur j.*

- Les formes de notation polaires et rectangulaires peuvent être reliées graphiquement sous la forme d'un triangle droit, avec l'hypothénuse représentant le vecteur lui-même (forme polaire : longueur de l'hypothénuse = magnitude; angle en fonction du côté horizontal = angle), le côté horizontal représente la composante rectangulaire "réelle" et le côté vertical représente la composante rectangulaire "imaginaire".

2.6 Arithmétique des nombres complexes

Comme les nombres complexes sont des entités mathématiques normales, comme ls nombres scalaires, ils peuvent être ajoutés, soustraits, multipliés, divisés, mis au carré, inversés, etc., comme les autres types de nombres. Quelques calculatrices scientifiques sont programmés pour effectuer ces opérations directement sur deux ou plusieurs nombres complexes mais ces opérations peuvent aussi être faites "à la main". Cette vous montrera comme les opérations de base peuvent être effectuées. Il est *hautement* recommendé que vous vous équipiez avec une calculatrice scientifique capable d'effectuer des fonctions arithmétiques facilement sur les nombres complexes. Cela rendra votre étude des circuits AC plus désagréable si vous êtes forcés d'effectuer tous les calculs de la manière la plus lente.

L'addition et la soustraction avec les nombres complexes sous forme rectangulaire est facile. Pour les additions, ajoutez simplement les composantes réelles des nombres complexes pour déterminer la composante réelle de la somme et ajoutez les composantes imaginaires des nombres complexes pour déterminer la composante imaginaire de la somme :

$$\begin{array}{r} 2 + j5 \\ + \ 4 - j3 \\ \hline \mathbf{6 + j2} \end{array} \qquad \begin{array}{r} 175 - j34 \\ + \ 80 - j15 \\ \hline \mathbf{255 - j49} \end{array} \qquad \begin{array}{r} -36 + j10 \\ + \ 20 + j82 \\ \hline \mathbf{-16 + j92} \end{array}$$

Lors de la soustraction des nombres complexes en forme rectangulaire, soustrayez simplement la composante réelle du second nombre complexe depuis la composante réelle de la première pour arriver à la composante réelle de la différence et soustrayez la composante imaginaire du second nombre complexe depuis la composante imaginaire de la première pour arriver à la composante imaginaire de la différence :

$$\begin{array}{r} 2 + j5 \\ - \ 4 - j3 \\ \hline \mathbf{-2 + j8} \end{array} \qquad \begin{array}{r} 175 - j34 \\ - \ 80 - j15 \\ \hline \mathbf{95 - j19} \end{array} \qquad \begin{array}{r} -36 + j10 \\ - \ 20 + j82 \\ \hline \mathbf{-56 - j72} \end{array}$$

Pour les multiplications et les divisions à la main, la forme polaire est la notation la plus judicieuse. Lors de la multiplication des nombres complexes sous forme polaire, *multipliez* simplement les magnitudes polaires des nombres complexes pour déterminer la magnitude polaire du produit et *ajoutez* les angles des nombres complexes pour déterminer l'angle du produit :

$(35 \angle 65^o)(10 \angle \text{-}12^o)$=$350\angle 53^o$

$(124 \angle 250^o)(11 \angle 100^o) = 1354 \angle \text{-}10^o$ ou $1354 \angle 350\ ^o$

$(3 \angle 30^o)(5 \angle \text{-}30^o)$=$15 \angle 0^o$

La division des nombres complexes à forme polaire est aussi facile : divisez simplement la magnitude polaire du premier nombre comlpexe par la magnitude polaire du second nombre complexe pour obtenir la magnitude polaire du quotient et soustrayez l'angle du second nombre complexe avec l'angle du premier nombre complexe pour obtenir l'angle du quotient :

$$\frac{35 \angle 65^o}{10 \angle \text{-}12^o} = \mathbf{3.5 \angle 77^o}$$

$$\frac{124 \angle 250^o}{11 \angle 100^o} = \mathbf{11.273 \angle 150^o}$$

$$\frac{3 \angle 30^o}{5 \angle \text{-}30^o} = \mathbf{0.6 \angle 60^o}$$

Pour obtenir la réciproque ou "inversion" (1/x), d'un nombre complexe, divisez simplement le nombre (sous forme polaire) sous une valeur scalaire de 1, qui est rien de moins qu'un nombre complexe sans composante imaginaire (angle = 0) :

$$\frac{1}{35 \angle 65^o} = \frac{1 \angle 0^o}{35 \angle 65^o} = \mathbf{0.02857 \angle \text{-}65^o}$$

$$\frac{1}{10 \angle \text{-}12^o} = \frac{1 \angle 0^o}{10 \angle \text{-}12^o} = \mathbf{0.1 \angle 12^o}$$

$$\frac{1}{0.0032 \angle 10^o} = \frac{1 \angle 0^o}{0.0032 \angle 10^o} = \mathbf{312.5 \angle \text{-}10^o}$$

Ce sont des opérations de base que vous aurez besoin de connaître de manière à manipuler les nombres complexes dans l'analyse des circuits AC. Les opérations avec des nombres complexes ne sont, néanmoins, pas seulement limités aux additions, soustractions, multiplications, divisions et inversions. Virtuellement toutes les opérations arithmétiques qui peuvent être effectuées avec des nombres scalaires peuvent être effectuées avec des nombres complexes, incluant les puissances, les racines, la résolution simultannée d'équations avec des coefficients complexes et même des fonctions trigonométriques (bien que celà implique une perspective complètement nouvelle en trigonométrie appelée *fonction hyperbolique* qui est bien au-delà du champ de cette discussion). Assurez-vous d'être familier avec les opérations arithmétiques de base d'additions, soustractions, multiplications, divisions, inversions et vous avez peu de problèmes avec l'analyse de circuits AC.

- **RÉSUMÉ :**

- Pour ajouter des nombres complexes sous forme rectangulaire, ajoutez les composantes réelles et ajoutes les composantes imaginaires. La soustraction est identique.

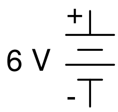
- Pour multiplier des nombres complexes sous forme polaire, multipliez les magnitudes et ajoutez les angles. Pour la division, divisez les magnitudes et soustrayez un angle de l'autre.

2.7 Plus sur la "polarité" AC

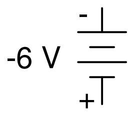
Les nombres complexes sont utiles pour l'analyse de circuits AC car elles fournissent une méthode adaptée de notation symbolique de décalage de phase entre des quantités AC comme la tension et le courant. Néanmoins, pour la plupart des personnes, l'équivalence entre des vecteurs abstraits et des circuits réels n'est pas facile à apréhender. Nous l'avons vu plus tôt dans ce chapitre comment les sources de tension AC sont des formes de tension données sous forme complexe (magnitude *et* angle de phase),de même que les marquages de polarité. Comm e le courant alternatif n'a pas de "polarité" comme peut l'avoir le courant continu, ces marques de polarité et leurs relations avec l'angle de phase tendent à être emmbrouillant. Cette section est écrite pour clarifier quelques uns de ces sujets.

La tension est, par nature, une quantité *relative*. Lorsque nous mesurons une tension, nous avons le choix sur la manière de connecter un voltmètre ou tout autre instrument de mesure de tension, car il existe deux points entre lesquels la tension existe et deux pointes de test sur l'instrument avec lequel nous effectuons la connexion. Dans les circuits DC, nous avons la polarité d'une source de tension et, explicitement, une chute de tension, en utilisant les symboles "+", "-" et en utilisant des pointes de test de mesure colorisées (rouge et noir). Si un voltmètre numérique indique une tension DC négative, nous savons que ses pointes de test sont connectées "à l'envers" sur la tension (la pointe rouge connectée au "-" et la pointe noire au "+").

Les batteries dont des polarités conçues par le biais de symboles intrinsèques : le côté de la batterie avec un trait court est toujours le côté négatif (-) et le côté avec la longue ligne est toujours le positif (+) :



Bien qu'il serait mathématiquement correct de représenter une tension de batterie comme une valeur négative avec des marquages de polarités inversées, cela serait incontestablement non conventionel :

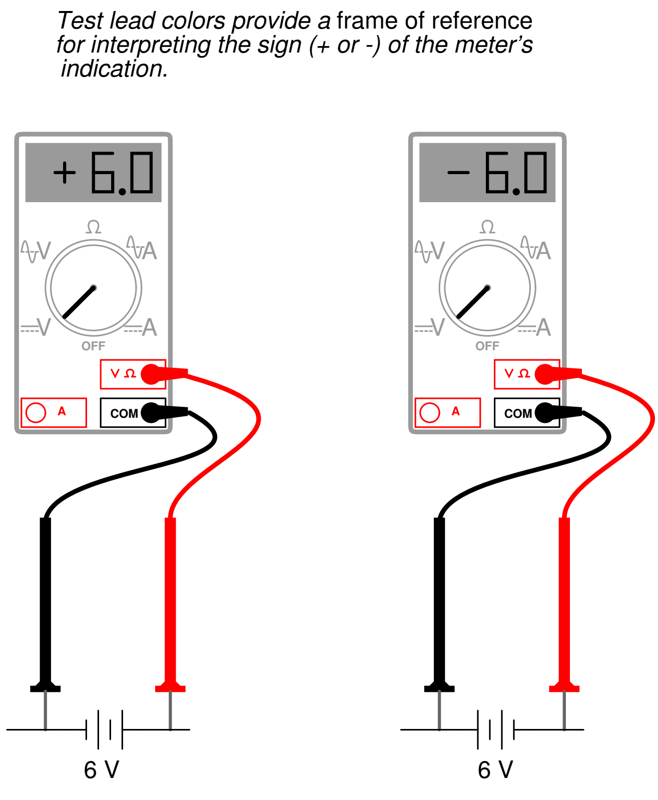


L'interprétation de cette notation peut être plus facile sur les marquages de polarités "+" et "-" vus comme des points de référence pour les pointes de test de voltmètres, le "+" signifiant "rouge" et le "-" signifiant "noir". Un voltmètre connecté à la batterie ci-dessus avec la pointe rouge sur la terminaison basse et la pointe noire à la terminaison haute devra alors indiquer une tension négative (-6 volts). Cette forme de notation et d'interprétation n'est pas réellement aussi inhabituelle que vous pourriez le penser : elle est habituellement rencontrée dans l'analyse de problèmes réseaux DC où les marques de polarités "+" et "-" sont initialement dessinées en fonction de suppositions formées et interprétées plus tard comme correctes ou "backward" selon le signe mathématique des chiffres calculés.

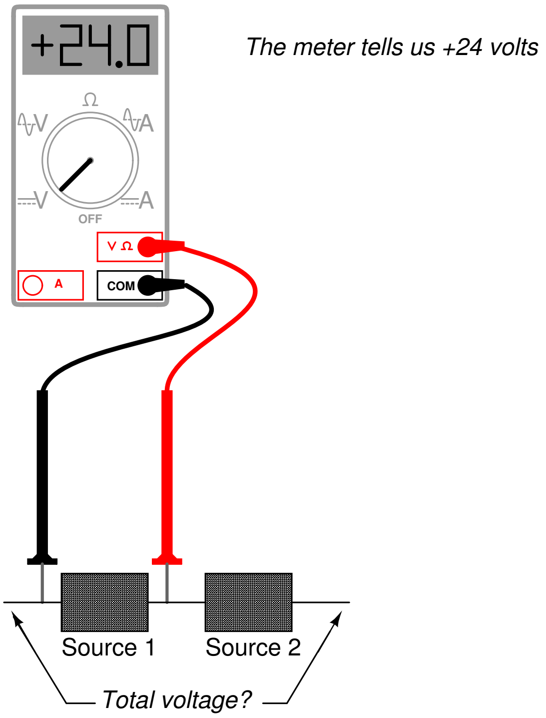
Dans les circuits AC, néanmoins, nous ne traitons pas de valeurs "négatives" de tension. À la place, nous décrivons à quel degré une tension aide ou s'oppose à une autre par la *phase* : le décalage temporel entre deux formes d'ondes. nous ne décrivons jamais une tension AC comme ayant un signe négatif car la facilité de la notation polaire permet aux vecteurs de pointer dans une direction opposée. Si une tension AC s'oppose directement à une autre tension AC, nous disons simplement que l'une est décalé de 180^o avec l'autre.

Toutefois, la tension est relative entre deux points et nous avons le choix sur la manière de connecter un instrument de mesure de tension entre ces deux points. Le signe mathématique d'une lecture de tension d'un voltmètre DC n'a de signification que dans le contexte de ses connexions de pointes de test : sur quels terminaux touchent les pointes de test rouge et noire. De même, l'angle de phase d'une tension AC n'a seulement de signification que dans le contexte de savoir lequel des deux points est considéré comme point de "référence". À cause de ce fait, les marques de polarités "+" et "-" sont souvent placées par les terminaux d'une tension AC dans les schémas pour donner à l'angle de phase une trame de référence.

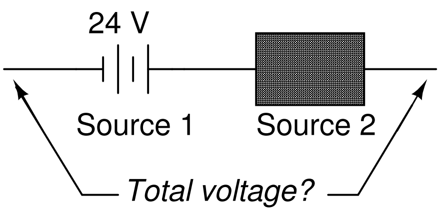
Résumons ces principes avec une aide graphique. D'abord, le principe de relier les connexions de pointes de test au signe mathématique d'une indication d'un voltmètre DC :



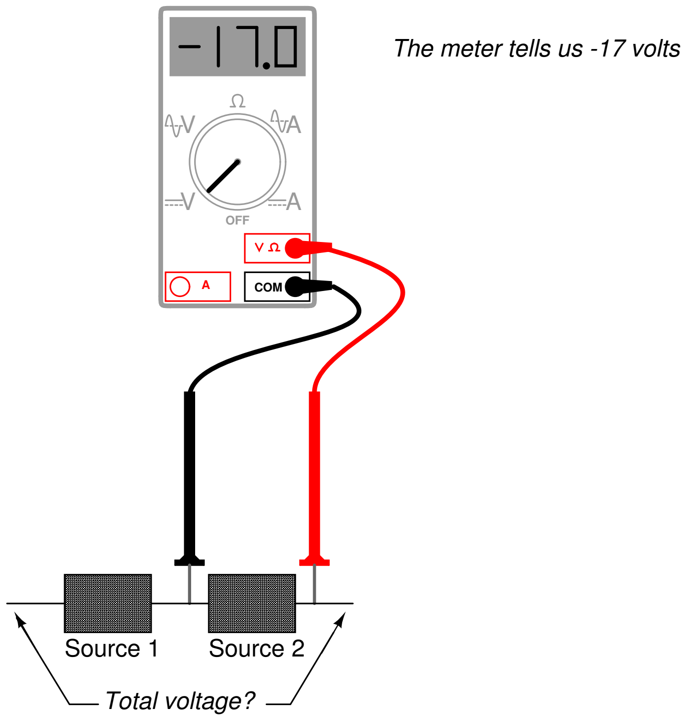
Le signe mathématique de l'affichage numérique d'un voltmètre DC n'a de signification que dans le contexte de ses pointes de test. Considérez l'utilisation d'un voltmètre DC en déterminant si les deux sources de tension DC s'ajoutent ou s'opposent l'une à l'autre, en supposant que les sources ne sont pas marquées, ni leur polarités. En utilisant le voltmètre pour mesurer la première source :



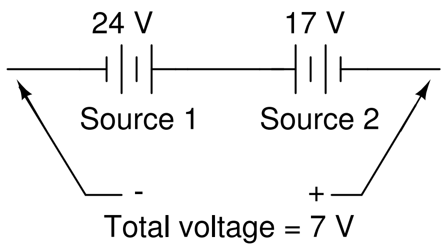
Cette première mesure de +24 sur la source de tension à gauche nous indique que la pointe noire du voltmètre touche réellement le côté négatif de la source de tension #1 et la pointe rouge touche réellement le positif. Maintenant, nous savons que la source #1 est une batterie avec cette orientation :



La mesure des autres sources de tension inconnues :

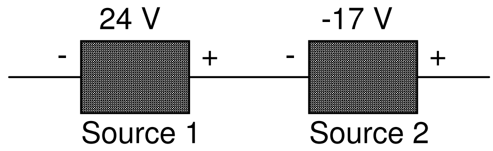


Cette lecture du second voltmètre, néanmoins, est un *négatif* (-) 17 volts, qui nous indique que la pointe de test noire touche réellement le côté positif de la source de tension #2, alors que la pointe de test rouge touche réellement la négative. Nous savons que la source #2 est une batterie avec une direction *opposée* :



Il devrait être évident à tout étudiant en électricité DC expérimenté que ces deux batteries s'opposent l'une à l'autre. Par définition, les tensions opposées se *soustraient* l'une à l'autre, nous soustrayons donc 17 volts à 24 volts pour obtenir une tension totale au travers des deux : 7 volts.

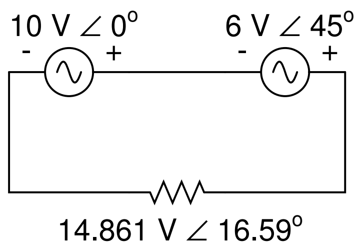
Nous pouvons, néanmoins, dessiner les deux sources indéfinies comme des boîtes, labelisées avec les chiffres de tension exacts obtenus par le voltmètre, les marques de polarité indiquent le placement des pointes de test du voltmètre :



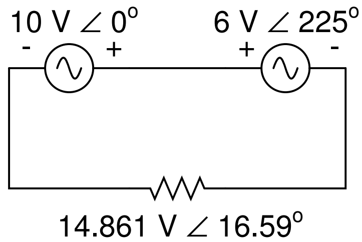
Selon ce schéma, les marques de polarités (qui indiquent le placement des pointes de test de l'instrument) indiquent que les sources *s'ajoutent* l'une à l'autre. Par définition, l'ajout de sources de tensions *aditionnées* l'une avec l'autre forment la tension totale, nous ajoutons donc 24 volts à -17 volts pour obtenir 7 volts : encore la bonne réponse. Si nous laissons les marquages de polarité guider notre décision pour soit ajouter ou soustraire des valeurs de tension – que ces marques de polarité représentent la *vraie* polarité ou seulement l'orientation des pointes de test de l'instrument – et inclure les signes mathématiques de ces valeurs de tensions dans nos calculs, le résultat sera toujours correct. Les marquages de polarité servent de *trames de référence* pour placer les signes mathématiques des valeurs de tension dans le contexte correct.

La même chose est vrai pour les tensions AC, excepté que les *angles de phase* se substituent aux *signes* mathématiques. De manière à lier des tensions AC multiples à des angles de phase différents, nous avons besoin de marquages de polarité pour fournir des trames de référence pour ces angles de phase de tensions.

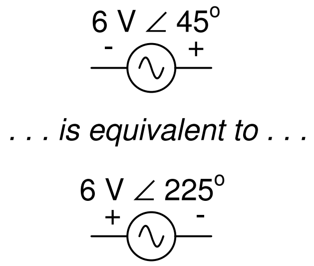
Prenons par exemple le circuit suivant :



Les marquages de polarité montrent ces deux sources de tension s'ajoutant l'une à l'autre, donc pour déterminer la tension totale sur la résistance, nous devons *ajouter* les valeurs de tension de 10 V $\angle$ 0^o et 6 V $\angle$ 45^o ensemble pour obtenir 14.861 V $\angle$ 16.59^o. Néanmoins, il serait parfaitement acceptable de représenter la source 6 volt comme 6 V $\angle$ 225^o, avec marquage de jeu de polarité inversé et encore obtenir la même tension totale :



6 V $\angle$ 45^o avec un négatif sur la gauche et un positif sur la droite est exactement le même que 6 V $\angle$ 225^o avec un positif sur la gauche et un négatif sur la droite : l'inversion des marquages de polarité complémentent parfaitement l'addition de 180^o à la désignation de l'angle de phase :

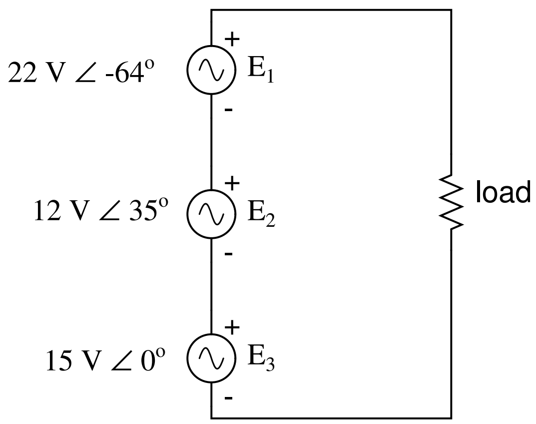


Contrairement aux sources de tension DC, dont les symboles définissent intrinsèquement les polarités par le moyen de lignes longues et courtes, les symboles de tensions AC n'ont pas de marquages de polarité intrinsèques. Donc, tout marquage de polarité doit être inclus comme symbole additionel sur le schéma et il n'existe pas de manière "correcte" pour les placer. Ils doivent, néanmoins, se correler avec un cun angle de phase donné pour représenter la vraie relation de phase de cette tension avec les autres dans le circuit.

- **RÉSUMÉ :**
- Les marquages de polarité sont quelque fois donnés au tensions AC dans les schémas de manière à fournir une trame de référence pour leurs angles de phase.

2.8 Quelques exemples avec des circuits AC

Connectons les trois sources de tension AC en série et utilisons les nombres complexes pour déterminer l'addition des tensions. Toutes les règles et lois apprises dans l'étude des circuits DC s'appliquent aussi aux circuits AC (la Loi d'Ohm, les Lois de Kirchhoff, les méthodes d'analyse de réseau), à l'exception des calculs de puissance (Loi de Joule). La seule qualification est que toutes les variables *doivent* être exprimées sous la forme complexe, en prenant en compte la phase de même que la magnitude et toutes les tensions, les courants doivent être de la même fréquence (de manière à ce que leurs relations de phase restent constantes).

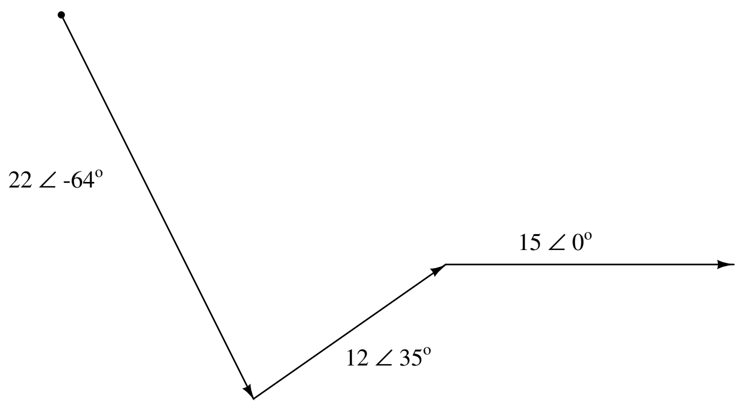


Les marquages de polarité pour toutes les trois sources de tension sont orientées d'une telle manière que leurs tensions établies doivent s'ajouter pour avoir la résistance totale aux bornes de la résistance de charge. Notez que bien que la magnitude et l'angle de phase soit donné pour chaque source de tension AC, aucune valeur de fréquence n'est spécifiée. Si c'est le cas, il est supposé que toutes les fréquences sont égales, et donc qu'elles satisfont à nos qualifications pour appliquer des règles DC à un circuit AC (toutes les valeurs sont données sous forme complexe, toutes possèdent la même fréquence). L'initialisation des équations pour trouver la tension totale apparaissent tel que :

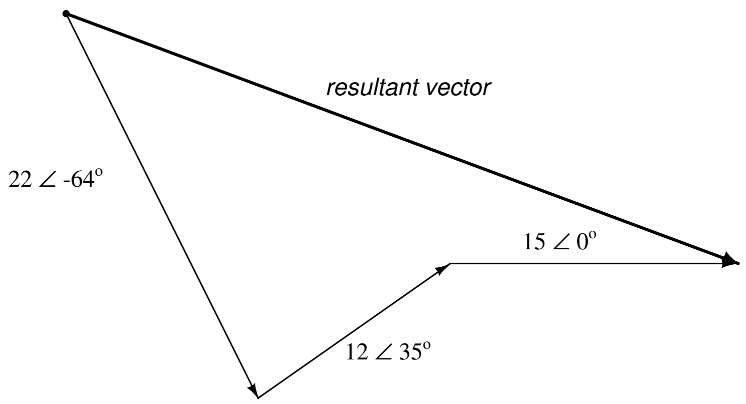
$E_{total} = E_1 + E_2 + E_3$

$E_{total} = (22V \angle \text{-}64^o) + (12V \angle 35^o) + (15V \angle 0^o)$

Les vecteurs s'ajoutent graphiquement de cette manière :



La somme de ces vecteurs sera un vecteur résultant avec un point de départ d'origine pour le vecteur de 22 volt (le point au bord haut-gauche du schéma) et terminant sur le point de terminaison pour le vecteur de 15 volt (la pointe de la flèche au milieu-droit du schéma) :



De manière à déterminer le vecteur magnitude et l'angle résultant sans faire appel aux graphiques, nous pouvons convertir chacun de ces nombres complexes à forme polaire en une forme rectangulaire et additionner. Rappelez-vous que nous *ajoutons* ces valeurs ensembles car les marquages de polarité pour ces trois sources de tension sont orientées d'une manière additive :

$15V \angle 0^o = 15 + j0V$

$12V \angle \text{-}35^o = 9.8298 + j6{,}8829V$

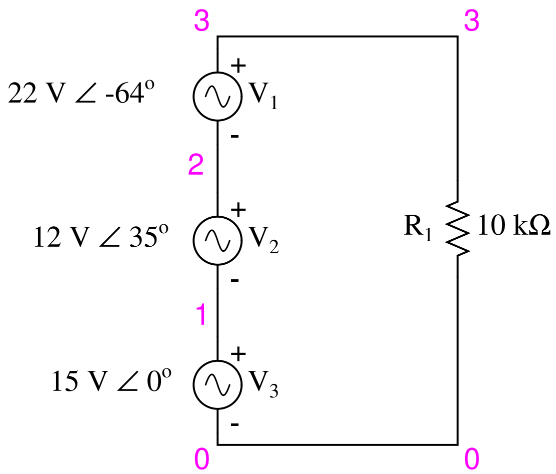
$22V \angle \text{-}64^o = 9{,}6442 + j19{,}7735V$

	15	+ j0 V
	9.8298	+ j6.8829 V
+	9.6442	- j19.7735 V
	34.4740	**- j12.8906 V**

Dans la forme polaire, cela équivaux à 36.8052 volts $\angle$ -20.5018^o. Ce qui signifie en termes réels que la tension mesurée aux bornes de ces trois sources de tension sera de 36.8052 volts, en retard sur le 15 volt (à 0^o de référence de phase) par 20.5018^o. Un voltmètre connecté à ces points dans un circuit réel indiquera seulement la magnitude polaire de la tension (36.8052 volts), pas l'angle. Un oscilloscope pourrait être utilisé pour afficher les deux formes d'ondes de tension et donc fournir une mesure de décalage de phase mais pas un voltmètre. Le même principe reste vrai pour les ampèremètres AC : ils indiquent la magnitude polaire du courant, pas l'angle de phase.

C'est extrèmement important pour lier les valeurs calculées de tension et de courant pour les circuits réels. Bien que la notation rectangulaire est adaptée pour l'addition et la soustraction et était donc l'étape finale dans notre problème d'exemple ici, elle n'est pas très applicable pour les mesures pratiques. Les valeurs rectangulaires doivent être converties en valeurs polaires (spécifiquement la *magnitude* polaire) avant qu'elles puissent être reliées aux mesures de circuit réel.

Nous pouvons utiliser SPICE pour vérifier l'exactitude de nos résultats. Dans ce circuit test, la valeur de la résistance de 10 kΩ est arbitraire. Elle a été placée de telle manière que SPICE ne déclare pas d'erreur de circuit ouvert et ne quitte l'analyse. De même, le choix des fréquences pour la simulation (60 Hz) est arbitraire car les résistances répondent uniformément pour toutes les fréquences de tensions et de courant AC. Il existe d'autres composants (particulièrement les capacités et les inductances) qui ne répondent pas uniformément aux différentes fréquences mais c'est un autre sujet!




```
ac voltage addition
v1 1 0 ac 15 0 sin
v2 2 1 ac 12 35 sin
v3 3 2 ac 22 -64 sin
```

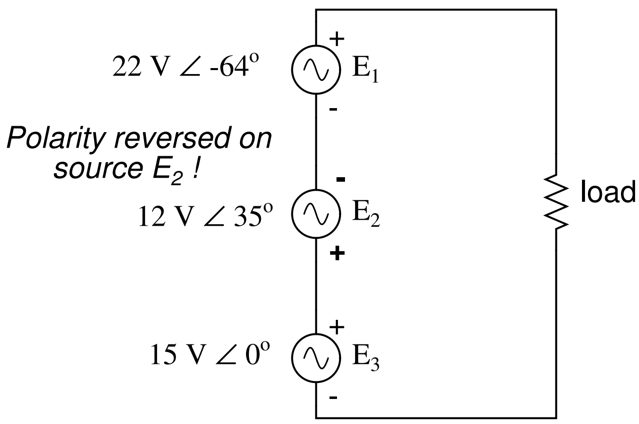
```
r1 3 0 10k
.ac lin 1 60 60           J'utilise une fréquence de 60 Hz
.print ac v(3,0) vp(3,0)   comme valeur par défaut
.end
```

```
freq          v(3)         vp(3)
6.000E+01     3.681E+01   -2.050E+01
```

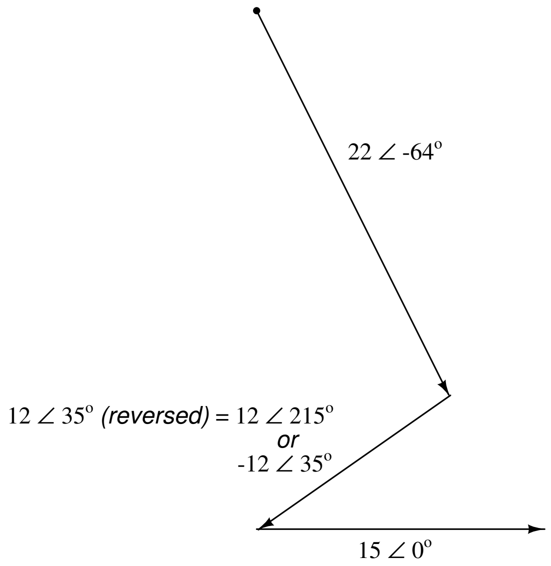
Nous obtenons donc une tension totale de 36.81 volts $\angle$ -20.5^o (avec comme référence la source de 15 volt, dont l'angle de phase était positionné arbitrairement à zéro degrés pour être la forme d'onde de "référence").

Au premier regard, cela n'est pas intuitif. Comment est-il possible d'obtenir une tension totale de plus de 36 volts avec des alimentations de 15 volt, 12 volt et 22 volt connectées en série? Avec le DC, ce serait impossible, car les valeurs de tension s'ajoutent ou se soustraient directement en fonction de leur polarité. Mais avec l'AC, notre "polarité" (le décalage de phase) peut varier entre le complément complet et l'opposition complète et ceci permet de telles sommes paradoxales.

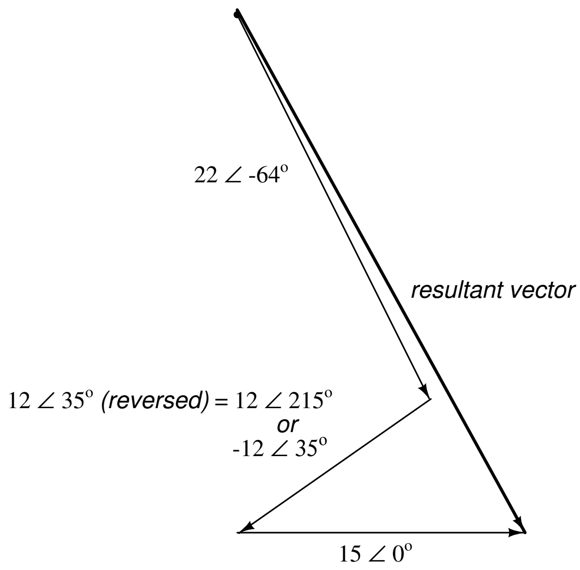
Que se passe-t-il lorsque nous prenons le même circuit et que nous inversons les connexions de l'alimentation? Sa contribution à la tension totale sera alors l'opposée de ce qu'elle était avant :



Notez comment l'angle de phase de l'alimentation de 12 volt a encore 35^o comme référence, même avec l'inversion des broches. Rappelez-vous que l'angle de phase de toute chute de tension est établie en référence à sa polarité notée. Même si l'angle est encore noté 35^o, le vecteur sera dessiné à 180^o à l'opposé de ce qu'il était avant :



Le vecteur résultant (somme) doit débuter au point haut-gauche (origine du vecteur de 22 volt) et se terminer sur la pointe de la flèche droite du vecteur de 15 volt :



L'inversion de connexion sur l'alimentation de 12 volt peut être représenté de deux manière différentes sous la forme polairee : par une addition de 180^o à son angle de vecteur (réalisant un 12 volts $\angle$ 215^o) ou en inversant le signe de la magnitude (réalisant un -12 volts $\angle$ 35^o). L'autre manière, la conversion vers une forme rectangulaire donne le même résultat :

12 V $\angle$ 35^o *(reversed)* = 12 V $\angle$ 215^o = **-9.8298 - j6.8829 V**

or

-12 V $\angle$ 35^o = **-9.8298 - j6.8829 V**

L'addtion de tension résultante sous la forme rectangulaire puis :

$$
\begin{array}{lrl}
 & 15 & + j0 \quad V \\
 & -9.8298 & - j6.8829\ V \\
+ & 9.6442 & - j19.7735\ V \\
\hline
 & \mathbf{14.8143} & \mathbf{- j26.6564\ V}
\end{array}
$$

Dans la forme polaire, cela équivaut à 30.4964 V $\angle$ -60.9368^o. Encore une fois, nous utilisons SPICE pour vérifier le résultat de nos calculs :

```
ac voltage addition
v1 1 0 ac 15 0 sin
v2 1 2 ac 12 35 sin    Notez l'inversion des numéros de noeuds 2 et 1
v3 3 2 ac 22 -64 sin  pour simuler l'inversion des connections
r1 3 0 10k
.ac lin 1 60 60
.print ac v(3,0) vp(3,0)
.end

freq          v(3)        vp(3)
6.000E+01     3.050E+01  -6.094E+01
```

- **RÉSUMÉ :**
- Toutes les lois et règles des circuits DC s'appliquent aux circuits AC, à l'exception des calculs de puissance (Loi de Joule), aussi longtemps que toutes les valeurs sont exprimées et manipulées sous forme complexe et toutes les tensions et courants sont à la même fréqence.
- Lors de l'inversion de la direction d'un vecteur (équivalent à inverser la polarité d'une source de tension AC en relation à d'autres sources de tension), il peut être exprimé de deux manières différences : ajouter 180^o à l'angle ou inverse le signe de la magnitude.
- Les mesures de multimètres dans un circuit AC correspondent aux *magnitudes polaires* des valeurs calculées. Les expressions rectangulaires des quantités complexes dans un circuit AC n'ont pas d'équivalent direct, empirique, bien qu'ils soient adaptés pour effectuer les additions et les soustractions comme le demandent les Loi de Tension et de Courant de Kirchhoff.

2.9 Contributeurs

Les contributeurs de ce chapitre sont listés dans l'ordre chronologique de leurs contributions, depuis le plus récent jusqu'au premier. Voyez l'Annexe 2 (Liste des contributeur) pour les dates et les informations de contact.

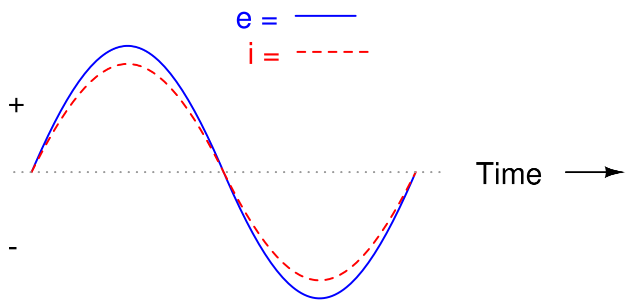
Jason Starck (June 2000) : HTML document formatting, which led to a much better-looking second edition.

Chapter 3

RÉACTANCE ET IMPÉDANCE – INDUCTIVE

3.1 Circuits résistants AC

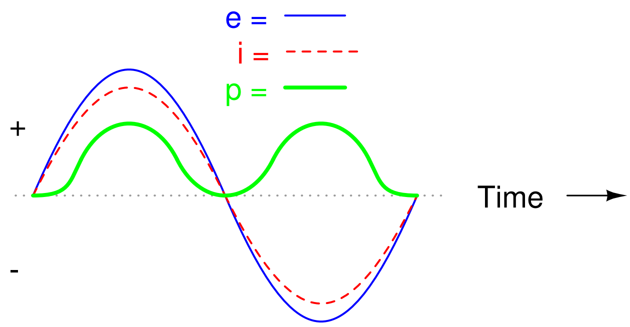
Si nous devions dessiner le courant et la tension pour un circuit AC très simple consistant en une source et une résistance, cela ressemblerait à quelque chose comme ceci :



Comme une résistance résiste simplement et directement au flux des électrons en permanence, la forme de l'onde de la chute de tension aux bornes de la résistance est exactement en phase avec la forme de l'onde du courant qui la traverse. Nous pouvons regarder à tout point du temps sur l'axe horizontal du tracé et comparer les valeurs du courant et de la tension l'un avec l'autre (tout point "instantané"des valeurs d'une onde est appelé *valeur instantanée*, signifiant que les valeurs sont à un *instant* dans le temps). Lorsque la valeur instantannée du courant est zéro, la tension instantanée aux bornes de la résistance est aussi zéro. De même, au moment où le courant dans la résistance est à son pic positif, la tension aux bornes de la résistance est aussi à son pic positif, ainsi de suite.

À tout point donné de l'onde, la Loi d'Ohm reste vraie pour toutes les valeurs instantannées de la tension et du courant.

Nous pouvons aussi calculer la puissance dissipée par cette résistance et tracez ces valeurs sur le même graphe :



Notez que la puissance n'est jamais une valeur négative. Lorsque le courant est positif (au-dessus de la ligne), la tension est aussi positive, le résultat d'une puissance est une valeur positive (p=ie). Inversement, lorsque le courant est négatif (en dessous de la ligne), la tension est aussi négative, ce qui a pour résultat une valeur positive de la puissance (un nombre négatif multiplié par un nombre négatif égale un nombre positif). Cette "polarité" cohérente de la puissance nous indique que la résistance dissipe toujours de la puissance, la prenant de la source et la relachant sous forme d'énergie. Que le courant soit positif ou négatif, une résistance dissipe toujours de l'énergie.

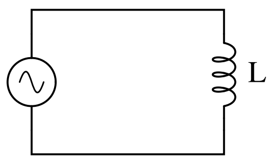
3.2 Circuit d'inductance AC

Les inductances ne se comportent pas comme des résistances. Alors que les résistances s'opposent simplement au flux des électrons qui les traverse (en faisant chuter une tension directement proportionnelle au courant),les inductances s'opposent aux *changements* en courant, en faisant chuter la tension directement proportionnellement au *taux de changement* du courant. En accord avec la *Loi de Lenz*, cette tension induite est toujours de telle polarité qu'elle tente de maintenir le courant à sa valeur présente. Ceci étant, si le courant augmente en magnitude, la tension induite va "s'opposer" au flux d'électron; si le courant diminue, la polarité sera inversée et "facilitera" le flux des électrons pour s'opposer à la diminution. Cette opposition au changement de courant est appelée *réactance*, plutôt que résistance.

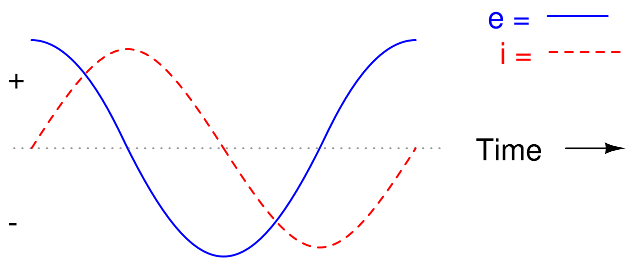
Exprimé mathématiquement, la relation entre la chute de tension aux bornes de l'inductance et le taux de changement du courant est tel que :

$$e = L\frac{di}{dt}$$

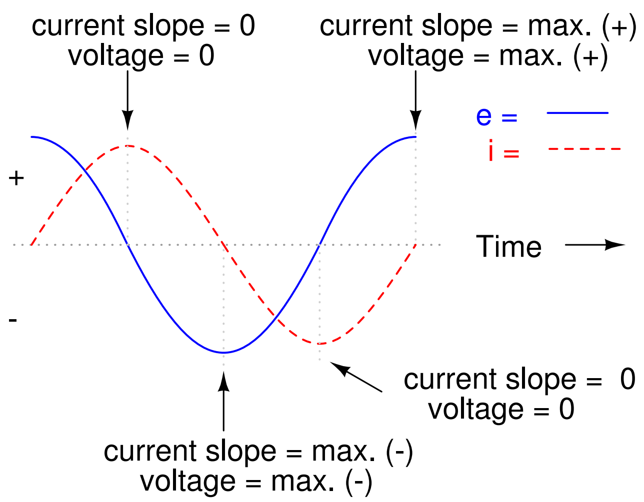
L'expression *di/dt* est arithmétique, signifiant que le taux de changement du courant instantanné (i) au cours du temps, en amps par seconde. L'inductance (L) est en Henrys et la tension instantannée (e) est, bien sûr, en volts. Vous trouverez quelques fois le taux de la tension instantannéeexprimée comme "v" au lieu de "e" (v = L di/dt) mais cela signifie exactement la même chose. Pour montrer ce qui se produit avec le courant alternatif, analysons un circuit avec inductance simple :



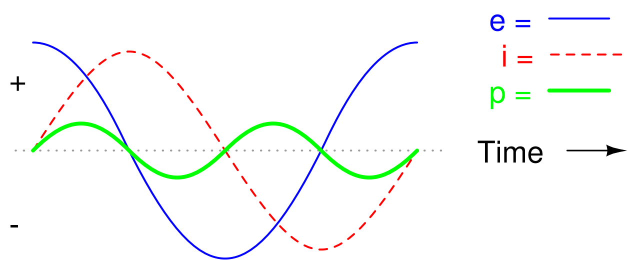
Si nous devions tracer le courant et la tension pour ce circuit très simple, il ressemblerait à quelque chose comme ceci :



Rappelez-vous que la chute de tension dans une inductance est une réaction contre le *changement* de courant qui y circule. C'est la raison pour laquelle la tension instantanée est à zéro alors que le courant instantané est au pic (passage au zéro ou pente de niveau, sur l'onde sinus de courant) et la tension instantanées est au pic lorsque le courant instantané est au changement maximum (le point où la pente est la plus raide sur l'onde courante, lorsqu'il passe le zéro). Cela donne comme résultat une onde de tension qui est déphasée de 90^o avec l'onde de courant. En regardant le graphe, l'onde de tension semble avoir un "démarrage haut" sur l'onde de courant; la tension est en "avance" sur le courant et le courant est en "retard" derrière la tension.



Les choses sont encore plus intéressantes lorsque nous traçons la puissance pour ce circuit :



Comme la puissance instantanée est le produit de la tension instantanée et du courant instantané (p=ie), la puissance vaut zéro lorsque le courant *ou* la tension instantané est à zéro. Lorsque le courant et la tension instantanés sont tous les deux positifs (au-dessus de la ligne), le puissance est positive. Comme avec l'exemple de la résistance, la puissance est aussi positive lorsque le courant et la tension instantanné sont tous les deux négatifs (en-dessous de la ligne). Néanmoins, comme les ondes de courant et de tension sont déphasés de 90^o, il existe des zones où l'une est positive alors que l'autre est négative, avec pour résultat des occurences de fréquences égales de *puissance instantanée négative*.

Mais que signifie une puissance *négative*? Cela signifie que l'inductance réinjecte la puissance dans le circuit, alors que la puissance positive signifie qu'il absorbe de la puissance depuis le circuit. Comme les cycles de puissance positive et négative sont égales en magnitude et durée dans le temps, l'inductance relâche juste la puissance dans le circuit absorbée lors du cycle complet. Ce qui signifie, au sens pratique, que la réactance d'une inductance dissipe une énergie nette de zéro, contrairement à une résistance, qui dissipe l'énergie sous forme de chaleur. Rappelez-vous que c'est seulement pour les inductances parfaite, qui n'ont aucune résistance de fil.

Comme l'opposition au changement du courant dans une inductance se transforme en une opposition au courant alternatif en général, qui est par définition toujours changeant en magnitude et direction instantanée. Cette opposition au courant alternatif est similaire à la résistance mais différent dans le fait qu'il resulte toujours en un décalage de phase entre le courant et la tensio et il ne dissipe pas de puissance. À cause de ces différences, elle a une un nom différent : *réactance*. La réactance à l'AC est exprimée en ohms, comme l'est la résistance, excepté que son symbole mathématique est X au lieu de R. Pour être spécifique, la réactance associée avec une inductance est habituellement symbolisée par la lettre capitale X avec une lettre L en indice, comme ceci : X_L.

Comme la chute de tension de l'inductance est proportionnelle au taux de changement de courant, elle chutera plus de tension pour les changements rapides de courant et moins de tension pour les changements de courants moins rapides. Cela signifie que la réactance en ohms pour toutes les inductances est directement proportionelle à la fréquence du courant alternatif. La formule exacte pour déterminer une réactance est la suivante :

X_L=2ΠfL

Si nous exposons une inductance de 10 mH aux fréquences de 60, 120 et 2500 Hz, elle aura les réactances suivantes :

Pour une inductance de 10 mH :

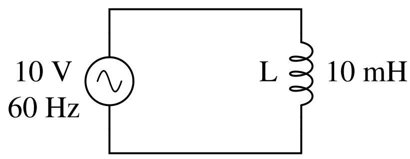
Fréquence (Hertz)	Réactance (Ohms)
60	3.7699
120	7.5398
2500	157.0796

Dans l'équation de réactance, le terme "2πf" (tout ce qui est du côté droit, excepté le L) a une signification spéciale en lui-même. C'est le nombre de radians par secondes dont "tourne" le courant alternatif, si vous imaginez qu'un cycle AC représente une rotation complète de cercle. Un *radian* est une unité de mesure angulaire : il y a 2π radians dans un cercle complet, de même qu'il y a 360^o dans un cercle complet. Si l'alternateur produisant l'AC est une unité à double pôle, il produira un cycle pour chaque tour complet de rotation de l'axe, à chaque 2π radians ou 360^o. Si cette constante de 2π est multipliée par la fréquence en Hertz (cycles par seconde), le résultat sera une valeur en radians par seconde, connu comme la *vitesse angulaire* du système AC.

La vitesse angulaire peut être représentée par l'expression 2πf ou peut être représentée par son propre symbole, la lettre Grecque, en minuscule, Omega qui apparaît similaire à notre "w" Romain en minuscule : ω. La formule de réactance $X_L = 2\pi fL$ peut donc aussi être écrite comme $X_L = \omega L$.

Il doit être compris que cette "vitesse angulaire" est une expression sur la rapidité de réalisation de cycles de formes d'onde AC, un cycle complet étant égal à 2π radians. Elle n'est pas nécessairement représentative de la vitesse réelle de l'axe de l'alternateur produisant l'AC. Si l'alternateur possède plus de deux pôles, la vitesse angulaire sera un multiple de la vitesses de l'axe. Pour cette raison, ω est quelque fois exprimée en unités de radians *électriques* par seconde plutôt qu'en radians simples par seconde, pour les distinguer du déplacement mécanique.

Quelque soir la manière dont nous exprimons la vitesse angulaire, il est apparent que c'est directement proportionel à la réactance dans une inductance. Lorsque la fréquence (ou la vitesse de l'axe de l'alternateur) augmente dans un système AC, une inductance offrira une opposition plus grande au passage du courant et vis-versa. L'alternance du courant dans un circuit inductif simple est égal à la tension (en volts) divisée par la réactance inductive (en ohms), juste comme le courant alternatif ou continu dans un dans un circuit résistif simple est égal à la tension (en volts) divisé par la résistance (en ohms). Un circuit d'exemple est montré ici :



(inductive reactance of 10 mH inductor at 60 Hz)

$$X_L = 3.7699\ \Omega$$

$$I = \frac{E}{X}$$

$$I = \frac{10\ \text{V}}{3.7699\ \Omega}$$

$$I = 2.6526\ \text{A}$$

Nous avons néanmoins besoin de garder en mémoire que la tension et le courant ne sont pas en phase ici. Comme nous l'avons montré ici, la tension possède un décalage de phase de $+90^o$ en fonction du courant. Si nous représentons ces angles de phase de tension et de courant mathématiquement sous la forme de nombres complexes, nous trouvons que l'opposition d'une inductance au courant possède aussi un angle de phase :

$$\text{Opposition} = \frac{\text{Voltage}}{\text{Current}}$$

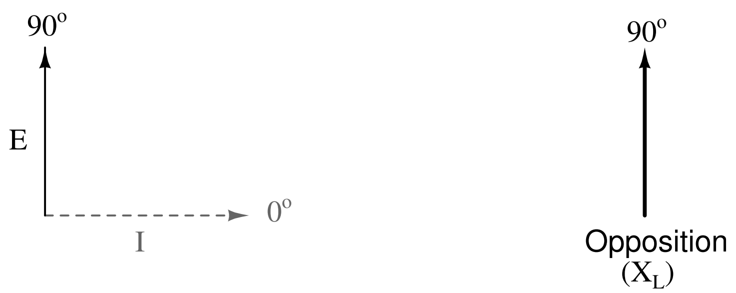
$$\text{Opposition} = \frac{10\ \text{V} \angle 90^o}{2.6526\ \text{A} \angle 0^o}$$

$$\text{Opposition} = 3.7699\ \Omega \angle 90^o$$

or

$$0 + j3.7699\ \Omega$$

For an inductor:



Mathématiquement, nous disons que l'angle de phase de l'opposition au courant d'une inductance est de 90^o, signifiant qu'une opposition au courant d'une inductance est une quantité imaginaire positive. Cet angle de phase d'oppposition réactive au courant devient critiquement important dans l'analyse de circuit, spécialement pour les circuits complexes AC où la réactance et la résistance interragissent. Il se révèlera bénéfique de représenter *toute* opposition du composant au courant en termes de nombres complexes plutôt qu'en valeurs scalaires de résistance et de réactance.

- **RÉSUMÉ :**

- La *réactance inductive* est l'opposition qu'une inductance offre au courant alternatif dû à son stockage et à sa restitution d'énergie, déphasée, dans son champ magnétique. La réactance est symbolisée par la majuscule "X" et est mesurée en ohms comme la résistance (R).

- La réactance inductive peut être calculée en utilisant la formule : $X_L = 2\pi fL$

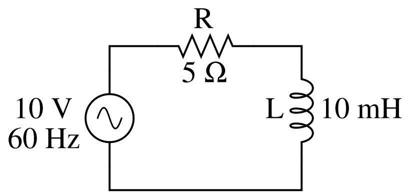
- La *vitesse angulaire* d'un circuit AC est une autre manière d'exprimer sa fréquence, en unités de radians électriques per seconde au lieu de cycles par seconde. Elle est symbolisée par la lettre minuscule Grecque "omega" ou ω.

- La réactance inductive *augmente* avec l'augmentation de fréquence. En d'autres mots, plus haute est la fréquence, plus elle s'oppose au flux AC des électrons.

3.3 Circuits de résistances-inductances séries

Dans la section précédente, nous avons exploré ce qui se passerait dans un circuit AC avec des circuits avec une seule résistance et une seule inductance. Nous allons maintenant mixer les deux composants ensemble sous la forme série et vérifier les effets.

Prenez ce circuits pour travailler avec :



La résistance offrira une résistance de 5 Ω au courant AC quelque soit la fréquence alors que l'inductance offrira 3.7699 Ω de réactance au courant AC à 60 Hz. Comme la résistance est un nombre réel de (5 Ω $\angle$ 0^o ou 5 + j0 Ω) et que la réactancede l'inductance est un nombre imaginaire (3.7699 Ω $\angle$ 90^o ou 0 + j3.7699 Ω), les effets combinés des effets des deux composants sera une opposition au courant égal à la somme complexe des deux nombres. Cette opposition combinée sera une combinaison de vecteurs de résistances et de réactance. De manière à exprimer succinctement cette opposition, nous avons besoin d'un terme plus facile à comprendre pour l'opposition au courant que soit une résistance, soit une réactance seule. Ce terme est appelé *impédance*, son symbole est Z et il est aussi exprimé dans l'unité des ohms, just comme la résistance et la réactance. Dans l'exemple ci-dessus, l'impédance de circuit totale est :

$$Z_{total} = (5\ \Omega \text{ resistance}) + (3.7699\ \Omega \text{ inductive reactance})$$

$$Z_{total} = 5\ \Omega\ (R) \quad + \quad 3.7699\ \Omega\ (X_L)$$

$$Z_{total} = (5\ \Omega \angle 0^o) + (3.7699\ \Omega \angle 90^0)$$

or

$$(5 + j0\ \Omega) + (0 + j3.7699\ \Omega)$$

$$Z_{total} = 5 + j3.7699\ \Omega \quad \textit{or} \quad 6.262\ \Omega \angle 37.016^o$$

L'impédance est reliée à la tension et au courant, comme vous pourriez vous y attendre, d'une manière similaire aux résistances de la Loi d'Ohm :

Ohm's Law for AC circuits:

$$\mathbf{E = IZ} \qquad \mathbf{I = \frac{E}{Z}} \qquad \mathbf{Z = \frac{E}{I}}$$

All quantities expressed in complex, not scalar, form

En fait, c'est une forme beaucoup plus étendue de la Loi d'Ohm qui a été apprise dans l'électronique DC (E=IR), juste comme l'impédance est une expression beaucoup plus étendue de l'opposition au flux des électrons que ne l'est une résistance. *Toute* résistance et toute réactance, séparément ou en combination (série/parallèle), peut être ou doit être représenté comme une impéance simple dans un circuit AC.

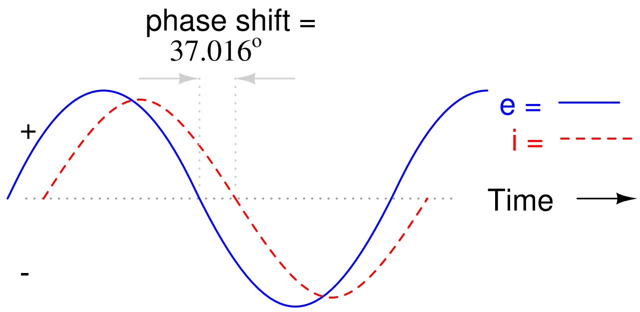
Pour calculer le courant dans le circuit ci-dessus, nous devons d'abord avoir un angle de phase de référence pour la source de tension, qui est généralement supposée être à zéro. (Les angles de phase de l'impédance résistive et inductive sont *toujours* à 0^o et $+90^o$, respectivement, en sans tenir compte des angles de phase pour la tension ou le courant).

$$I = \frac{E}{Z}$$

$$I = \frac{10\ V \angle 0^o}{6.262\ \Omega \angle 37.016^o}$$

$$I = 1.597\ A \angle -37.016^o$$

Comme avec un circuit purement inductif, l'onde de courant est en retard sur l'onde de tension (de la source), bien que cette fois, le retard ne soit pas aussi important : seulement 37.016^o à rapprocher d'un complet 90^o comme dans le cas d'un circuit purement inductif.



Pour la résistance et l'inductance, les relations de phase entre la tension et le courant n'ont pas changé. La tension sur la résistance est en phase (décalage de 0^o) avec le courant qui y passe; et la tension dans l'inductance est décalée de $+90^o$ avec le courant qui y passe. Nous pouvons le vérifier mathématiquement :

$$E = IZ$$

$$E_R = I_R Z_R$$

$$E_R = (1.597 \text{ A} \angle -37.016^o)(5\ \Omega \angle 0^o)$$

$$E_R = 7.9847 \text{ V} \angle -37.016^o$$

Notice that the phase angle of E_R is equal to the phase angle of the current.

La tension dans la résistance a exactement le même angle de phase que le courant, nous indiquant que E et I sont en phase (pour la résistance seulement).

$$E = IZ$$

$$E_L = I_L Z_L$$

$$E_L = (1.597 \text{ A} \angle -37.016^o)(3.7699\ \Omega \angle 90^o)$$

$$E_L = 6.0203 \text{ V} \angle 52.984^o$$

Notice that the phase angle of E_L is exactly 90^o more than the phase angle of the current.

La tension dans l'inductance possède un angle de phase de 52.984^o, alors que le courant qui passe dans l'inductance possède un angle de phase de -37.016^o, une différence de exactement 90^o entre les deux. Cela nous indique que E et I sont encore déphasés de 90^o (pour l'inductance seulement).

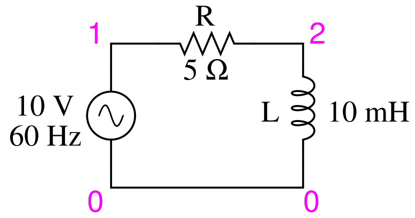
Nous pouvons aussi prouver mathématiquement que ces valeurs complexes s'ajoutent ensemble pour faire la tension totale, juste comme la Loi de Tension de Kirchhoff l'avait prédit :

$$E_{total} = E_R + E_L$$

$$E_{total} = (7.9847 \text{ V} \angle -37.016^o) + (6.0203 \text{ V} \angle 52.984^o)$$

$$E_{total} = 10 \text{ V} \angle 0^o$$

Contrôlons la validité de nos calculs avec SPICE :




```
ac r-l circuit
v1 1 0 ac 10 sin
r1 1 2 5
l1 2 0 10m
.ac lin 1 60 60
.print ac v(1,2) v(2,0) i(v1)
.print ac vp(1,2) vp(2,0) ip(v1)
.end
```

```
freq          v(1,2)       v(2)         i(v1)
6.000E+01     7.985E+00    6.020E+00    1.597E+00
freq          vp(1,2)      vp(2)        ip(v1)
6.000E+01    -3.702E+01    5.298E+01    1.430E+02
```

Interpreted SPICE results

$$E_R = 7.985 \text{ V} \angle -37.02^o$$

$$E_L = 6.020 \text{ V} \angle 52.98^o$$

$$I = 1.597 \text{ A} \angle 143.0^o$$

Notez qu'à l'identique des circuits DC, SPICE sort les valeurs de courant comme si elles étaient négatives (déphasage de 180^o) avec la tension d'alimentation. Au lieu d'un angle de phase de -37.016^o, nous avons un anglde phase de courant de 143^o (-37^o + 180^o). C'est seulement une particularité de SPICE et ne représente rien de significatif dans la simulation de circuit elle-même. Notez comment les lectures de phase de tension de la résistance et de l'inductance correspondent à nos calculs (-37.02^o et 52.98^o, respectivement), juste comme nous les avions supposés.

Avec tous ces chiffres à garder, même pour un circuit aussi simple que celui-ci, il nous serait bénéfique d'utiliser la méthode de la "table". L'application d'une table à ce simple circuit de résistance-inductance série sera fera comme suit. Nous dessinons d'abord une table pour les valeurs E/I/Z et nous y placons toutes les valeurs des composants en ces termes (en d'autres mots, n'insérez pas les valeurs réelles de la résistance ou de de l'inductance en Ohms et Henrys, respectivement dans la table; nous les convertissons plutôt en valeurs complexes d'impédance et écrivez-les dedans) :

	R	L	Total	
E			**10 + j0** **10** $\angle$ 0°	Volts
I				Amps
Z	**5 + j0** **5** $\angle$ 0°	**0 + j3.7699** **3.7699** $\angle$ 90°		Ohms

Bien que cela ne soit pas nécessaire, je trouve que cela aide d'écrire les *deux* formes, rectangulaires et polaires de chaque valeur dans la table. Si vous utilisez une calculatrice qui a la possibilité d'effectuer de l'arithmétique complexe sans le besoin de conversion entre les formes rectangulaires et polaires alors ces données suplémentaires ne sont pas nécessaires. Si vous êtes néanmoins forcés d'effectuer de l'arithmétique complexe "à la main" (les additions et les soustractions sous forme rectangulaire et les multiplications et divisions sous forme polaire), en écrivant chaque valeur dans les deux formes sera donc utile.

Maintenant que les valeurs "données" sont écrites dans leur emplacements respectifs de la table, nous pouvons le traiter comme avec le DC : déterminer l'impédance totale depuis les impédances individuelles. Comme c'est un circuit série, nous savons que l'opposition au flux des électrons (résistance *ou* impédance) s'ajoute pour forme l'opposition totale :

	R	L	Total	
E			10 + j0 10 $\angle$ 0°	Volts
I				Amps
Z	5 + j0 5 $\angle$ 0°	0 + j3.7699 3.7699 $\angle$ 90°	**5 + j3.7699** **6.262** $\angle$ 37.016°	Ohms

Rule of series circuits

$Z_{total} = Z_R + Z_L$

Maintenant que nous connaissons la tension et l'impédance totale, nous pouvons appliquer la Loi Ohm (I=E/Z) pour déterminer le courant total :

	R	L	Total	
E			10 + j0 10 ∠ 0°	Volts
I			**1.2751 - j0.9614** **1.597** ∠ -37.016°	Amps
Z	5 + j0 5 ∠ 0°	0 + j3.7699 3.7699 ∠ 90°	5 + j3.7699 6.262 ∠ 37.016°	Ohms

Ohm's Law

$$I = \frac{E}{Z}$$

Comme avec le DC, le courant total dans un circuit AC série est partagé également par touts les composants. Cela est encore vrai car dans un circuit série, il n'existe qu'un chemin pour le flux des électrons et le taux qui y passe reste uniforme. En conséquence, nous pouvons transférer les valeurs du courant dans la colonne pour la résistance et l'inductance :

	R	L	Total	
E			10 + j0 10 ∠ 0°	Volts
I	**1.2751 - j0.9614** **1.597** ∠ -37.016°	**1.2751 - j0.9614** **1.597** ∠ -37.016°	1.2751 - j0.9614 1.597 ∠ -37.016°	Amps
Z	5 + j0 5 ∠ 0°	0 + j3.7699 3.7699 ∠ 90°	5 + j3.7699 6.262 ∠ 37.016°	Ohms

Rule of series circuits:

$$I_{total} = I_R = I_L$$

Ce qui manque maintenant est la chute de tension dans la résistance et l'inductance, respectivement. On peut les trouver par l'utilisation de la Loi d'Ohm (E=IZ), appliquée verticalement dans chaque colonne de la table :

	R	L	Total	
E	**6.3756 - j4.8071** **7.9847** ∠ -37.016°	**3.6244 + j4.8071** **6.0203** ∠ 52.984°	10 + j0 10 ∠ 0°	Volts
I	1.2751 - j0.9614 1.597 ∠ -37.016°	1.2751 - j0.9614 1.597 ∠ -37.016°	1.2751 - j0.9614 1.597 ∠ -37.016°	Amps
Z	5 + j0 5 ∠ 0°	0 + j3.7699 3.7699 ∠ 90°	5 + j3.7699 6.262 ∠ 37.016°	Ohms
	Ohm's Law E = IZ	*Ohm's Law* E = IZ		

Et avec ceci, la table est complète. Les mêmes règles que nous avons appliqué dans l'analyse des circuits DC, s'appliquent aussi aux circuits AC, avec comme avertissement, que toutes les valeurs doivent représentées et calculées sous forme polaire plutôt que scalaire. Aussi longtemps que le décalage de phase est correctement représenté dans dans nos calculs, il n'y a pas de différence fondamentale sur la manière d'approcher l'analyse de base de circuit AC par rapport au DC.

Il est maintenant temps de revoir la relation entre ces valeurs calculées et les lectures données par les instruments de mesure réels pour la tension et le courant. Ici, les valeurs, directement reliées aux mesures réelles sont celles en *notation polaire*, pas rectangulaires! En d'autres mots, si vous connectez un voltmètre sur la résistance de ce circuit, il indiquera **7.9847** volts, pas 6.3756 (rectangulaire réel) ou 4.8071 (rectangulaire imaginaire) volts. Pour le décrire en termes graphiques, les instruments de mesure vous indiquent simplement quelles est la longueur du vecteur pour cette valeur particulière (tension ou courant).

La notation rectangulaire, si elle est adaptée pour les additions et les soustractions arithmétiques, est une forme de notation plus abstraite que la polaire pour les mesures réelles. Comme je l'ai dit plutôt, j'indiquerais, et la forme polaire, et rectangulaire de chaque valeur dans mes tables de circuits AC simplement pour faciliter les calculs mathématiques. Ce n'est pas absolument nécessaire mais peut être utile pour ceux qui nous suivent sans avoir de calculatrice élaborée. Si nous devions nous cantoner à l(utilisation d'une forme de notation, le meilleur choix serait le polaire car c'est le seul qui peut être directement correlé avec les mesures réelles.

- **RÉSUMÉ :**
- *l'Impédance* est la mesure totale de l'opposition électrique au courant et est la somme complexe (vecteur) de la résistance ("réelle") et de la réactance ("imaginaire"). Elle est symbolisée par la lettre "Z" et est mesurée en ohms, juste comme la résistance (R) et la réactance (X).
- Les impédances (Z) sont gérées comme les résistances (R) dans l'analyse d'un circuit série : les impédances séries s'ajoutent pour former l'impédance totale. Assurez -vous d'effectuer tous les calculs sous la forme complexe (pas scalaire)! $Z_{Total} = Z_1 + Z_2 + \ldots Z_n$
- Une impédance purement résistive aura toujours un angle de phase d'exactement 0^o (Z_R = R $\Omega \angle 0^o$).
- Une impédance purement inductive aura toujours un angle de phase d'exactement $+90^o$ ($Z_L = X_L\ \Omega \angle 90^o$).

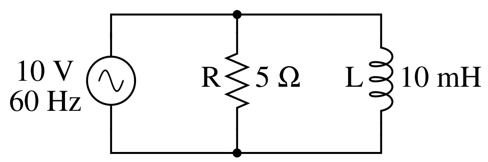
- La Loi d'Ohm pour les circuits AC : E = IZ ; I = E/Z ; Z = E/I

- Lorsque les résistances et les inductances sont mélangées dans des circuits, l'impédance totale aura un angle de phase entre 0^o et $+90^o$. Le courant dans le circuit aura un angle de phase entre 0^o et -90^o.

- Les circuits séries AC montrent les même propriétés fondamentales comme dans les circuits DC série : le courant est uniforme dans le circuit, les chutes de tension s'ajoutent pour former la tension totale et les impédances s'ajoutent pour former l'impédance totale.

3.4 Circuits résistance-inductance parallèles

Prenons les même composants de notre exemple série et connectons-les en parallèle :



Comme la source d'alimentation a la même fréquence que dans le circuit d'exemple série et la résistance et l'inductance ont tous les deux les mêmes valeurs de résistance et d'inductance; respectivement, ils doivent donc avoir les même valeurs d'impédance. Nous pouvons donc débuter notre table d'analyse avec les même valeurs "données" :

	R	L	Total	
E			**10 + j0** **10** ∠ 0°	Volts
I				Amps
Z	**5 + j0** **5** ∠ 0°	**0 + j3.7699** **3.7699** ∠ 90°		Ohms

La seule différence dans notre technique d'analyse, cette fois, est que nous appliquons les règles des circuits parallèles au lieu des règles séries des circuits. L'approche est fondamentalement la même que pour le DC. Nous savons que la tension est partagée uniformément par tous les composants dans un circuit parallèle, nous pouvons donc transférer les valeurs de la tension totale (10 volts $\angle$ 0^o) à toutes les colonnes des composants :

	R	L	Total	
E	10 + j0 $10 \angle 0^{\circ}$	10 + j0 $10 \angle 0^{\circ}$	10 + j0 $10 \angle 0^{\circ}$	Volts
I				Amps
Z	5 + j0 $5 \angle 0^{\circ}$	0 + j3.7699 $3.7699 \angle 90^{\circ}$		Ohms

Rule of parallel circuits:
$E_{total} = E_R = E_L$

Nous appliquons maintenant la Loi d'Ohm (I=E/Z) verticalement aux deux colonnes de la table, en calculant le courant dans la résistance et le courant dans l'inductance :

	R	L	Total	
E	10 + j0 $10 \angle 0^{\circ}$	10 + j0 $10 \angle 0^{\circ}$	10 + j0 $10 \angle 0^{\circ}$	Volts
I	**2 + j0** **2** $\angle 0^{\circ}$	**0 - j2.6526** **2.6526** $\angle -90^{\circ}$		Amps
Z	5 + j0 $5 \angle 0^{\circ}$	0 + j3.7699 $3.7699 \angle 90^{\circ}$		Ohms

Ohm's Law $I = \frac{E}{Z}$ *Ohm's Law* $I = \frac{E}{Z}$

Juste comme dans les circuits DC, les branches du courant dans le circuit AC parallèle s'ajoutent pour former le courant total (la Loi de Courant de Kirchhoff reste encore vrai pour l'AC comme il l'était pour le DC) :

	R	L	Total	
E	10 + j0 $10 \angle 0^{\circ}$	10 + j0 $10 \angle 0^{\circ}$	10 + j0 $10 \angle 0^{\circ}$	Volts
I	2 + j0 $2 \angle 0^{\circ}$	0 - j2.6526 $2.6526 \angle -90^{\circ}$	**2 - j2.6526** **3.3221** $\angle -52.984^{\circ}$	Amps
Z	5 + j0 $5 \angle 0^{\circ}$	0 + j3.7699 $3.7699 \angle 90^{\circ}$		Ohms

Rule of parallel circuits:
$I_{total} = I_R + I_L$

Finalement, une impédance totale peut être calculée en utilisant la Loi d'Ohm (Z=E/I) verticalement dans la colonne "Total". Incidement, l'impédance parallèle peut aussi être calculée en utilisant une formule réciproque identique à celle utilisée dans le calcul des résistances en parallèle.

$$Z_{parallel} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \ldots \frac{1}{Z_n}}$$

Le seul problème, avec l'utilisation de cette formule, est qu'elle implique typiquement beaucoup de calculs pour en finir. Et si vous êtes déterminé à réaliser ces calculs "à la main," préparez-vous à avoir beaucoup de travail! Mais, comme dans les circuits DC, nous avons souvent plusieurs options dans le calcul des valeurs de notre table d'analyse et cet exemple n'est pas différent. Quelque soit la manière dont nous calculons l'impédance totale (Loi d'Ohm ou formule réciproque), vous obtiendrez les mêmes valeurs :

	R	L	Total	
E	10 + j0 10 ∠ 0°	10 + j0 10 ∠ 0°	10 + j0 10 ∠ 0°	Volts
I	2 + j0 2 ∠ 0°	0 - j2.6526 2.6526 ∠ -90°	2 - j2.6526 3.322 ∠ -52.984°	Amps
Z	5 + j0 5 ∠ 0°	0 + j3.7699 3.7699 ∠ 90°	**1.8122 + j2.4035** **3.0102** ∠ 52.984°	Ohms

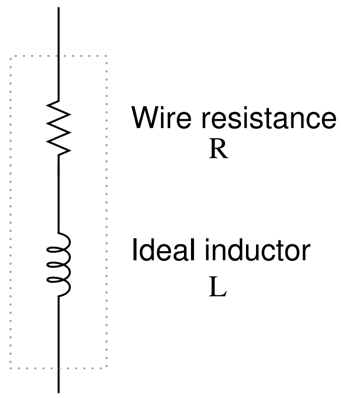
Ohm's Law $Z = \frac{E}{I}$ *or* *Rule of parallel circuits:* $Z_{total} = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_L}}$

- **RÉSUMÉ :**
- Les impédances (Z) sont gérées comme des résistances (R) dans les analyses de circuits parallèles : les impédances parallèles diminuent pour former l'impédance totale, en utilisant la formule réciproque. Assurez-vous juste d'effectuer tous les calculs en forme complexe (pas scalaire)! $Z_{Total} = 1/(1/Z_1 + 1/Z_2 + \ldots 1/Z_n)$
- La Loi d'Ohm pour les circuits AC : $E = IZ$; $I = E/Z$; $Z = E/I$
- Lorsque les résistances et les inductances sont mélangées ensembles dans les circuits parallèles (just comme dans les circuits série), l'impédance totale aura un angle de phase entre 0^o et $+90^o$. Le courant du circuit aura un angle de phase entre 0^o and -90^o.
- Les circuits parallèles montrent les même propriétés fondamentales que dans les circuits DC parallèles : la tension est uniforme dans le circuit, les branches de courant s'ajoutent pour former le courant total et les impédances diminuent (par la formule réciproque) pour former l'impédance totale.

3.5 Bizarreries de l'inductance

Dans un cas idéal, une inductance agit comme un composant purement réactif. Ceci étant, son opposition au courant AC est strictement basée sur la réaction inductive au au changements du courant et pas la friction des électrons comme avec les résistances. Les inductances ne sont, néanoins, pas si pures dans leur comportement réactif. Pour commencer, elles sont faites de fils et nous savons que tous les fils possèdent une résistance mesurable (à moins que le fils ne soit supraconducteur). Cette résistance interne agit comme si elle était connectée en série avec l'inductance parfaite de la bobine, comme ceci :

Equivalent circuit for a real inductor



En conséquence, l'impédance de toute inductance réelle sera toujours une combinaison complexe de résistance et de réactance inductive.

La composition de ce problème est quelque fois appelée *l'effect de peau*, qui est la tendance de l'AC à circuler sur les bords externes de la section d'un conducteur au lieu du milieu. Lorsque les électrons circulent dans une seule direction (DC), ils utilisent la zone complète de la section du conducteur pour se déplacer. Les changements de flux des électrons, d'un autre côté, tendent à éviter de circuler dans le milieu d'un conducteur, limitant la zone de la section disponible. L'effet de peau devient plus prononcé avec l'augmentation de fréquence.

De même, l'alternance de champ magnétique d'une inductance énergisée avec de l'AC peut irradier dans l'espace sous forme d'onde électromagnétique, spécialement si l'AC est en haute fréquence. Cette énergie irradiée ne retourne pas dans l'inductance et elle se manifeste elle-même comme une résistance (dissipation de puissance) dans le circuit.

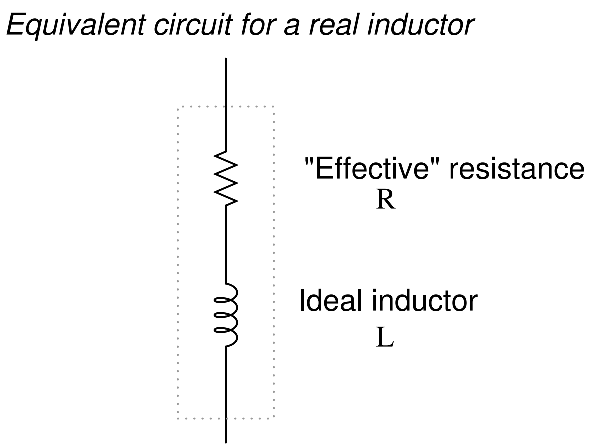
En plus des pertes résistives et des radiations, il existe d'autres effets dans les inducteurs en fer qui se manifestent eux-même comme une résistance addditionnelle entre les broches. Lorsqu'une inductance est énergisée en AC, les champs magnétiques alternatifs produits ont tendance à induire des courants circulant dans le coeur en fer connus comme des *courants eddy.* Ces courants électriques dans le coeur de fer dépassent la résistance électrique offerte par le fer, qui n'est pas aussi bon conducteur que le cuivre. Les pertes dûes aux courants Eddy sont contrecarrées en divisant le le coeur de fer en plusieurs fines lamelles (laminations), chacune séparée de l'autre par une fine couche de vernis d'isolation électrique. Avec la section centrale du coeur divisée en plsueirs sections électriques isolées, le courant ne peut pas circuler dans cette section et il n'y aura pas (ou très peu) de pertes résistives dûes à cet effet.

Comme vous pouvez vous y attendre, les pertes par courant eddy dans les coeurs d'inductances métalliques se manifestent elles-mêmes sous forme de chaleur. L'effet est plus prononcé aux hautes fréquences et peut être si extrême qu'il est quelques fois exploité dans les processus de fabrication pour chauffer les objets en métal! En fait, ce processus de "chauffage inductif" est souvent utilisé dans les opérations de de fonte de métal à haute pureté, où les éléments métalliques et les alliages doivent être chauffés dans un environnement sous vide pour éviter la contamination par l'air et où donc la technologie de chauffage par combustion standard serait inappliquable. C'est une technologie "sans-contact", la substance à chauffer ne devant pas toucher la(les) bobine(s) produisant le champ magnétique.

En haute fréquence, les courants eddy peuvent même se développer à l'intérieur d'un fil lui-même, en contribuant aux effets résistifs additionnels. Pour contrer cette tendance, des fils spéciaux, très fins, fait de brins isolés individuellement, appelés *fils de Litz* (raccourcis pour *Litzendraht*) peuvent être utilisés. L'isolation séparant les fils les uns des autres empêche les courants eddy de circuler dans toute la zone de section du fil.

En plus, toute hystérésis magnétique, qui a besoin d'être dépassée avec chaque inversion du champ magnétique, constitue une dépense d'énergie qui se manifeste elle-même comme une résistance dans le circuit. Quelques matériaux de noyau (tel que la ferrite) sont particulièrement notoire pour leur effet d'hystérésie. Contrer cet effet est réalisé plus facilement en choisissant un matériau de noyau plus adapté et limiter l'intensité pic du champ magnétique généré avec chaque cycle.

Ensemble, les propriétés résistives parasites d'une inductance réelle (résistance de fil, pertes par radiation, courants eddy et les pertes hystérésis) sont exprimées sous l'unique terme de "résistance effective :"



Il est important de noter que l'effet de peau et les pertes de radiation s'appliquent autant aux longueurs de fils dans les circuits AC qu'ils le font pour un fil bobiné. Leur effet est habituellement trop faible pour être noté mais aux fréquences radio, il peut être assez important. Une antenne de transmission radio, par exemple, est conçue avec l'objectif désigné de dissiper le plus d'énergie possible sous forme de radiation électromagnétique.

La résistance effective dans une inductance peut être un élément important pour les concepteurs de circuits AC. Pour aider à quantifier la valeur relative de la résistance effective dans une inductance, une autre valeur existe, appelée le *facteur Q* ou "facteur de qualité" qui est calculée comme suit :

$$Q = \frac{X_L}{R}$$

Le symbole "Q" n'a rien à voir avec la charge électrique (coulombs), ce qui tend à être perturbant. Bizarement, Ceux Qui Décident ont pris la même lettre de l'alphabet pour indiquer une valeur totalement différente.

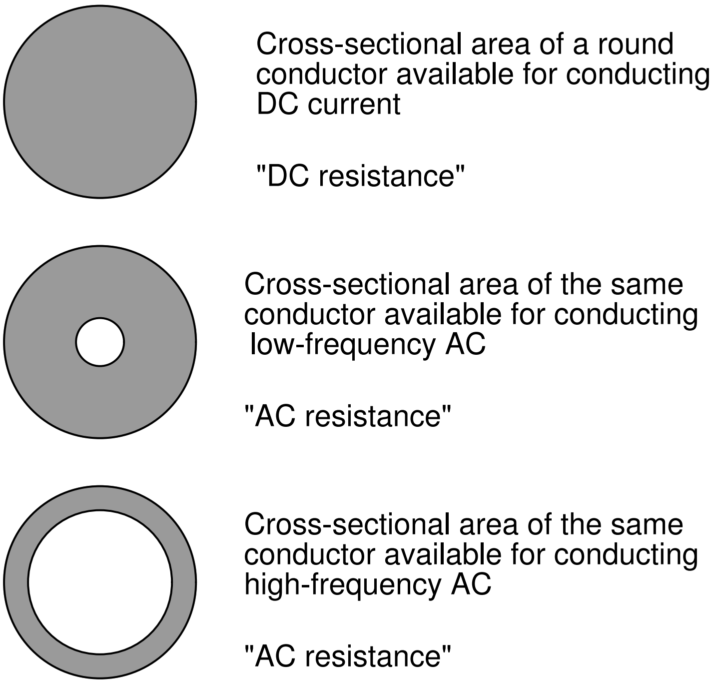
Plus haute est la valeur de "Q," plus l'inductance est "pure". Comme il n'est pas facile d'ajouter de résistance additionnelle si nécessaire, une inductance avec un haut Q est meilleure qu'une avec un Q plus bas pour les objectifs de conception. Une inductance idéale aurait un Q infini, avec une résistance effective nulle.

Comme la réactance inductive (X) varie avec la fréquence, Q variera aussi. Néanmois, comme l'effet résistif des inductances (résistance de fil, pertes par radiation, courants eddy et les pertes hystérésis) varient aussi avec la fréquence, Q varie proportionellement avec la réactance. De manière à obtenir une signification précise de Q, il doit être spécifié à une fréquence de test particulière.

La résistance parasite n'est pas la seule anomalie de l'inductance à laquelle nous devons faire attention. À cause du fait que de multiples tours de fils composant l'inductance sont séparés les uns des autres par un isolant (air, vernis ou tout autre type d'isolation électrique), une capacité a le potentiel de se développer entre les tours. La capacitance AC sera exploré dans le chapitre suivant mais il est suffisant de dire, à ce point, qu'il se comporte très différement d'une inductance AC et donc "teinte" la pureté réactive des inductances réelles.

3.6 Développement de "l'effet de peau"

Comme mentionné précédement, l'effet de peau est lorsque le courant alternatif tend à éviter le voyage dans le centre d'un conducteur solide, se limitant à la conduction proche de la surface. Cela limitera effectivement Cela limite effectivement la zone de conduction disponible pour transporter le flux alternatif des électrons, augmentant la résistance de ce conducteur au-dessus ce ce qu'il serait normalement pour un courant direct :



La résistance électrique du conducteur avec toute cette zone utile est connue comme la "résistance DC", la "résistance AC" du même conducteur se réfèrent aux hautes valeurs résultant de l'effet de peau. Comme vous pouvez le voir, aux hautes fréquences, le courant AC évite le voyage dans la majeure partie du conducteur. Pour le courant conducteur, le fil pourrait aussi bien être creux!

Dans quelques applications radio (antennes, la plupart du temps) cet effet est exploité. Comme les courants AC radio-fréquence ("RF") ne traversent pas le milieu du conducteur, pourquoi ne pas utiliser des bras de métal creux au lieu de fils de métal solide et préserver du poids et des coûts? La plupart des structures d'antennes et de conducteurs RF de puissance sont faits de tubes de métal évidés pour cette raison.

Dans la photographie suivante, vous pouvez voir quelques inductances utilisées dans un circuit de transmission radio de 50 kW. Les inductances sont des tubes de cuivre évidés enduits d'argent, pour son excellente conductivité sur la "peau" du tube :

La valeur à laquelle la fréquence affecte la résistance effective d’un fil conducteur plein est impacté par la section de ce fil. La règle est que les fils à large section ont un effet de peau plus prononcé (changement de la résistance depuis le DC) que les fils à plus petite section pour une fréquence donnée. L’équation pour approximer l’effet de peau aux hautes fréquences (à plus de 1 MHz) est comme suit :

$$R_{AC} = (R_{DC})(k)\sqrt{f}$$

Where,

R_{AC} = AC resistance at given frequency "f"

R_{DC} = Resistance at zero frequency (DC)

k = Wire gage factor (see table below)

f = Frequency of AC in MHz (MegaHertz)

La table suivante donne les valeurs approximatives du facteur ”k” pour divers diamètres de fils :

```
Taille de gauge  Facteur k
=====================
4/0 ------------------ 124.5
2/0 ------------------- 99.0
```

```
1/0 ------------------- 88.0
2 ---------------------- 69.8
4 ---------------------- 55.5
6 ---------------------- 47.9
8 ---------------------- 34.8
10 -------------------- 27.6
14 -------------------- 17.6
18 -------------------- 10.9
22 --------------------- 6.86
```

Par exemple, une longueur de fil avec une gauge numéro 10 avec une résistance DC terminale de 25 Ω aura une résistance AC (effective) de 2.182 kΩ à la fréquence de 10 MHz :

$$R_{AC} = (R_{DC})(k)\sqrt{f}$$

$$R_{AC} = (25\ \Omega)(27.6)\ \sqrt{10}$$

$$R_{AC} = 2.182\ k\Omega$$

Rappelez-vous que cette valeur *n'est pas* une impédance et elle *ne doit pas* être considérée comme ayant des effets réactifs, inductifs ou capacitifs. C'est simplement une valeur estimée de la résistance pure du conducteur (qui s'oppose au flux AC des électrons qui *dissipe* la puissance sous forme de chaleur), corrigée pour l'effet de peau. La réactance, les effets combinés de la réactance et de la résistance (impédance), sont des éléments différents.

3.7 Contributeurs

Les contributeurs de ce chapitre sont listés dans l'ordre chronologique de leurs contributions, depuis le plus récent jusqu'au premier. Voyez l'Annexe 2 (Liste des contributeur) pour les dates et les informations de contact.

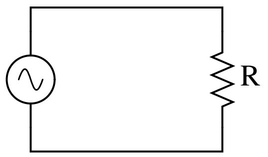
Jim Palmer (June 2001) : Identified and offered correction for typographical error in complex number calculation.

Jason Starck (June 2000) : HTML document formatting, which led to a much better-looking second edition.

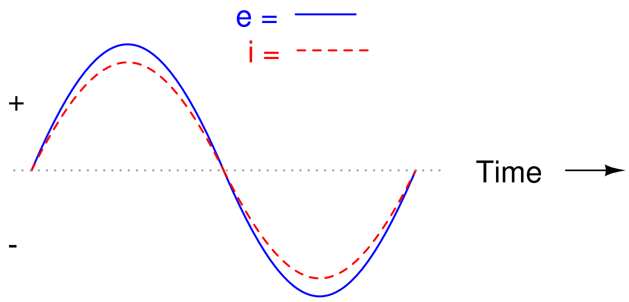
Chapter 4

RÉACTANCE ET IMPÉDANCE – CAPACITIVE

4.1 Circuits résistants AC



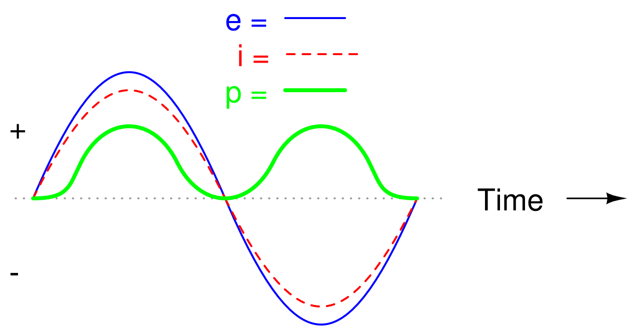
Si nous devions dessiner le courant et la tension pour un circuit AC très simple consistant en une source et une résistance, cela ressemblerait à quelque chose comme ceci :



Comme une résistance résiste simplement et directement au flux des électrons en permanence, la forme de l'onde de la chute de tension aux bornes de la résistance est exactement en phase avec la forme de l'onde du courant qui la traverse. Nous pouvons regarder à tout point du temps sur l'axe horizontal du tracé et comparer les valeurs du courant et de la tension l'un avec l'autre (tout point "instantané" des valeurs d'une onde est appelé *valeur instantanée*, signifiant que les valeurs sont à un *instant* dans le temps). Lorsque la valeur instantannée du courant est zéro, la tension instantanée aux bornes de la résistance est aussi zéro. De même, au moment où le courant dans la résistance est à son pic positif, la tension aux bornes de la résistance est aussi à son pic positif, ainsi de suite.

À tout point donné de l'onde, la Loi d'Ohm reste vraie pour toutes les valeurs instantannées de la tension et du courant.

Nous pouvons aussi calculer la puissance dissipée par cette résistance et tracez ces valeurs sur le même graphe :



Notez que la puissance n'est jamais une valeur négative. Lorsque le courant est positif (au-dessus de la ligne), la tension est aussi positive, le résultat d'une puissance est une valeur positive (p=ie). Inversement, lorsque le courant est négatif (en dessous de la ligne), la tension est aussi négative, ce qui a pour résultat une valeur positive de la puissance (un nombre négatif multiplié par un nombre négatif égale un nombre positif). Cette "polarité" cohérente de la puissance nous indique que la résistance dissipe toujours de la puissance, la prenant de la source et la relachant sous forme d'énergie. Que le courant soit positif ou négatif, une résistance dissipe toujours de l'énergie.

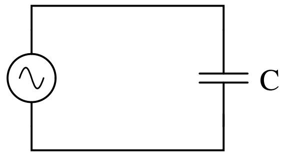
4.2 Circuits AC à condensateurs

Les capacités ne se comportent pas comme des résistances. Alors que les résistances permettent un flux d'électrons directement proportionnel à la chute de tension, les condensateurs s'opposent aux *changements* en tension en tirant ou fournissant du courant lorsqu'ils montent ou descendent à leur nouveau niveau de tension. Le flux d'électrons "aux bornes" d'une capacité est directement proportionnel à la *valeur de changement* de la tension aux bornes du condensateur. Cette opposition au changement de tension est une autre forme de *réactance* mais d'un type précisément oppposé à celui montré par les inductances.

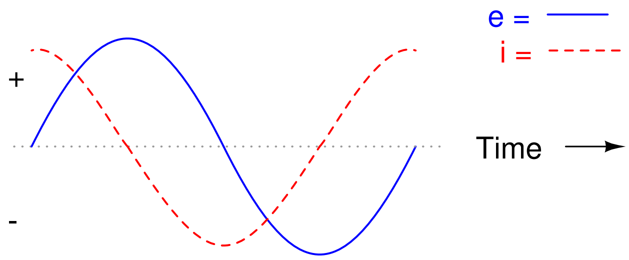
Exprimé mathématiquement, la relation entre le courant "traversant" le condensateur et le taux de changement de tension aux bornes de ce dernier est tel que :

$$i = C\,\frac{de}{dt}$$

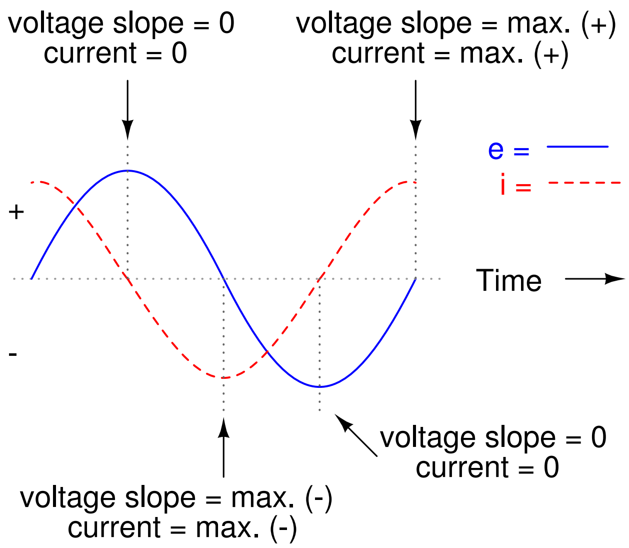
L'expression *de/dt* est une forme arithmétique, signifiant que le taux de changement de la tension instantannée (e) pendant une durée, est en volts par secondes. Le condensateur (C) est en Farads et le courant instantané (i) est bien sûr en ampères. Vous trouverez quelquefois le taux de changement de tension instantanée dans le temps exprimée comme dv/dt au lieu de de/dt : en utilisant la lettre minuscule "v" au lieu de "e" pour représenter la tension mais cela représente exactement les mêmes vakeurs. Pour montrer ce qui se produit avec le courant alternatif, analysons un simple circuit à condensateur :



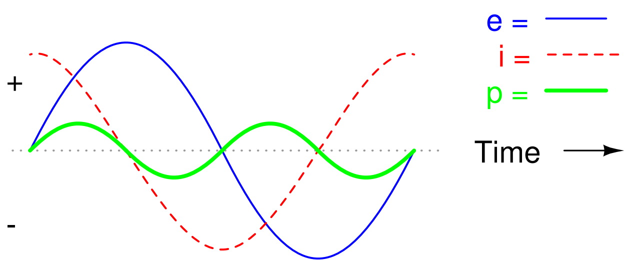
Si nous devions tracer le courant et la tension pour ce circuit très simple, il ressemblerait à quelque chose comme ceci :



Rappelez-vous, le courant dans un condensateur est une réaction contre le *changement* de tension qui la traverse. C'est pourquoi le courant instantanné est à zéro pendant que la tension instantanée est à un pic (passage à zéro ou pente de niveau, sur l'onde de tension sinus) et le courant instantanné est à son pic lorsque la tension instantannée est à son changement maximum (les points de pente la plus raide sur l'onde de tension, où il coupe le zéro). Ce résultat dans une onde de tension est un décalage de phase du courant de -90^o. En regardant le graphe, l'onde de courant semble avoir un "head start" sur l'onde de tension; le courant est en "avance" sur la tension et la tension "traîne" derrière le courant.



Comme vous l'avez deviné, la même onde de puissance inhabituelle que nous avons vu avec le circuit à simple inducteur est aussi présent dans le circuit à condensateur simple :



Comme pour un circuit à inductance simple, les résultats du décalage de phase à 90 degrés entre la tension et le courant sont une onde de puissance qui alterne à égalité entre le positif et le négatif. Ceci signifie qu'un condensateur ne dissipe pas de puissance car il réagit aux changements de tension; il absorbe et envoie simplement de la puissance, alternativement.

L'opposition au changement de tension d'un condensateur se transmet en une opposition générale de la tension alternative, ce qui est, par définition, toujours un changement de la magnitude instantannée et de la direction. Pour toute magnitude de tension altenative à une fréquence donnée, un condensateur de capacité donnée "conduira" une certaine magnitude de courant AC. De la même manière que le courant dans une résistance est en fonction de la tension à ses bornes et le frein offert par la résistance, le courant AC aux bornes du condensateur est une fonction de la tension AC et de la *réactance* offerte par le condensateur. Comme pour les inductances, la réactance du condensateur est exprimée en ohms et est symbolisée par la lettre X (ou X_C pour être plus spécifique).

Comme les condensateurs "conduisent" le courant en proportion du taux de changement de tension, ils passeront plus de courant pour des changements de tension plus rapides (comme ils se chargent et se déchargent aux même pics de tension en moins de temps) et moins de courant pour les changements plus lents de tension. Cela signifie que la réactance en ohms pour tout condensateur est *inversement* proportionel à la fréquence du courant alternatif :

$$X_C = \frac{1}{2\pi f C}$$

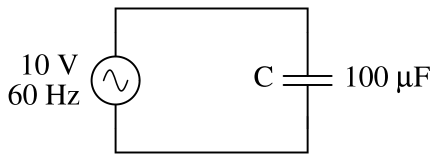
Pour un condensateur de 100 uF :

Fréquence (Hertz)	Réactance (Ohms)
60	26.5258
120	13.2629
2500	0.6366

Veuillez noter que la relation d'une réactance capacitive avec la fréquence est exactement opposée à celle d'une réactance inductive. La réactance capacitive (en ohms) diminue avec l'augmentation de fréquence AC. De fait, la réactance inductive (en ohms) augmente avec la fréquence AC. Les inductances opposent des changements de courant plus rapides en produisant de plus grandes chutes de tension; les condensateurs s'oppose aux changements rapides de tension en permettant de plus grands courants.

Comme pour les inductances, le terme de l'équation de réactance 2πf peut être remplacé par la lettre minuscule Grèque Omega (ω) qui signifie la *vitesse angulaire* du circuit AC. Donc, l'équation $X_C = 1/(2\pi fC)$ peut aussi être écrite comme $X_C = 1/(\omega C)$, avec ω donné en unité de *radians par secondes.*

Le courant alternatif dans un circuit à condensateur simple est égal à la tension (en volts) divisé par la réactance capacitive (en ohms), de la même manière qu'un courant altenatif ou continu dans un circuit résitif simple est égal à la tension (en volts) divisée par la résistance (en ohms). Le circuit suivant illustre, par exemple, cette relation mathématique :



$$X_C = 26.5258\ \Omega$$

$$I = \frac{E}{X}$$

$$I = \frac{10\ V}{26.5258\ \Omega}$$

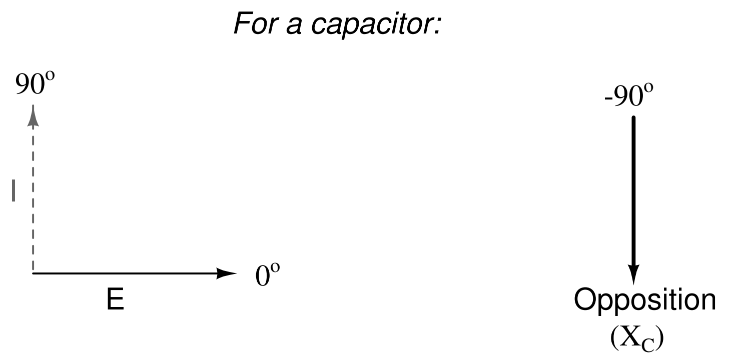
$$I = 0.3770\ A$$

Nous devons néanmoins garder en méméoire que la tension et le courant n'est pas en phase ici. Comme montré précédement, le courant a un décalage de phase de $+90^o$ par rapport à la tension. Si nous représentons les angles de phase de la tension et du courant mathématiquement, noous pouvons calculer l'angle de phase de l'opposition réactive de l'inductance au courant.

$$\text{Opposition} = \frac{\text{Voltage}}{\text{Current}}$$

$$\text{Opposition} = \frac{10\ V \angle 0^o}{0.3770\ A \angle 90^o}$$

$$\text{Opposition} = 26.5258\ \Omega \angle -90^o$$



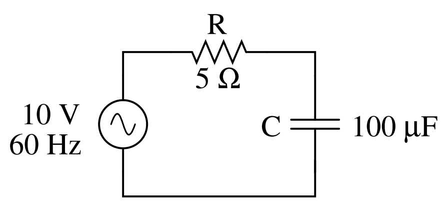
Mathématiquement, nous disons que l'opposition de l'angle de phase d'un condensateur est de -90^o, signifiant que l'opposition d'un condensateur au courant est une quantité imaginaire négative. Cet angle de phase d'opposition réactive au courant devient d'une importance critique dans l'analyse de circuit, spécialement pour les circuits AC complexes où la réactance et la résistance interagissent. Il serait bénéfique de représenter *toute* opposition de composant au courant en des termes de nombres complexes et pas seulement en quantités scalaires de résistance et réactance.

- **RÉSUMÉ :**
- La *réactance capacitive* est l'opposition qu'offre un condensateur au courant alternatif dû à son stockage décalé en phase et la fourniture d'énergie dans son champ électrique. La réactance est symbolisée par la lettre capitale "X" et est mesurée en ohms comme les résistances (R).
- La réactance capacitive peut être calculée en utilisant cette formule : $X_C = 1/(2\pi fC)$
- La réactance capacitive *diminue* avec l'augmentation de fréquence. En d'autres mots, plus la fréquence est élevée, moins elle s'oppose (plus elle "conduit") le flux AC des électrons.

4.3 Circuits à résistance-condensateur série

Dans la dernière section, nous avons appris ce qui se passait dans un dans un circuit AC à résistance ou condensateur simple. Nous allons maintenant combiner les deux composants ensemble en série et examiner les effets.

Prenons ce circuit comme un exemple à analyser :



La résistance offre 5 Ω contre le passage du courant AC indépendamment de la fréquence, alors que le condensateur offre une réactance de 26.5258 Ω au courant AC à 60 Hz. Comme la valeur de la résistance est un ombre réel (5 Ω ∠ 0^o ou 5 + j0 Ω) et la réactance du condensateur est un nombre imaginaire (26.5258 Ω ∠ -90^o ou 0 - j26.5258 Ω), l'effet combiné des deux composants sera une opposition au courant égale à la somme complexe des deux nombres. Le terme de cette opposition complexe au courant est l'*impedance*, son symbole est Z et il est aussi exprimé dans l'unité des ohms, comme les résistances et les réactances. Dans l'exemple ci-dessous, l'impédance totale du circuit est :

$$Z_{total} = (5\ \Omega \text{ resistance}) + (26.5258\ \Omega \text{ capacitive reactance})$$

$$Z_{total} = 5\ \Omega\ (R) + 26.5258\ \Omega\ (X_C)$$

$$Z_{total} = (5\ \Omega \angle 0^o) + (26.5258\ \Omega \angle -90^o)$$

or

$$(5 + j0\ \Omega) + (0 - j26.5258\ \Omega)$$

$$Z_{total} = 5 - j26.5258\ \Omega \quad \textit{or} \quad 26.993\ \Omega \angle -79.325^o$$

L'impédance est associé à la tension et au courant comme vous pouvez vous y attendre, de la même manière que la résistance dans la Loi d'Ohm :

Ohm's Law for AC circuits:

$$\mathbf{E = IZ} \qquad \mathbf{I = \frac{E}{Z}} \qquad \mathbf{Z = \frac{E}{I}}$$

All quantities expressed in complex, not scalar, form

En fait, c'est une forme beaucoup plus étendue de la Loi Ohm qui a été aprise dans le manuel DC (E=IR), comme l'impédance est une expression beaucoup plus étendue de l'opposition au flux des électrons que ne l'est une simple résistance. Toute résistance et toute réactance, séparément ou en combination (série/parallèle), peut être et doit être représenté comme une simple impédance.

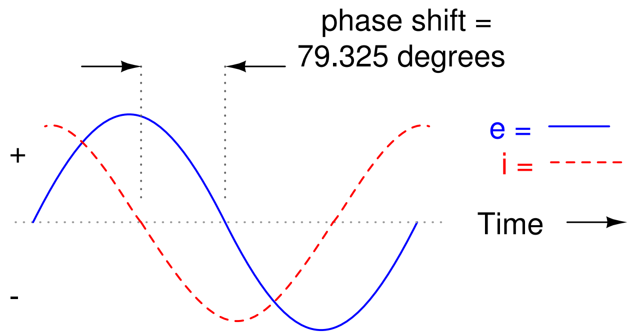
Pour calculer le courant dans le circuit ci-dessus, nous avons d'abord besoin de donner une référence d'angle de phase pour la source de tension, qui est supposée être à zéro. (Les angles de phase de l'impédance résistive et capacitive sont *toujours* 0^o et -90^o, respectivement, indépendament des angles de phase sonnés pour la tension ou le courant).

$$I = \frac{E}{Z}$$

$$I = \frac{10\ V \angle 0^o}{26.933\ \Omega \angle -79.325^o}$$

$$I = 370.5\ mA \angle 79.325^o$$

Comme pour le circuit purement capacitif, l'onde de courant est en avance sur l'onde de tension (de la source), bien que cette fois, la différence soit de 79.325^o au lieu de 90^o.



Comme nous l'avons appris dans le chapitre sur l'inductance AC, la méthode de la "table" de l'organisation des quantités d'un circuit est un outil très utile pour l'analyse AC de la même manière que pour l'analyse DC. Plaçons les chiffres connus pour ce circuit série dans la table et continuons l'analyse en utilisant cet outil :

	R	C	Total	
E			10 + j0 10 ∠ 0°	Volts
I			68.623m + j364.06m 370.5m ∠ 79.325°	Amps
Z	5 + j0 5 ∠ 0°	0 - j26.5258 26.5258 ∠ -90°	5 - j26.5258 26.993 ∠ -79.325°	Ohms

Le courant dans un circuit série est partagé équitablement entre tous les composants, les chiffres sont donc placés dans la colonne "Total" car le courant peut être distribué de même dans toutes les autres colonnes :

	R	C	Total	
E			10 + j0 10 ∠ 0°	Volts
I	**68.623m + j364.06m** **370.5m** ∠ 79.325°	**68.623m + j364.06m** **370.5m** ∠ 79.325°	68.623m + j364.06m 370.5m ∠ 79.325°	Amps
Z	5 + j0 5 ∠ 0°	0 - j26.5258 26.5258 ∠ -90°	5 - j26.5258 26.993 ∠ -79.325°	Ohms

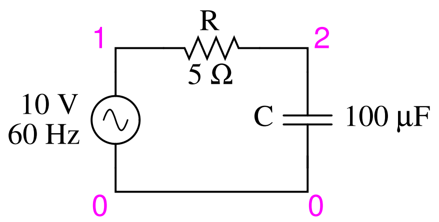
Rule of series circuits:
$I_{total} = I_R = I_C$

En continuant avec notre analyse, nous pouvons appliquer la Loi d'Ohm (E=IR) verticalement pour déterminer le tension aux bornes de la résistance et du condensateur :

	R	C	Total	
E	**343.11m + j1.8203** **1.8523** ∠ 79.325°	**9.6569 - j1.8203** **9.8269** ∠ -10.675°	10 + j0 10 ∠ 0°	Volts
I	68.623m + j364.06m 370.5m ∠ 79.325°	68.623m + j364.06m 370.5m ∠ 79.325°	68.623m + j364.06m 370.5m ∠ 79.325°	Amps
Z	5 + j0 5 ∠ 0°	0 - j26.5258 26.5258 ∠ -90°	5 - j26.5258 26.993 ∠ -79.325°	Ohms
	Ohm's Law E = IZ	*Ohm's Law* E = IZ		

Notez que la tension aux bornes de la résistance a le même angle de phase que le courant, nous indiquant que E et I sont en phase (pour la résistance seulement). La tension aux bornes du condensateur a un angle de phase de -10.675^o, exactement 90^o *moins* que l'angle de phase du courant dans le circuit. Ceci nous indique que la tension et le courant du condensateur sont encore décalés de 90^o l'un avec l'autre.

Contrôlons nos calculs avec SPICE :




```
ac r-c circuit
v1 1 0 ac 10 sin
r1 1 2 5
c1 2 0 100u
.ac lin 1 60 60
.print ac v(1,2) v(2,0) i(v1)
.print ac vp(1,2) vp(2,0) ip(v1)
.end
```

```
freq          v(1,2)      v(2)        i(v1)
6.000E+01     1.852E+00   9.827E+00   3.705E-01
freq          vp(1,2)     vp(2)       ip(v1)
6.000E+01     7.933E+01  -1.067E+01  -1.007E+02
```

Interpreted SPICE results

E_R = 1.852 V ∠ 79.33°

E_C = 9.827 V ∠ -10.67°

I = 370.5 mA ∠ -100.7°

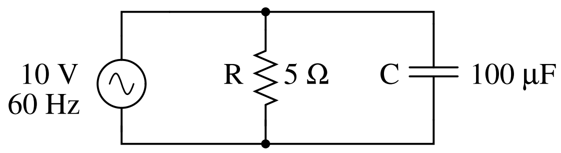
Une fois encore, SPICE affiche d'une manière désordonnée l'actuel angle de phase à une valeur égale à l'angle de phase réel plus 180^o (ou moins 180^o). Néanmoins, il est très facile de corriger ce chiffre et de contrôler si notre travail est correct. Dans ce cas, les -100.7^o sortis par SPICE pour l'angle de phase actuel valent 79.3^o, ce qui correspond à notre chiffre précédement calculé de 79.325^o.

Il faut encore insister que les chiffres calculés correspondent à des mesures de tensions et de courant réels qui sont sous la forme *polaire*, pas rectangulaire! Par exemple, si nous devions réellement construire ce circuit série résistance-condensateur et mesurer la tension aux bornes de la résistance, notre voltmètre indiquerait **1.8523** volts, pas 343.11 millivolts (en rectangulaire réel) ou 1.8203 volts (rectangulaire imaginaire). Les instruments réels connectés à des circuits réels fournissent des indications correspondants à la longueur de vecteur (magnitude) des chiffres calculés. Alors la forme rectangulaire de la notation des nombres complexes est utile pour effectuer des additions et des soustractions, c'est une forme plus abstraite que la polaire qui, seule, a une correspondence directe aux vraies mesures.

- **RÉSUMÉ :**
- *L'impédance* est la mesure totale de l'opposition au courant électrique et est la somme complexe (vectorielle) de la résistance ("réelle") et de la réactance ("imaginaire").
- Les impédances (Z) sont gérées comme des résistances (R) dans l'analyse des circuits série : les impédances série s'ajoutent pour former l'impédance totale. Assurez-vous simplement d'effectuer les calculs dans la forme complexe (et non scalaire)! $Z_{Total} = Z_1 + Z_2 + \ldots Z_n$
- Veillez noter que les impédances s'ajoutent toujours en série, quelque soit le type de composant qui forme l'impédances. Ceci étant, les impédances résistives, inductives et capacitives doivent être traitées mathématiquement de la même manière.
- Une impédance purement résistive aura toujours exactement un angle de phase de 0^o (Z_R = R Ω ∠ 0^o).
- Une impédance purement capacitive aura toujours exactement un angle de phase de -90^o (Z_C = X_C Ω ∠ -90^o).
- La Loi d'Ohm pour les circuits AC : E = IZ ; I = E/Z ; Z = E/I
- Lorsque les résistances et les condensateurs sont mélangés ensemble dans un circuit, l'impédance totale aura un angle de phase quelque part entre 0^o and -90^o.
- Les circuits AC en série montrent les même propriétés fondamentales que les circuits série DC : le courant est uniforme dans le circuit, les chutes de tension s'ajoutent pour former la tension globale et les impédances s'ajoutent pour former l'impédance globale.

4.4 Circuits résistance-condensateur parallèle

En utilisant des composants avec la même valeur de notre circuit série d'exemple, nous les connecterons en parallèle et regardons ce qui se passe :



Comme la source de puissance a la même fréquence que dans l'exemple du circuit série; et la résistance et le condensateur ont les même valeurs et ils doivent aussi avoir les même valeurs d'impedance. Nous pouvons donc démarrer notre table d'analyse avec les mêmes valeurs "données" :

	R	C	Total	
E			$10 + j0$ $10 \angle 0^o$	Volts
I				Amps
Z	$5 + j0$ $5 \angle 0^o$	$0 - j26.5258$ $26.5258 \angle -90^o$		Ohms

C'est maintenant un circuit parallèle, nous savons que la tension est partagée équitablement par tous les composants, nous plaçons donc le chiffre pour la tension complète (10 volts $\angle$ 0^o) dans toutes les colonnes :

	R	C	Total	
E	**$10 + j0$** **$10 \angle 0^o$**	**$10 + j0$** **$10 \angle 0^o$**	$10 + j0$ $10 \angle 0^o$	Volts
I				Amps
Z	$5 + j0$ $5 \angle 0^o$	$0 - j26.5258$ $26.5258 \angle -90^o$		Ohms

Rule of parallel circuits:
$E_{total} = E_R = E_C$

Nous pouvons maintenant appliquer la Loi d'Ohm (I=E/Z) verticalement aux deux colonnes dans la table, en calculant le courant au travers de la résistance et du condensateur :

	R	C	Total	
E	$10 + j0$ $10 \angle 0^o$	$10 + j0$ $10 \angle 0^o$	$10 + j0$ $10 \angle 0^o$	Volts
I	**$2 + j0$** **$2 \angle 0^o$**	**$0 + j376.99m$** **$376.99m \angle 90^o$**		Amps
Z	$5 + j0$ $5 \angle 0^o$	$0 - j26.5258$ $26.5258 \angle -90^o$		Ohms

Ohm's Law $I = \frac{E}{Z}$ *Ohm's Law* $I = \frac{E}{Z}$

Comme avec le circuit DC, les branches de courant dans un circuit AC parallèle s'ajoutent pour former le courant total (encore la Loi du Courant de Kirchhoff) :

	R	C	Total	
E	$10 + j0$ $10 \angle 0^o$	$10 + j0$ $10 \angle 0^o$	$10 + j0$ $10 \angle 0^o$	Volts
I	$2 + j0$ $2 \angle 0^o$	$0 + j376.99m$ $376.99m \angle 90^o$	**$2 + j376.99m$** **$2.0352 \angle 10.675^o$**	Amps
Z	$5 + j0$ $5 \angle 0^o$	$0 - j26.5258$ $26.5258 \angle -90^o$		Ohms

Rule of parallel circuits: $I_{total} = I_R + I_C$

Finalement, l'impédance totale peut être calculée en utilisant la Loi d'Ohm (Z=E/I) verticalement dans la colonne "Total". Comme nous l'avons vu dans le chapitre sur les inductances AC, l'impédance parallèle peut aussi être calculée en utilisant une formule réciproque identique à celle utilisée dans le calcul des résistances parallèles. Il est à noter que la loi sur l'impédance parallèle reste vraie quelque soient les impédances placées en parallèle. En d'autres mots, il n'y a pas de différences dans le calcul d'un circuit composé de résistances, condensateurs, inductances en parallèle ou d'une combinaison : sous la forme d'impédances (Z), tous les termes sont communs et peuvent être appliqués uniformément à la même formule. Une fois encore, la formule de l'impédance parallèle ressemble à ceci :

$$Z_{parallel} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \ldots \frac{1}{Z_n}}$$

Le seul inconvénient de l'utilisation de cette équation est la somme de travail nécessaire pour la travailler, spécialement sans l'assistance d'un calculateur capable de manipuler des quantités complexes. Indépendament de la manière de calculer l'impédance totale pour notre circuit parallèle (soit par la Loi d'Ohm ou la formule réciproque), nous arrivons au même chiffre :

	R	C	Total	
E	10 + j0 10 ∠ 0°	10 + j0 10 ∠ 0°	10 + j0 10 ∠ 0°	Volts
I	2 + j0 2 ∠ 0°	0 + j376.99m 376.99m ∠ 90°	2 + j376.99m 2.0352 ∠ 10.675°	Amps
Z	5 + j0 5 ∠ 0°	0 - j26.5258 26.5258 ∠ -90°	**4.8284 - j910.14m** **4.9135** ∠ -10.675°	Ohms

Ohm's Law *or* *Rule of parallel circuits:*

$$Z = \frac{E}{I} \qquad Z_{total} = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_C}}$$

- **RÉSUMÉ :**
- Les impédances (Z) sont gérées comme les résistances (R) en parallèle dans l'analyse d'un circuit : les impédances parallèles diminuent pour former l'impédance totale, en utilisant la formule réciproque. Assurez-vous d'effectuer tous les calculs sous la forme complexe (et non scalaire)! $Z_{Total} = 1/(1/Z_1 + 1/Z_2 + \ldots 1/Z_n)$
- La Loi d'Ohm pour les circuits AC : $E = IZ$; $I = E/Z$; $Z = E/I$
- Lorsque kes résistances et les condensateurs sont mélangés ensemble en parallèle dans des circuits (comme dans les circuits série), l'impédance totale aura un angle de phase compris entre 0^o et -90^o. Le courant du circuit aura un angle de phase compris entre 0^o et $+90^o$.
- Les circuits AC parallèles montrent les même propriétés fondamentales que les circuits DC parallèles : la tension est uniforme dans le circuit, les branches de courant s'ajoutent pour former le courant total et l'impédance diminue (via la formule réciproque) pour former l'impédance totale.

4.5 Particularités des condensateurs

Comme avec les inducteurs, le condensateur idéal est un composant purement réactif, possédant des effets résistifs absoluments nuls (dissipation de puissance). Dans le monde réel, bien sûr, rien n'est aussi parfait. Néanmoins, les condensateurs ont la vertu d'être des composants réactifs plus *purs* que les inductances. Il est beaucoup plus facile de concevoir et de construire un condensateur avec une faible résistance série qu'il n'est facile de le faire avec une inductance. Le résultat pratique de ceci est que ce condensateur réel possède typiquement un angle de phase approchant les 90^o (réellement, -90^o) que les inducteurs. En conséquence, ils auront tendance à dissiper moins de puissance que l'inducteur équivalent.

Les condensateurs tendent aussi à être plus petits et plus légers que les inductances équivalentes et comme leurs champs électriques sont presque totalement contenus entre leurs broches (contrairement aux inducteurs, dont les champs magnétiques tendent naturellement à s'étendre en dehors des dimensions de leur coeur), ils sont moins prompt à transmettre ou recevoir du "bruit" élecctromagnétique de/vers les autres composants. Pour ces raisons, les concepteurs de circuits tendent à favoriser les condensateurs sur les inductances lorsqu'une conception permet l'alternative.

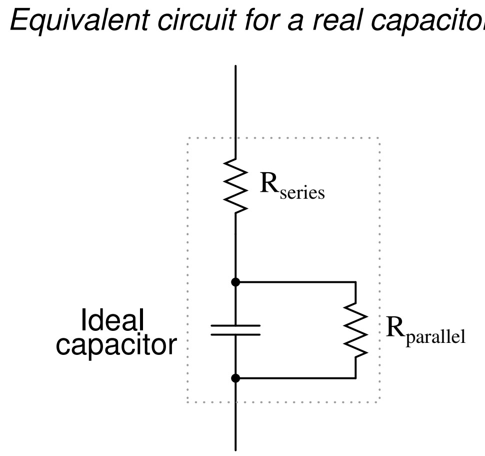
Les condensateurs avec un effet résistif sont dit *lossy*, en référence à leur tendance à dissiper ("perdre") de la puissance comme une résistance. La source de la perte d'un condensateur est habituellement le matériel diélectrique plutôt que la résistance du fil car la longueur du fil dans un condensateur est très minime.

Le matériel diélectrique tend à réagir au changement de champ électrique en produisant de la chaleur. Cet effet de chaleur représente une perte de puissance et est équivalent à la résistance dans un circuit. L'effet est plus prononcé à hautes fréquences et peut donc être extrême, ce qui est quelque fois exploité dans les processus industriels pour chauffer des matériaux comme le plastique! L'objet plastique devant être chauffé est placé entre deux plaques de métal, connectées à une source de tension à haute-fréquence AC. La température est contrôlée en faisant varier la tension ou la fréquence de la source et les plaques ne doivent jamais être au contact de l'objet devant être chauffé.

Cet effet est indésirable pour les condensateurs où nous souhaitons que le composant se comporte comme un élément de circuit purement *réactif*. Une des manières de mitiger l'effet de "perte" du diélectrique est de choisir un matériau diélectrique moins susceptible à cet effet. Tous les matériaux diélectriques ne sont pas également "lossy". Une échelle relative des pertes diélectriques du moins au plus est donnée ici :

```
Vacuum --------------- (Faible Perte)
Air
Polystyrene
Mica
Glass
Low-K ceramic
Plastic film (Mylar)
Paper
High-K ceramic
Aluminum oxide
Tantalum pentoxide --- (Forte Perte)
```

La résistivité diélectrique se manifeste d'elle-même comme une résistance série et parallèle avec la capacitance pure :



Heureusement, ces résistances aléatoires ont habituellement un faible impact (faible résistance série et haute résistance parallèle), bien moins significatif que les résistances présentes dans une inductance moyenne.

Les condensateurs électrolytiques, connus pour leur relativement haute capacitance et leur faible tension de travail, sont aussi connus pour leurs pertes, dues à la fois aux caractéristiques du film diélectrique microscopiquement fin et la pâte électrolytique. À moins d'être spécialement fabriqué pour le service AC, les condensateurs électrolytiques ne doivent jamais être utilisés avec de l'AC à moins qu'il soit mélangé (biaisé) avec une tension DC constante prévenant le condensateur de voir une tension négative. Même alors, leur caractéristique résistive peut être une contrainte trop importance pour l'application.

4.6 Contributeurs

Les contributeurs à ce chapitre sont listés dans l'ordre chronologique de leurs contributions, depuis le plus récent au premier. Voyez l'Appendice 2 (Contributor List) pour les dates et les informations de contact.

Jason Starck (June 2000) : HTML document formatting, which led to a much better-looking second edition.

Chapter 5

RÉACTANCE ET IMPÉDANCE – R, L ET C

5.1 Résumé des R, X et Z

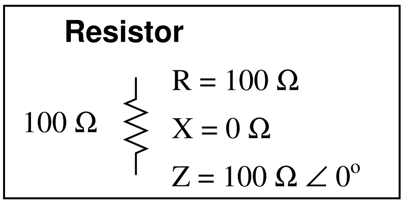
Avant que nous commencions à explorer les effets des résistances, inductances et condensateurs, connectés ensemble dans les mêmes circuits AC, reprenons rapidement quelques termes basiques et quelques éléments.

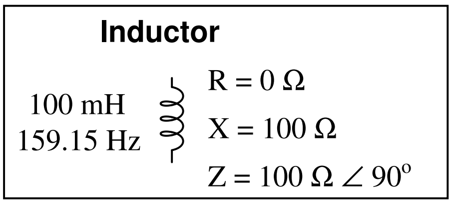
La **résistance** est essentiellement une *friction* contre le déplacement des électrons. Elle est présente dans tous les conducteurs, dans une certaine mesure (excepté pour les *supra*conducteurs!), mais plus particulièrement dans les résistances. Lorsqu'un courant alternatif passe dans une résistance, une chute de tension est produite, en phase avec le courant. La résistance est symbolisée mathématiquement par la lettre "R" et est mesurée dans l'unité des ohms (Ω).

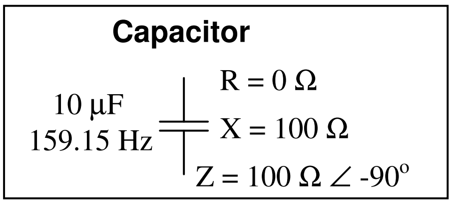
La **réactance** est essentiellement *inertielle* contre le déplacement des électrons. Elle est présente partout où des champs électriques ou magnétiques sont développés en proportion de l'application d'une tension ou d'un courant, respectivement; mais plus notablement dans des condensateurs et de inductances. Lorsqu'un courant alternatif passe dans une réactance pure, une chute de tension est produite, ayant un décalage de phase de 90^o avec le courant. La réactance est mathématiquement symbolisé par la lettre "X" et est mesurée en ohms (Ω).

L'**impédance** est une l'expression étendue de toute forme d'opposition au flux des électrons, incluant la résistance et la réactance. Elle est présente dans tous les circuits et tous les composants. Lorsque le courant alternatif passe dans une impédance, une chute de tension est produite avec un décalage de phase entre 0^o et 90^o avec le courant. L'impédance est symbolisée mathématiquement par la lettre "Z" et est mesurée dans l'unité des ohms (Ω), dans la forme complexe.

Les résistances parfaites possèdent une résistance mais pas de réactance. Les inductances et les condensateurs parfaits possèdent une réactance mais pas de résistance. Tous les composants possèdent une impédance et à cause de cette qualité universelle, il est sensé de traduire toutes les valeurs de composants (résistance, inductance, capacitance) en des termes communs d'impédance comme une première étape dans l'analyse d'un circuit AC.







L'angle de phase de l'impédance pour tous les composants est le décalage de phase entre la chute de tension et le courant aux bornes du composant. Pour une résistance parfaite, la chute de tension et le courant sont *toujours* en phase l'un avec l'autre et donc l'angle d'impédance pour une résistance est de 0^o. Pour une inductance parfaite, la chute de tension est en avance sur le courant de 90^o et donc l'angle de phase de l'impédance de l'inductance est de $+90^o$. Pour un condensateur parfait, la chute de tension est en retard sur le courant de 90^o et donc l'angle de phase de l'impédance du condensateur est de -90^o.

Les impédances en AC se comportent d'une manière analogues aux résistances dans les circuits DC : elles s'ajoutent en série et elles diminuent en parallèle. Une version révisée de la Loi d'Ohm, basée sur l'impédance plutôt que sur la résistance, ressemble à ceci :

Ohm's Law for AC circuits:

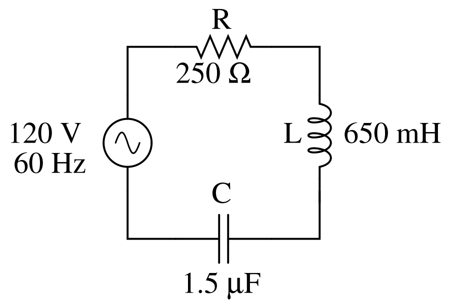
$$\mathbf{E = IZ} \qquad \mathbf{I = \frac{E}{Z}} \qquad \mathbf{Z = \frac{E}{I}}$$

All quantities expressed in complex, not scalar, form

Les Lois de Kirchhoff et toutes les méthodes d'analyse de réseau et les théorèmes sont vrais pour les circuits AC tant que les quantités sont représentées dans leur forme complexe plutôt que scalaire. Alors que cette équivalence qualifiée peut être arithmiquement stimulante, elle est conceptuellement simple et élégante. La seule difference réelle entre les calculs dans les circuits DC et AC est en regard de la *puissance*. Comme la réactance ne dissipe pas de puissance comme le fait une résistance, le concept de puissance dans les circuits AC est radicalement different de celui des circuits DC. Vous en saurez plus sur ce sujet dans un prochain chapitre!

5.2 R, L, C série

Prenons le circuit d'exemple suivant et analysons-le :



La première étape est de déterminer les réactances (en ohms) pour l'inductance et le condensateur.

$$X_L = 2\pi f L$$

$$X_L = (2)(\pi)(60\ \text{Hz})(650\ \text{mH})$$

$$X_L = 245.04\ \Omega$$

$$X_C = \frac{1}{2\pi f C}$$

$$X_C = \frac{1}{(2)(\pi)(60\ \text{Hz})(1.5\ \mu\text{F})}$$

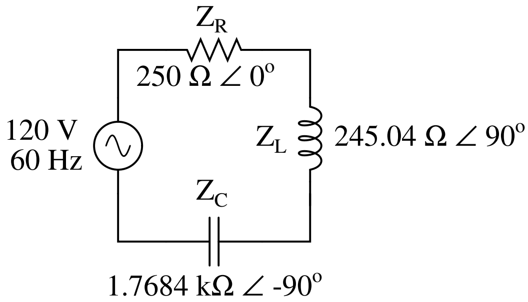
$$X_C = 1.7684\ \text{k}\Omega$$

L'étape suivante permet d'exprimer toutes les résistances et réactances dans une forme mathématique commune : l'impédance. Rappelez-vous qu'une réactance inductive se traduit par une impédance imaginaire positive (ou une impédance à +90^o), alors qu'une réactance capacitive se traduit par une impédance imaginaire négative (une impédance à -90^o). La résistance, bien sûr, est toujours considérée comme une impédance purement "réelle" (angle polaire de 0^o) :

$$Z_R = 250 + j0\ \Omega \quad \textit{or} \quad 250\ \Omega \angle 0^o$$

$$Z_L = 0 + j245.04\ \Omega \quad \textit{or} \quad 245.04\ \Omega \angle 90^o$$

$$Z_C = 0 - j1.7684k\ \Omega \quad \textit{or} \quad 1.7684\ \text{k}\Omega \angle -90^o$$



Maintenant que toutes les quantités d'opposition au courant électrique est exprimé dans un format de nombre complexe, commun (comme les impédances et pas comme des résistances ou des réactances), ils peuvent être traité de la même manière que des résistances dans un circuit DC. C'est le temps idéal pour dresser une table d'analyse pour ce circuit et insérons toutes les valeurs "données" (la tension totale et les impédances de résistances, inductances et condensateurs).

	R	L	C	Total	
E				**120 + j0** **120 ∠ 0°**	Volts
I					Amps
Z	**250 + j0** **250 ∠ 0°**	**0 + j245.04** **254.04 ∠ 90°**	**0 - j1.7684k** **1.7684k ∠ -90°**		Ohms

À moins que cela ne soit spécifié, la source de tension sera notre référence pour le décalage de phase et aura un angle de phase de 0^o. Rappelez-vous qu'il n'existe pas déphasage d'angle de phase "absolu" pour une tension ou d'un courant car c'est toujours une quantité reliée à une autre onde. Les angles de phase pour l'impédance, néanmoins (comme ceux de la résistance, de l'inductance et du condensateur), sont absoluments connus car les relations de phase entre la tension et le courant de chaque composant sont absolument définis.

Notez que je suppose une inductance et un condensateur parfaitement réactifs, avec des angles de phase d'impédances de exactement +90 et -90^o, respectivement. Bien que les composants réels ne sont pas parfaits de ce point de vue, ils doivent en être très proches. Pour simplifier, je suppose que les inductances et les condensateurs sont parfaits dans mes exemples de calculs sauf indication contraire.

Comme l'exemple précédent est un circuit série, nous savons que l'impédance totale du circuit est égale à la somme des parties, donc :

$$Z_{total} = Z_R + Z_L + Z_C$$

$$Z_{total} = (250 + j0\ \Omega) + (0 + j245.04\ \Omega) + (0 - j1.7684k\ \Omega)$$

$$Z_{total} = 250 - j1.5233k\ \Omega \quad \textit{or} \quad 1.5437\ k\Omega \angle -80.680^o$$

Inserons cette valeur de l'impédance totale dans notre table :

	R	L	C	Total	
E				120 + j0 120 $\angle$ 0°	Volts
I					Amps
Z	250 + j0 250 $\angle$ 0°	0 + j245.04 254.04 $\angle$ 90°	0 - j1.7684k 1.7684k $\angle$ -90°	**250 - j1.5233k** **1.5437k** $\angle$ -80.680°	Ohms

Rule of series circuits:
$Z_{total} = Z_R + Z_L + Z_C$

Nous pouvons maintenant appliquer la Loi d'Ohm (I=E/R) verticalement dans la colonne "Total" pour trouver le courant total de circuit série :

	R	L	C	Total	
E				120 + j0 120 $\angle$ 0°	Volts
I				**12.589m + 76.708m** **77.734m** $\angle$ 80.680°	Amps
Z	250 + j0 250 $\angle$ 0°	0 + j245.04 254.04 $\angle$ 90°	0 - j1.7684k 1.7684k $\angle$ -90°	250 - j1.5233k 1.5437k $\angle$ -80.680°	Ohms

Ohm's Law
$I = \frac{E}{Z}$

Dans un circuit série, le courant doit être égal dans tous les composants. Nous pouvons donc prendre la valeur obtenue pour le courant total et le distribuer dans chacune des colonnes :

	R	L	C	Total	
E				120 + j0 120 $\angle$ 0°	Volts
I	**12.589m + 76.708m** **77.734m** $\angle$ 80.680°	**12.589m + 76.708m** **77.734m** $\angle$ 80.680°	**12.589m + 76.708m** **77.734m** $\angle$ 80.680°	12.589m + 76.708m 77.734m $\angle$ 80.680°	Amps
Z	250 + j0 250 $\angle$ 0°	0 + j245.04 254.04 $\angle$ 90°	0 - j1.7684k 1.7684k $\angle$ -90°	250 - j1.5233k 1.5437k $\angle$ -80.680°	Ohms

Rule of series circuits:
$I_{total} = I_R = I_L = I_C$

Nous sommes prêts pour appliquer la Loi d'Ohm (E=IZ) à chacunes des colonnes des composants individuels dans la table, pour déterminer les chutes de tension :

	R	L	C	Total	
E	**3.1472 + j19.177** **19.434** ∠ 80.680°	**-18.797 + j3.0848** **19.048** ∠ 170.68°	**135.65 - j22.262** **137.46** ∠ -9.3199°	120 + j0 120 ∠ 0°	Volts
I	12.589m + 76.708m 77.734m ∠ 80.680°	12.589m + 76.708m 77.734m ∠ 80.680°	12.589m + 76.708m 77.734m ∠ 80.680°	12.589m + 76.708m 77.734m ∠ 80.680°	Amps
Z	250 + j0 250 ∠ 0°	0 + j245.04 254.04 ∠ 90°	0 - j1.7684k 1.7684k ∠ -90°	250 - j1.5233k 1.5437k ∠ -80.680°	Ohms
	↑ *Ohm's Law* E = IZ	↑ *Ohm's Law* E = IZ	↑ *Ohm's Law* E = IZ		

Veuillez noter quelque chose d'étrange ici : bien que notre tension d'alimentation est seulement de 120 volts, la tension aux bornes du condensateur est de 137.46 volts! Comment cela peut-il être? La réponse tient dans l'interaction entre les réactances inductives et capacitives. Exprimé comme impédance, nous pouvons voir que l'inductance s'oppose au courant d'une manière précisément opposée à celle d'un condensateur. Exprimé sous forme rectangulaire, l'inpédance de l'inducteur possède un terme imaginaire positif et le condensateur possède un terme imaginaire négatif. Lorsque ces deux impédances contraires sont ajoutées (en série), elles tendent à s'annuler l'une l'autre! Bien qu'elles soient encore *ajoutées ensemble* pour produire une somme, cette dernière est réellement *moindre* que l'une des deux impédances seules (capacitive ou inductive). Cela ressemble à l'addition de nombres positifs et négatifs (scalaires) : la somme est une quantité moindre que la valeur absolue de l'un des deux.

Si l'impédance totale dans un circuit série avec des éléments à la fois inductifs et capacitifs est moindre que l'impédance des éléments séparément alors le courant total dans ce circuit doit être *plus important* que celui qui passerait dans les éléments inductifs ou capacitifs seuls. Avec ce courant anormalemebt haut au travers de chaque composant, des tensions plus importantes que la tension de source peuvent être obtenues au travers des composants individuels! Les autres conséquences des réactances opposées des inductances et des condensateurs dans le même circuit seront explorées dans le chapitre suivant.

Une fois que vous avez maîtrisé la technique de réduction de toutes les valeurs de composants en impédances (Z), l'analyse de circuits AC est seulement de la même dificulté que l'analyse de circuits DC, excepté que les quantités traitent de vecteurs au lieu de scalaires. A l'exception des équations traitant de puissance (P), les équations dans les circuits AC sont les même que celles dans les circuits DC, utilisant les impédances (Z) au lieu des résistances (R). La Loi d'Ohm (E=IZ) reste vraie, de même que les Lois de Kirchhoff et du Courant.

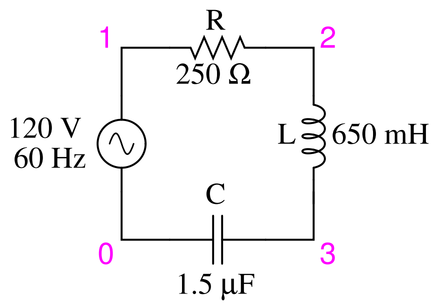
Pour démontrer la Loi de Tension de Kirchhoff dans un circuit AC, nous pouvons regarder les réponses que nous avons tirées des chutes de tension des composants dans le circuit précédent. KVL (Kirchhoff's Voltage Law) nous indique que la somme algébrique des chutes de tension aux bornes des résistances, inductances et condensateur doit être égale à la tension appliquée par la source. Même si cela semble faux au premier regard, une petite addition de nombre complexe prouve le contraire :

$E_R + E_L + E_C$ *should equal* E_{total}

$$
\begin{array}{rll}
 & 3.1472 + j19.177 \text{ V} & E_R \\
 & -18.797 + j3.0848 \text{ V} & E_L \\
+ & 135.65 - j22.262 \text{ V} & E_C \\
\hline
 & 120 + j0 \quad \text{V} & E_{total}
\end{array}
$$

Excepté les erreurs d'arrondi, la somme de ces chutes de tension est de 120 volts. Effectué sur un calculateur (préservant toutes les valeurs), la réponse que vous allez recevoir doit être *exactement* de 120 + j0 volts.

Nous pouvons aussi utiliser SPICE pour vérifier les valeurs de notre circuit :




```
ac r-l-c circuit
v1 1 0 ac 120 sin
r1 1 2 250
l1 2 3 650m
c1 3 0 1.5u
.ac lin 1 60 60
.print ac v(1,2) v(2,3) v(3,0) i(v1)
.print ac vp(1,2) vp(2,3) vp(3,0) ip(v1)
.end
```

```
freq          v(1,2)      v(2,3)      v(3)        i(v1)
6.000E+01     1.943E+01   1.905E+01   1.375E+02   7.773E-02
freq          vp(1,2)     vp(2,3)     vp(3)       ip(v1)
6.000E+01     8.068E+01   1.707E+02  -9.320E+00  -9.932E+01
```

Interpreted SPICE results

E_R = 19.43 V ∠ 80.68°

E_L = 19.05 V ∠ 170.7°

E_C = 137.5 V ∠ -9.320°

I = 77.73 mA ∠ -99.32° (*actual phase angle* = 80.68°)

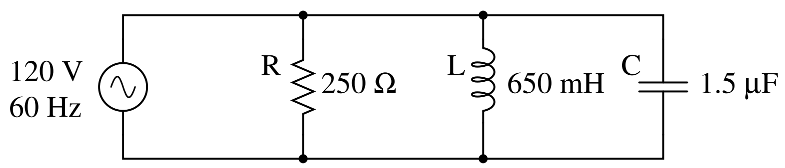
La simulation SPICE montre que nos calculs manuels sont corrects.

Comme vous pouvez le voir, il y a pet de différence entre l'analyse de circuit AC et DC, excepté que toutes les quantités de tension, courant et résistance (*l'impedance*) doivent réellement être traitées sous leur forme complexe au lieu de scalaire pour prendre en compte l'angle de phase. C'est bien car cela signifie que ce que vous avez appris à propos des circuits electriques DC s'applique à ce que vous apprenez ici. La seule exception à cette cohérence est le calcul de la puissance, qui est si unique qu'il a son propre chapitre.

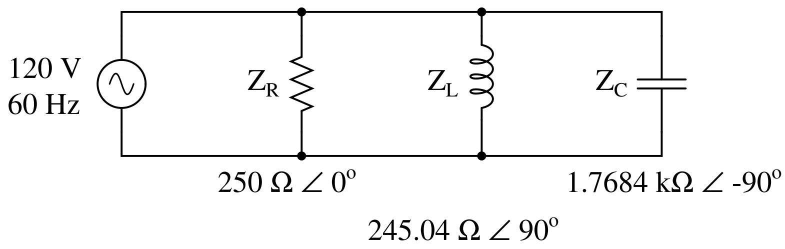
- **RÉSUMÉ :**
- Les impédances de tout type s'ajoutent en série : $Z_{Total} = Z_1 + Z_2 + \ldots Z_n$
- Bien que les impédances s'ajoutent en série, l'impédance totale pour un circuit contenant à la fois une inductance et un condensateur peut être inférieure ou supérieur qu'une impédance individuelle car les impédances inductives et capacitives séries tendent à s'annuler l'une l'autre. Ceci peut mener à des chutes de tension au travers des composants excedant la tension d'alimentation!
- Toutes les règles et les lois des circuits DC s'appliquent aux circuits AC, tant que les valeurs sont exprimées sous forme complexe plutôt que scalaire. La seule exception à ce principe est le calcul de la *puissance*, qui différente de celle de l'AC.

5.3 R, L, C parallèle

Nous pouvons prendre les mêmes composants du circuit série et les réaranger en une configuration parallèle pour obtenir facilement un exemple de circuit :



Le fait que ces composants soient maintenant connectés en parallèle au lieu de série n'a absolument aucun effet sur leur impédance individuelle. Aussi longtemps que l'alimentation est à la même fréquence qu'avant, les réactances inductives et capacitives ne changent pas :



Avec toutes les valeurs de composants exprimées comme des impédances (Z), nous pouvons initialiser une table d'analyse et de procéder comme dans le précédent problème d'exemple, excepté cette fois en suivant les règles des circuits parallèles au lieu de série :

	R	L	C	Total	
E				**120 + j0** **120 ∠ 0°**	Volts
I					Amps
Z	**250 + j0** **250 ∠ 0°**	**0 + j245.04** **254.04 ∠ 90°**	**0 - j1.7684k** **1.7684k ∠ -90°**		Ohms

En sachant que la tension est partagée également par tous les composants dans un circuit parallèle, nous pouvons transférer la valeur de la tension complète dans toutes les colonnes de composants de la table :

	R	L	C	Total	
E	**120 + j0** **120 ∠ 0°**	**120 + j0** **120 ∠ 0°**	**120 + j0** **120 ∠ 0°**	120 + j0 120 ∠ 0°	Volts
I					Amps
Z	250 + j0 250 ∠ 0°	0 + j245.04 254.04 ∠ 90°	0 - j1.7684k 1.7684k ∠ -90°		Ohms

Rule of parallel circuits:
$E_{total} = E_R = E_L = E_C$

Nous pouvons maintenant appliquer la Loi d'Ohm (I=E/Z) verticalement dans chaque colonne pour déterminer le courant dans chaque composant :

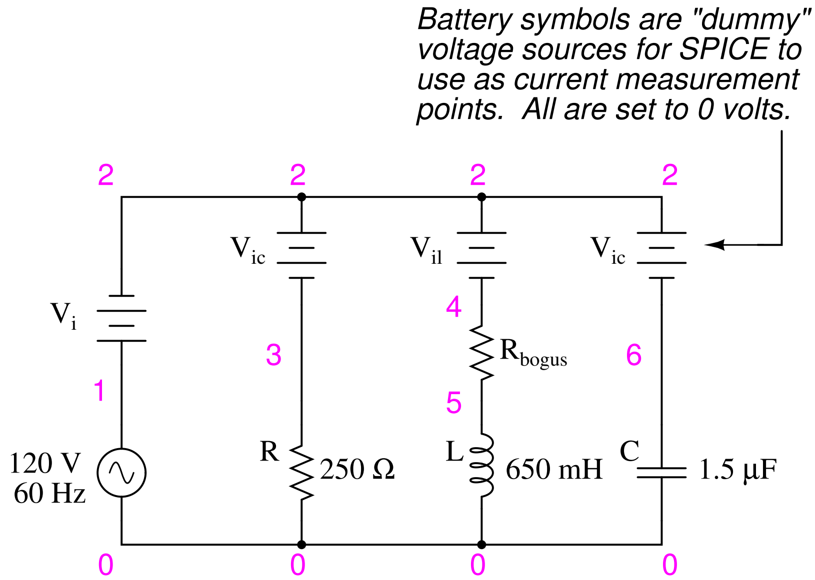
	R	L	C	Total	
E	120 + j0 120 ∠ 0°	120 + j0 120 ∠ 0°	120 + j0 120 ∠ 0°	120 + j0 120 ∠ 0°	Volts
I	**480m + j0** **480** ∠ 0°	**0 - j489.71m** **489.71m** ∠ -90°	**0 + j67.858m** **67.858m** ∠ 90°		Amps
Z	250 + j0 250 ∠ 0°	0 + j245.04 254.04 ∠ 90°	0 - j1.7684k 1.7684k ∠ -90°		Ohms

	R	L	C
	↑ *Ohm's Law* $I = \frac{E}{Z}$	↑ *Ohm's Law* $I = \frac{E}{Z}$	↑ *Ohm's Law* $I = \frac{E}{Z}$

Il existe deux stratégies pour calculer le courant et l'impédance total. Nous pouvons d'abord calculer l'impédance totale depuis toutes les impédances individuelles en parallèle ($Z_{Total} = 1/(1/Z_R + 1/Z_L + 1/Z_C)$ et calcule alors le courant total en divisant la tension source par l'impédance totale (I=E/Z). Néanmoins, travailler des équations des impédances parallèles avec les nombres complexes n'est pas une tâche facile, avec toutes les réciproques (1/Z). C'est spécialement vrai si vous n'avez pas la chance d'avoir une calculatrice qui traite les nombres complexes et que vous êtes forcés de tout faire à la main (effectuer une réciproque des impédances individuelles sous la forme polaire et les convertir tous en sus sous la forme polaire pour l'inversion finale puis faire l'inversion). La seconde manière de calculer le courant et l'impédance total est d'ajouter toutes les branches de courant pour obtenir un courant total (le courant total dans un circuit parallèle – AC ou DC – est égal à la somme des branches de courant), puis utilisons la Loi d'Ohm pour déterminer l'impédance totale de la tension totale et du courant total (Z=E/I).

	R	L	C	Total	
E	120 + j0 120 ∠ 0°	120 + j0 120 ∠ 0°	120 + j0 120 ∠ 0°	120 + j0 120 ∠ 0°	Volts
I	480m + j0 480 ∠ 0°	0 - j489.71m 489.71m ∠ -90°	0 + j67.858m 67.858m ∠ 90°	**480m - j421.85m** **639.03m** ∠ -41.311°	Amps
Z	250 + j0 250 ∠ 0°	0 + j245.04 254.04 ∠ 90°	0 - j1.7684k 1.7684k ∠ -90°	**141.05 + j123.96** **187.79** ∠ 41.311°	Ohms

Les deux méthodes, effectuées correctement fourniront les réponses correctes. Analysons ce circuit avec SPICE et voyons ce qui arrive :




```
ac r-l-c circuit
v1 1 0 ac 120 sin
vi 1 2 ac 0
vir 2 3 ac 0
vil 2 4 ac 0
rbogus 4 5 1e-12
vic 2 6 ac 0
r1 3 0 250
l1 5 0 650m
c1 6 0 1.5u
.ac lin 1 60 60
.print ac i(vi) i(vir) i(vil) i(vic)
.print ac ip(vi) ip(vir) ip(vil) ip(vic)
.end
```

```
freq          i(vi)       i(vir)      i(vil)      i(vic)
6.000E+01     6.390E-01   4.800E-01   4.897E-01   6.786E-02
freq          ip(vi)      ip(vir)     ip(vil)     ip(vic)
6.000E+01    -4.131E+01   0.000E+00  -9.000E+01   9.000E+01
```

Interpreted SPICE results

I_{total} = 639.0 mA ∠ -41.31°

I_R = 480 mA ∠ 0°

I_L = 489.7 mA ∠ -90°

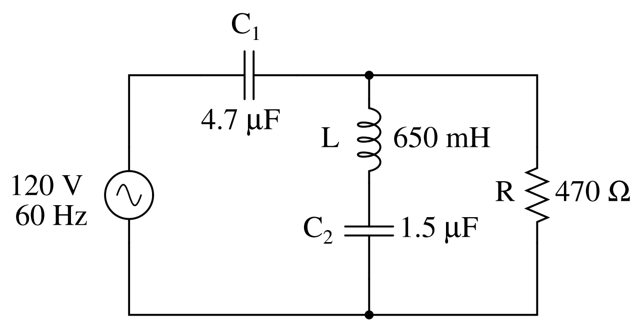
I_C = 67.86 mA ∠ 90°

Il y a un petit truc pour faire fonctionner SPICE comme il faut sur ce circuit (en installant des sources de tension "factices" dans chaque branche pour obtenir les valeurs du courant et positionner une résistance "factice" dans la branche de l'inductance pour éviter une boucle de source directe inductance-vers-tension, ce que ne peut tolérer SPICE) mais nous avons les bonnes lectures. En plus de ça, en installant les sources de tension factices (mesure de courant) dans les bonnes directions, nous pouvons limiter les idiosyncrasies de SPICE qui affiche des valeurs de courant déphasés de 180^o. De cette manière, nos lecture de phase de courant correspondent exactement à nos calculs manuels.

5.4 R, L, C série-parallèle

Maintenant que nous avons vu que l'analyse de circuits AC série et parallèle n'est pas fondamentalement differente de l'analyse de circuit DC, il ne devrait pas être surprenant que l'analyse série-parallèle soit identique, simplement en utilisant les nombres complexes au lieu de scalaires pour représenter la tension, le courant et l'impédance.

Prenons ce circuit série-parallèle par exemple :



La première chose à faire, comme d'habitude, est de déterminer les valeurs de l'impédance (Z) pour tous les les composants basés sur la fréquence de la source de puissance AC. Pour ce faire, nous avons d'abord besoin de déterminer les valeurs de la réactance (X) pour tous les inductances et les condensateurs, il faut alors convertir les valeurs de réactance (X) et de résistance (R) sous la forme d'impédance (Z) :

Reactances and Resistances:

$X_{C1} = \frac{1}{2\pi f C_1}$ $X_{C1} = \frac{1}{(2)(\pi)(60\text{ Hz})(4.7\ \mu\text{F})}$ $X_{C1} = 564.38\ \Omega$	$X_L = 2\pi f L$ $X_L = (2)(\pi)(60\text{ Hz})(650\text{ mH})$ $X_L = 245.04\ \Omega$
$X_{C2} = \frac{1}{2\pi f C_2}$ $X_{C2} = \frac{1}{(2)(\pi)(60\text{ Hz})(1.5\ \mu\text{F})}$ $X_{C2} = 1.7684\text{ k}\Omega$	$R = 470\ \Omega$

$Z_{C1} = 0 - j564.38\ \Omega$ *or* $564.38\ \Omega \angle -90^{o}$

$Z_L = 0 + j245.04\ \Omega$ *or* $245.04\ \Omega \angle 90^{o}$

$Z_{C2} = 0 - j1.7684k\ \Omega$ *or* $1.7684\text{ k}\Omega \angle -90^{o}$

$Z_R = 470 + j0\ \Omega$ *or* $470\ \Omega \angle 0^{o}$

Nous pouvons maintenant initilisaliser les valeurs de notre table :

	C_1	L	C_2	R	Total	
E					120 + j0 $120 \angle 0^{o}$	Volts
I						Amps
Z	0 - j564.38 $564.38 \angle -90^{o}$	0 + j245.04 $245.04 \angle 90^{o}$	0 - j1.7684k $1.7684k \angle -90^{o}$	470 + j0 $470 \angle 0^{o}$		Ohms

Dans un circuit qui fait une *combinaison* série-parallèle, nous devons réduire l'impédance totale en plus d'une étape. La première étape est de combiner L et C_2 comme une combinaison série d'impédances, en ajoutant leur impédances ensemble. Cette impédance sera alors combinée en parallèle avec l'impédance de la résistance, pour arriver à une autre combinaison d'impédances. Finalement, cette quantité sera ajoutée à l'impédance de C_1 pour obtenir l'impédance totale.

De manière à ce que notre table puisse suivre toutes ces étapes, il sera nécessaire de lui ajouter des colonnes additionnelles de telle manière que chaque étape soit représentée. L'ajout de colonnes horizontales à la table montrée ci-dessus ne serait pas pratique pour des raisons de formatage, je vais donc placer une nouvelle ligne en dessous, chaque colonne étant désignée par sa combinaison respective de composants :

	L -- C_2	R // (L -- C_2)	*Total* C_1 -- [R // (L -- C_2)]	
E				Volts
I				Amps
Z				Ohms

Calculer ces nouvelles impédances (combinaison) nécessitera une addition complexe pour les combinaisons série et la formule "réciproque" pour les impédances complexes en parallèle. Ce fois, l'utilisation de la formule réciproque ne peut être évitée : les valeurs nécessaires ne peuvent être obtenus d'une autre manière!

	L -- C_2	R // (L -- C_2)	*Total* C_1 -- [R // (L -- C_2)]	
E			120 + j0 120 $\angle$ 0°	Volts
I				Amps
Z	**0 - j1.5233k** **1.5233k** $\angle$ -90°	**429.15 - j132.41** **449.11** $\angle$ -17.147°	**429.15 - j696.79** **818.34** $\angle$ -58.371°	Ohms

Rule of series circuits:
$Z_{L\text{--}C2} = Z_L + Z_{C2}$

Rule of parallel circuits:
$$Z_{R//(L\text{--}C2)} = \frac{1}{\frac{1}{Z_R} + \frac{1}{Z_{L\text{--}C2}}}$$

Rule of series circuits:
$Z_{total} = Z_{C1} + Z_{R//(L\text{--}C2)}$

Avec le colonne "Total" de la seconde table, nous pouvons l'enlever sans danger cette colonne de la première table. Cela nous donne une table avec quatre colonnes et une autre avec trois colonnes.

Maintenant que nous connaissons l'impédance totale de (818.34 Ω $\angle$ -58.371^o) et la tension (120 volts $\angle$ 0^o), nous pouvons appliquer la Loi d'Ohm (I=E/Z) verticalement dans la colonne "Total" pour obtenir la valeur du courant total :

	$L -- C_2$	$R // (L -- C_2)$	*Total* C_1 -- [$R // (L -- C_2)$]	
E			120 + j0 120 ∠ 0°	Volts
I			**76.899m + j124.86m** **146.64m** ∠ 58.371°	Amps
Z	0 - j1.5233k 1.5233k ∠ -90°	429.15 - j132.41 449.11 ∠ -17.147°	429.15 - j696.79 818.34 ∠ -58.371°	Ohms

Ohm's Law

$$I = \frac{E}{Z}$$

A ce point, nous pourrions nous poser la question : existe-t-il des composants ou des combinaisons de composants qui partagent soit le la tension soit le courant? Dans ce cas, et C_1 et la combinaison parallèle de R//(L−−C_2) partagent le même courant (total) car l'impédance totale est composée de deux jeux d'impédances en série. Nous pouvons donc transférer la valeur du courant total dans les deux colonnes :

	C_1	L	C_2	R	
E					Volts
I	**76.899m + j124.86m** **146.64m** ∠ 58.371°				Amps
Z	0 - j564.38 564.38 ∠ -90°	0 + j245.04 245.04 ∠ 90°	0 - j1.7684k 1.7684k ∠ -90°	470 + j0 470 ∠ 0°	Ohms

Rule of series circuits:

$I_{total} = I_{C1} = I_{R//(L--C2)}$

	$L -- C_2$	$R // (L -- C_2)$	*Total* C_1 -- [$R // (L -- C_2)$]	
E			120 + j0 120 ∠ 0°	Volts
I		**76.899m + j124.86m** **146.64m** ∠ 58.371°	76.899m + j124.86m 146.64m ∠ 58.371°	Amps
Z	0 - j1.5233k 1.5233k ∠ -90°	429.15 - j132.41 449.11 ∠ -17.147°	429.15 - j696.79 818.34 ∠ -58.371°	Ohms

Rule of series circuits:

$I_{total} = I_{C1} = I_{R//(L--C2)}$

Nous pouvons maintenant calculer les chutes de tension aux bornes de C_1 et de la combinaison série-parallèle de R//(L−−C_2) en utilisant la Loi d'Ohm (E=IZ) verticalement dans ces colonnes de la table :

	C_1	L	C_2	R	
E	**70.467 - j43.400** **82.760** ∠ -31.629°				Volts
I	76.899m + j124.86m 146.64m ∠ 58.371°				Amps
Z	0 - j564.38 564.38 ∠ -90°	0 + j245.04 245.04 ∠ 90°	0 - j1.7684k 1.7684k ∠ -90°	470 + j0 470 ∠ 0°	Ohms

Ohm's Law
E = IZ

	L -- C_2	R // (L -- C_2)	*Total* C_1 -- [R // (L -- C_2)]	
E		**49.533 + j43.400** **65.857** ∠ 41.225°	120 + j0 120 ∠ 0°	Volts
I		76.899m + j124.86m 146.64m ∠ 58.371°	76.899m + j124.86m 146.64m ∠ 58.371°	Amps
Z	0 - j1.5233k 1.5233k ∠ -90°	429.15 - j132.41 449.11 ∠ -17.147°	429.15 - j696.79 818.34 ∠ -58.371°	Ohms

Ohm's Law
E = IZ

Un rapide double contrôle de notre travail à ce point serait de voir si les chutes de tension aux bornes de C_1 et la combinaison série-parallèle de R//(L−−C_2) s'ajoutent réellement au total. Selon la Loi de Tension de Kirchhoff, elles devraient!

E_{total} *should be equal to* $E_{C1} + E_{R//(L--C2)}$

$$\begin{array}{r} 70.467 - j43.400 \text{ V} \\ +\ 49.533 + j43.400 \text{ V} \\ \hline 120 + j0 \text{ V} \end{array} \longleftarrow \textit{Indeed, it is!}$$

La dernière étape était purement une précaution. Dans un problème avec autant d'étapes que celui-là, il y a beaucoup d'oportunités d'erreurs. Des contrôles croisés occasionels comme celui-ci peuvent économiser à une personne beaucoup de travail et des frustrations non nécessaires en identifiant les problèmes avec l'étape finale de la recherche.

Après avoir résolu les chutes de tension aux bornes de C_1 et la combinaison R//(L−−C_2), nous nous reposons la question : quels autres composants partagent les même valeurs de tension ou courant? Dans ce cas, la résistance (R) et la combinaison de l'inducteur et du seconde condensateur (L−−C_2) partagent la même tension car ces jeux d'impédances sont en parallèle l'un avec l'autre. Nous pouvons donc transférer la valeur de la tension que nous venons de résoudre dans les deux colonnes pour R et L−−C_2 :

	C_1	L	C_2	R	
E	70.467 - j43.400 82.760 ∠ -31.629°			**49.533 + j43.400** **65.857** ∠ 41.225°	Volts
I	76.899m + j124.86m 146.64m ∠ 58.371°				Amps
Z	0 - j564.38 564.38 ∠ -90°	0 + j245.04 245.04 ∠ 90°	0 - j1.7684k 1.7684k ∠ -90°	470 + j0 470 ∠ 0°	Ohms

Rule of parallel circuits:

$E_{R//(L--C2)} = E_R = E_{L--C2}$

	L -- C_2	R // (L -- C_2)	*Total* C_1 -- [R // (L -- C_2)]	
E	**49.533 + j43.400** **65.857** ∠ 41.225°	49.533 + j43.400 65.857 ∠ 41.225°	120 + j0 120 ∠ 0°	Volts
I		76.899m + j124.86m 146.64m ∠ 58.371°	76.899m + j124.86m 146.64m ∠ 58.371°	Amps
Z	0 - j1.5233k 1.5233k ∠ -90°	429.15 - j132.41 449.11 ∠ -17.147°	429.15 - j696.79 818.34 ∠ -58.371°	Ohms

Rule of parallel circuits:

$E_{R//(L--C2)} = E_R = E_{L--C2}$

Nous avons maintenant tout ce qu'il faut pour calculer le courant aux bornes de la résistance et de la combinaison série L$--$C$_2$. Tout ce ue nous avons à faire est d'appliquer la Loi d'Ohm (I=E/Z) verticalement dans les deux colonnes :

	C_1	L	C_2	R	
E	70.467 - j43.400 82.760 ∠ -31.629°			49.533 + j43.400 65.857 ∠ 41.225°	Volts
I	76.899m + j124.86m 146.64m ∠ 58.371°			**105.39m + j92.341m** **140.12m** ∠ 41.225°	Amps
Z	0 - j564.38 564.38 ∠ -90°	0 + j245.04 245.04 ∠ 90°	0 - j1.7684k 1.7684k ∠ -90°	470 + j0 470 ∠ 0°	Ohms

Ohm's Law

$I = \frac{E}{Z}$

	$L -- C_2$	$R // (L -- C_2)$	*Total* $C_1 -- [R // (L -- C_2)]$	
E	49.533 + j43.400 65.857 ∠ 41.225°	49.533 + j43.400 65.857 ∠ 41.225°	120 + j0 120 ∠ 0°	Volts
I	**-28.490m + j32.516m** **43.232m** ∠ 131.22°	76.899m + j124.86m 146.64m ∠ 58.371°	76.899m + j124.86m 146.64m ∠ 58.371°	Amps
Z	0 - j1.5233k 1.5233k ∠ -90°	429.15 - j132.41 449.11 ∠ -17.147°	429.15 - j696.79 818.34 ∠ -58.371°	Ohms

Ohm's Law

$$I = \frac{E}{Z}$$

Un autre double contrôle rapide de notre travail serait de voir si les valeurs de courant pour $L--C_2$ et R s'ajoutent au courant total. Selon la Loi de Courant de Kirchhoff, on devrait avoir :

$I_{R//(L--C2)}$ *should be equal to* $I_R + I_{(L--C2)}$

$$\begin{array}{rl} & 105.39\text{m} + j92.341\text{m} \\ + & -28.490\text{m} + j32.516\text{m} \\ \hline & 76.899\text{m} + j124.86\text{m} \end{array} \longleftarrow \textit{Indeed, it is!}$$

Comme L et C_2 sont connectés en série et comme nous connaissons le courant dans la combinaison de leur impédance série, nous pouvons distribuer cette valeur de courant dans les colonnes L et C_2 suivant la règle des circuits série où les composants série partagent le même courant :

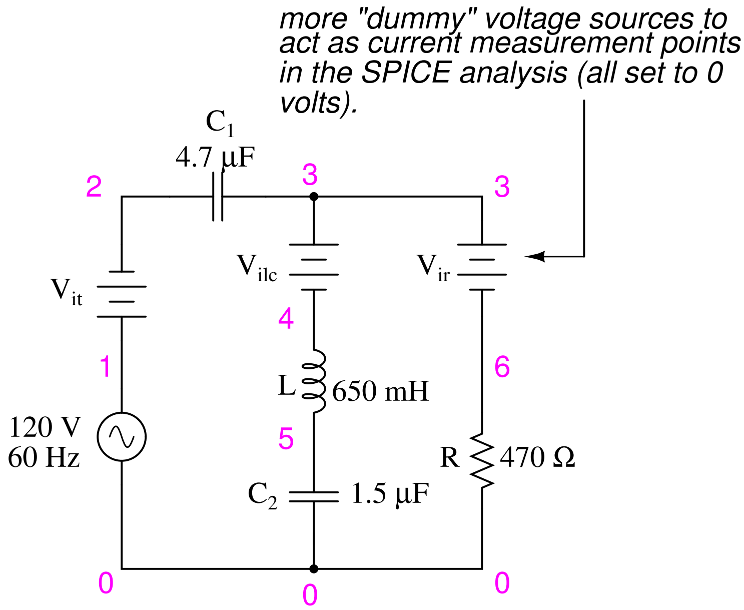
	C_1	L	C_2	R	
E	70.467 - j43.400 82.760 ∠ -31.629°			49.533 + j43.400 65.857 ∠ 41.225°	Volts
I	76.899m + j124.86m 146.64m ∠ 58.371°	**-28.490m + j32.516m** **43.232m** ∠ 131.22°	**-28.490m + j32.516m** **43.232m** ∠ 131.22°	105.39m + j92.341m 140.12m ∠ 41.225°	Amps
Z	0 - j564.38 564.38 ∠ -90°	0 + j245.04 245.04 ∠ 90°	0 - j1.7684k 1.7684k ∠ -90°	470 + j0 470 ∠ 0°	Ohms

Rule of series circuits:
$I_{L--C2} = I_L = I_{C2}$

Avec la dernière étape (deux calculs, réellement), nous pouvons compléter notre table d'analyse pour ce circuit. Avec les valeurs d'impédance et de courant en place pour L et C_2, tout ce que nous avons à faire est d'appliquer la Loi d'Ohm (E=IZ) verticalement dans ces deux colonnes pour calculer les chutes de tension.

	C_1	L	C_2	R	
E	70.467 - j43.400 82.760 ∠ -31.629°	**-7.968 - j6.981** **10.594** ∠ 221.22°	**57.501 + j50.382** **76.451** ∠ 41.225	49.533 + j43.400 65.857 ∠ 41.225°	Volts
I	76.899m + j124.86m 146.64m ∠ 58.371°	-28.490m + j32.516m 43.232m ∠ 131.22°	-28.490m + j32.516m 43.232m ∠ 131.22°	105.39m + j92.341m 140.12m ∠ 41.225°	Amps
Z	0 - j564.38 564.38 ∠ -90°	0 + j245.04 245.04 ∠ 90°	0 - j1.7684k 1.7684k ∠ -90°	470 + j0 470 ∠ 0°	Ohms
		Ohm's Law E = IZ	*Ohm's Law* E = IZ		

Allons maintenant avec SPICE pour vérifier notre travail avec un ordinateur :




```
ac series-parallel r-l-c circuit
v1 1 0 ac 120 sin
vit 1 2 ac 0
vilc 3 4 ac 0
vir 3 6 ac 0
c1 2 3 4.7u
l 4 5 650m
c2 5 0 1.5u
r 6 0 470
.ac lin 1 60 60
.print ac v(2,3) vp(2,3) i(vit) ip(vit)
.print ac v(4,5) vp(4,5) i(vilc) ip(vilc)
.print ac v(5,0) vp(5,0) i(vilc) ip(vilc)
```

```
.print ac v(6,0) vp(6,0) i(vir) ip(vir)
.end

freq          v(2,3)      vp(2,3)     i(vit)      ip(vit)     C1
6.000E+01     8.276E+01  -3.163E+01   1.466E-01   5.837E+01

freq          v(4,5)      vp(4,5)     i(vilc)     ip(vilc)    L
6.000E+01     1.059E+01  -1.388E+02   4.323E-02   1.312E+02

freq          v(5)        vp(5)       i(vilc)     ip(vilc)    C2
6.000E+01     7.645E+01   4.122E+01   4.323E-02   1.312E+02

freq          v(6)        vp(6)       i(vir)      ip(vir)     R
6.000E+01     6.586E+01   4.122E+01   1.401E-01   4.122E+01
```

Chaque ligne de listing de sortie de SPICE donne la tension, l'angle de phase de la tension, le courant, et l'angle de phase du courant pour C_1, L, C_2 et R, dans cet ordre. Comme vous pouvez le voir, ces valeurs correspondent avec nos valeurs calculées à la main dans notre table d'analyse du circuit.

Aussi effrayante que peut apparaître une analyse de circuit AC en série-parallèle, il doit être souligné qu'il n'y a rien de réellement nouveau ici, excepté l'utilisation de nombres complexes. La Loi d'Ohm (dans sa nouvelle forme de E=IZ) reste encore vraie, de même que les Lois de tension et de courant de Kirchhoff. Alors qu'il est possible de faire potentiellement plus d'erreurs dans les calculs nécessaires des nombres complexes, les principes et les techniques de réduction de circuits série-parallèle sont exactement les mêmes.

- **RÉSUMÉ :**
- L'analyse de circuits série-parallèle AC est identique à celle des circuits série-parallèle DC. La seule différence substancielle est que toutes les valeurs et les calculs sont dans la forme complexe (non scalaire).
- Il est important de se rappeler qu'avant la réduction série-parallèle (simplification) ne démarre, vous devez déterminer l'impédance (Z) de chaque résistance, inductance et condensateur. De cette manière, tous les valeurs de composants seront exprimés dans des termes habituels (Z) au lieu d'un mélange incompatible de résistances (R), inductances (L) et condensateurs (C).

5.5 Susceptance et Admittance

Dans l'étude des circuits DC, l'étudiant en électricité aborde un terme signifiant l'opposé de la résistance : la *conductance*. C'est un terme utile lors de l'exploration de la formule mathématique pour les résrstances parallèles : $R_{parallel} = 1 / (1/R_1 + 1/R_2 + \ldots 1/R_n)$. Contrairement à la résistance, qui diminue alors que plus de composants sont inclus dans un circuit, la conductance les ajoute simplement. Mathématiquement, la conductance est la réciproque de la résistance et chaque terme 1/R dans la "formule de résistance parallèle" est réellement une conductance.

Alors que le terme "résistance" denote l'amplitude de l'opposition au flux des électrons dans un circuit, la "conductance" réprésente la facilité avec laquelle les électrons peuvent passer. La résistance est la mesure de combien un circuit *résiste* au courant, alors que la conductance est la mesure de combien un circuit *conduit* le courant. La conductance est utilisée pour être mesurée dans l'unité des *mhos* ou "ohms" épelé à l'envers. Maintenant, la bonne unité de mesure est le *Siemens*. Lorsqu'il est symbolisé dans une formule mathématique, la lettre correcte à utiliser pour la conductance est "G".

Les composants réactifs tels que les inductances et les condensateurs s'pposent au flux des électrons en fonction du temps, plutôt que d'une manière constante, sans changer de quantité de frottement comme le font les résistances. Nous appelons cette opposition basée sur le temps, *reactance* et comme la résistance, nous la mesurons dans l'unité des *ohms*.

De la même manière que la conductance est le complément de la résistance, il existe aussi une expression complémentaire de la réactance, appelée *susceptance*. Elle est égale, mathématiquement, à 1/X, au réciproque de la réactance. Comme la conductance, il est habituel de la mesurer dans l'unité des mhos mais elle maintenant en mesurée en Siemens. Son symbole mathématique est "B", malheureusement le même symbole utilisé pour représenter la densité de flux magnétique.

Les termes "réactance" et "susceptance" ont une certaine logique linguistique en eux, comme résistance et conductance. Alors que la réactance est la mesure de combien un circuit *réagit* contre le changement du courant dans le temps, la susceptance est la mesure de combien un circuit est *susceptible* de conduire un changement de courant.

Si l'on a pour tâche de déterminer l'effet total de plusieurs réactances pures, connectées en parallèle, l'on pourrait convertir chaque réactance (X) en un susceptance (B) puis ajouter les susceptances plutôt que diminuer les réactances : $X_{parallel} = 1/(1/X_1 + 1/X_2 + \ldots 1/X_n)$. Comme les conductances (G), les susceptances (B) s'ajoutent en parallèle et se retranchent en série. De même que la conductance, la susceptance est une quantité scalaire.

Lorsque des composants résistifs et réactifs sont connectés, leurs effets combinés ne peuvent plus être analisés avec les quantités scalaires des résistances (R) et des réactances (X). De même, les valeurs de conductances (G) et de susceptances (B) sont lus utiles dans les circuits où les deux types d'opposition ne sont pas mélangés, i.e. soit un circuit purement résistif (conductif) ou un circuit purement réactif (susceptif). De manière à exprimer et quantifier les effets du mélange des composants résistifs et réactifs, nous devons avoir un nouveau terme : *impédance*, measured in ohms and symbolized by the letter "Z".

Pour être cohérents, nous avons besoin d'une mesure complémentaire représentant la réciproque d'une impédance. Le nom de cette mesure est *admittance*. L'admittance est measurée en (devinez quoi ?) l'unité de Siemens et son symbole est "Y". Comme l'impédance, l'admittance est une quantité complexe plutôt que scalaire. Nous voyons encor eune certaine logique à la signification de ce nouveau terme : alors que l'impédance est une mesure de combiende courant alternatif est *empéché* dans un circuit, l'admittance est une mesure de combien de courant est *admis*.

Avec une calculatrice scientifique capable de traiter les nombres arithmétiques complexes sous forme polaire et rectangulaire, vous ne devriez jamais avoir à travailler avec des valeurs de susceptance (B) ou d'admittance (Y). Soyez tout de même au courant de leur existence et de leur signification.

5.6 Résumé

Avec l'exception notable des calculs de puissance (P), tous les calculs de circuits AC sont basés sur les même principes généraux que les calculs pour les circuits DC. La seule différence significative est le fait que les calculs AC utilisent des quantités complexes alors que les calculs DC utilisent des quantités scalaires. La Loi d'Ohm, de Kirchhoff et même les théorèmes de réseau appris en DC restent vrais pour l'AC lorsque la tension, le courant et l'impédance sont tous exprimés avec des nombres complexes. Les mêmes stratégies de dépanage appliquées pour les circuits DC sont utilisées pour l'AC, bien que ce dernier puisse être beaucoup plus difficile à travailler à cause des angles de phase qui ne sont pas affichés par les multimètres manuels .

La puissance est un problème entièrement différent et sera couvert par son propre chapitre dans ce livre. Comme la puissance dans un circuit réactif est à la fois absorbée et émise – pas seulement dissipée comme pour les résistances –son traitement mathématique nécessite des applications de trigonométriques plus directes pour la résolution.

Lorsque l'on est confronté à l'analyse d'un circuit AC, la première étape est de convertir toutes les valeurs de résistances, inductances et condensateurs en impédances (Z), basées sur la fréquence de la source de puissance. Après ceci, il faut procéder avec les même étapes et stratégies apprises pour l'analyse de circuits DC, en utilisant la "nouvelle" forme de la Loi d'Ohm : E=IZ ; I=E/Z ; et Z=E/I

Rappelez-vous que seules les valeurs calculées, exprimées sous forme *polaire*, s'appliquent directement à des mesures empiriques de tension et de courant. La notation rectangulaire est plus utile comme outil pour ajouter et soustraire des valeurs complexes ensemble. La notation polaire, où la magnitude (longueur de vecteur) est directement reliée à la magnitude de la mesure de la tension ou du courant et l'angle est directement relié au décalage de phase en degrés, est la forme la plus pratique pour exprimer des valeurs complexes pour l'analyse de circuit.

5.7 Contributeurs

Les contributeurs de ce chapitre sont listés dans l'ordre chronologique de leurs contributions, depuis le plus récent jusqu'au premier. Voyez l'Annexe 2 (Liste des contributeur) pour les dates et les informations de contact.

Jason Starck (June 2000) : HTML document formatting, which led to a much better-looking second edition.

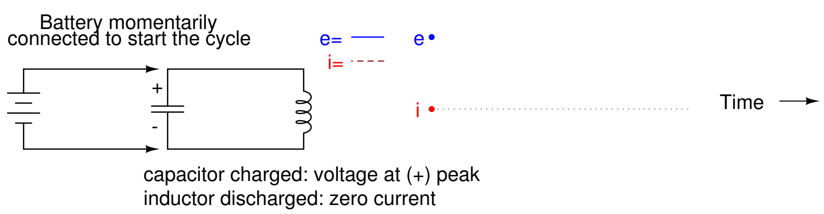
Chapter 6

RÉSONANCE

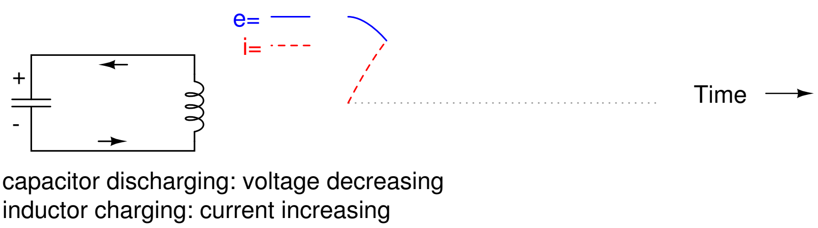
6.1 Un pendule électrique

Les condensateurs stockent leur énergie sous forme de champ électrique et manifestent électriquement cette énergie stockée comme un potentiel : *tension statique*. Les inductances stockent de l'énergie sous forme de champ magnétique et la manifestent électriquement sous forme de déplacement cinétique des électrons : le *courant*. Les condensateurs et les inductances sont les deux côtés de la même pièce réactive, stockant et libérant de l'énergie dans des modes complémentaires. Lorsque ces deux types de composants réactifs sont connectés directement ensemble, leur tendance complémentaire à stocker de l'énergie produira un résultat inhabituel.

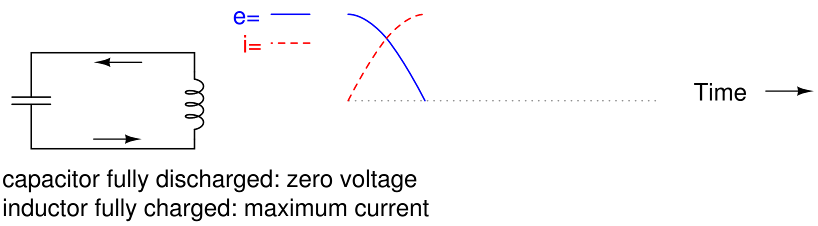
Si le condensateur ou l'inductance démarre dans un état chargé, les deux composants échangent de l'énergie, d'un côté à l'autre, créant leur propre tension AC et leurs cycles de courant. Si nous supposons que les deux composants sont soumis à des applications soudaines de tension (disons une batterie connectée momentanément), le condensateur se chargera rapidement et l'inductance s'opposera au changement de courant, laissant le condensateur dans l'état de charge et l'inductance déchargée :



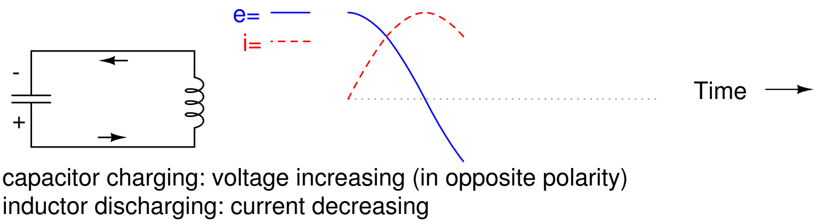
Le condensateur va commencer à se décharger, faisant baisser sa tension. Pendant ce temps, l'inductance commencera à se créer une "charge" sous la forme d'un champ magnétique alors que le courant augmente dans le circuit :



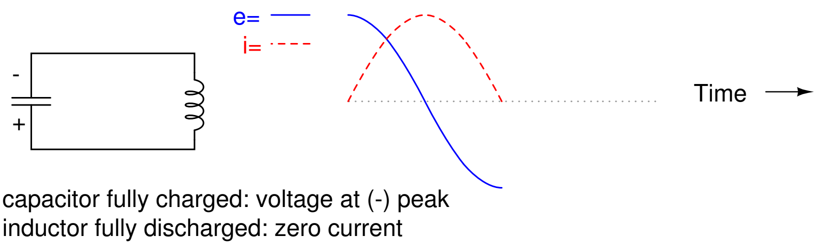
L'inductance, encore chargée, gardera le flux des électrons dans le circuit jusqu'à ce que le condensateur se soit complètement déchargé, n'ayant plus de tension à ses bornes :



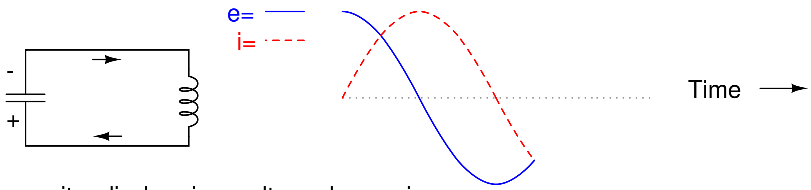
L'inductance maintiendra le flux du courant même s'il n'y a pas de tension appliquée. En fait, elle génèrera une tension (comme une batterie) de manière à maintenir le courant dans la même direction. Le condensateur, étant le récipient du courant, commencera à accumuler une charge de la polarité opposée à celle d'avant :



Lorsque l'inductance est finalement vidée de sa réserve d'énergie et que les électrons s'arrêtent, le condensateur aura atteint une charge (de tension) complète de polarité opposée comme lors du démarrage :

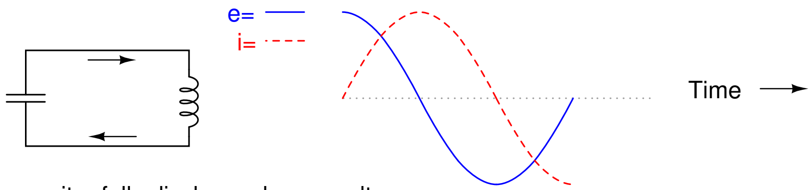


Nous sommes maintenant dans une condition très similaire à celle où nous avons débuté : le condensateur à pleine charge et pas de courant dans le circuit. Le condensateur, comme plutôt, débutera sa décharge dans l'inductance, créant une augmentation de courant (dans la direction opposée, comme ci-dessus) et une baisse de tension avec la perte de sa propre énergie de réserve :



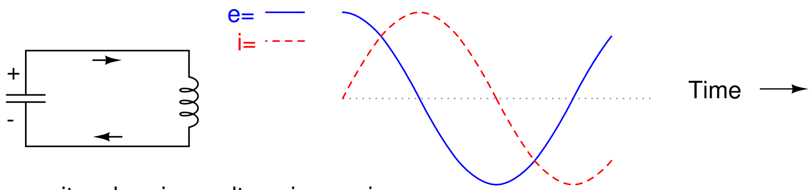
capacitor discharging: voltage decreasing
inductor charging: current increasing

Le condensateur se déchargera réellement à zéro volts, laissant l'inductance complètement chargée faisant passer la totalité du courant à ses bornes :



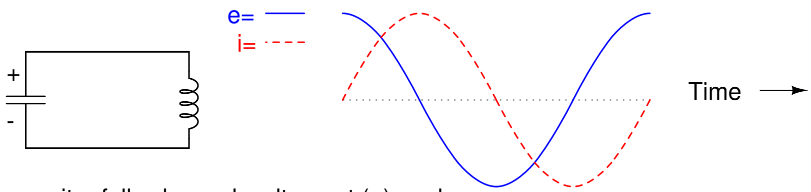
capacitor fully discharged: zero voltage
inductor fully charged: current at (-) peak

L'inductance, désirant maintenir le courant dans la même direction, agira encore comme une source, générant une tension comme une batterie pour continuer le flux. En le faisant, le condensateur commencera à se charger et le courant diminuera en magnitude :



capacitor charging: voltage increasing
inductor discharging: current decreasing

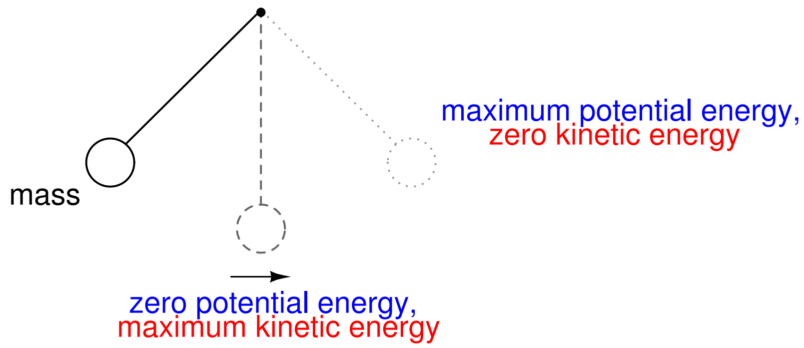
Le condensateur deviendra encore réellement chargé comme l'inductance dépense toute ses réserves d'énergie pour tenter de maintenir le courant. La tension sera encore une fois à son pic positif et le courant à zéro. Cela complète un cycle complet d'échange d'énergie entre le condensateur et l'inductance :

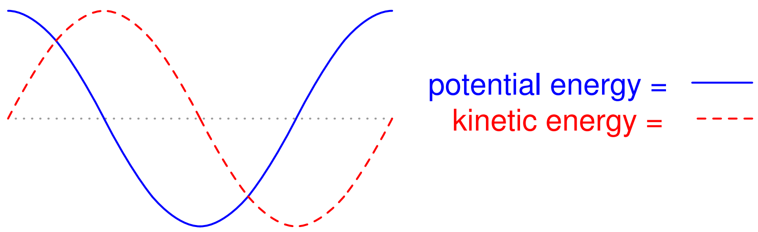


capacitor fully charged: voltage at (+) peak
inductor fully discharged: zero current

Cette oscillation continuera avec une amplitude diminuant d'une manière continue à cause de la perte de puissance dûe aux diverses résistances dans le circuit, jsuqu'à ce que le processus ne s'arrête de lui-même. D'une manière générale; ce comportement est proche de celui d'un pendule : lorsque la masse du pendule va et vient, il existe une transformation d'énergie de cinétique (déplecement) vers potentiel (hauteur), d'une manière similaire à la manière dont l'énergie est transférée dans le circuit avec condensateur/inductance d'un côté vers l'autre sous la forme d'alternance de courant (déplacement cinétique des électrons) et de tension (énergie potentielle électrique).

Au pic de hauteur de chaque battement du pendule, la masse s'arrête brièvement et change de direction. C'est à ce point que l'énergie potentielle (hauteur) est à son maximum et que l'énergie cinétique (déplacement) est à zéro. Comme la masse revient de l'autre côté, elle passe rapidement par un point où la corde est droite. À ce point, l'énergie potentielle (hauteur) est à zéro et l'énergie cinétique (déplacement) est au maximum. Comme le circuit, les oscillations de va et vient d'un pendule continuent avec une amplitude diminuant, le résultat de la friction de l'air (résistance) dissipant de l'énergie. De même, comme le circuit, la position du pendule et les mesures de vélocité tracent deux ondes sinus (déphasées de 90 degrés) au cours du temps :





En physique, ce type d'oscillation naturelle pour un système mécanique est appelé *déplacement harmonique simple*. Les mêmes principes sous-jacents gouvernent et les oscillations d'un circuit avec condensateur/inductance et l'action d'un pendule, d'où la similarité des effets. C'est une propriété intéressante de tout pendule est que sa période est gouvernée par la longueur de la corde portant la masse et pas pas la masse elle-même. C'est pourquoi un pendule gardera la même fréquence lors de la diminition de l'amplitude des oscillations. Le taux d'oscillation est indépendant de la *somme* d'énergie qu'il stocke.

La même chose est vraie pour le circuit de condensteur/inductance. Le taux d'oscillation est strictement dépendant de la taille du condensateur et de l'inductance, pas sur la valeur de la tension (ou du courant) à chaque pic respctif des ondes. L'aptitude d'un tel circuit à stocker l'énergie sous la forme d'une tension et d'un courant oscillant lui a donné le nom de *circuit réservoir*. Sa propriété de maintient d'une seule fréquence naturelle indépendament du peu d'énergie qui y est stocké lui donne une signification spéciale dans la conception de circuits électriques.

Néanmoins, cette tendance à osciller ou *résonner*, à une fréquence particulière n'est pas limitée exclusivement aux circuits conçus dans ce but. En fait, à peu près tous les circuits AC avec une combinaison de condensateur et d'inductance (appelé commmunément un "circuit LC") tendra à manifester des effets inhabituels lorsque la source de fréquence AC de puissance approche cette fréquence naturelle. C'est vrai quelque soit la destination voulue d'un circuit.

Si la fréquence de l'alimentation de puissance pour un circuit correspond exactement à la fréquence naturelle de la combinaison LC du circuit, le circuit est dit être dans un état de *résonance*. Les effets inhabituels atteindront un maximum dans dans cette condition de résonance. Pour cette raison, nous avons besoin d'être capables de prédire quelle fréquence de résonance sera obtenue pour diverses combinaisons de L et C et être au courant de quels sont effets de la résonance.

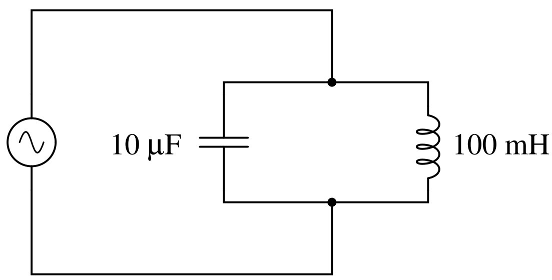
- **RÉSUMÉ :**

- Un condensateur et une inductance sont directement connectés ensemble pour former quelque chose appelé un *circuit réservoir*, qui oscille (ou *résone*) à une fréquence particulière. À cette fréquence, l'énergie est alternativement déplacée entre le condensateur et l'inductance sous la forme d'une tension et d'un courant alternatif avec un déphasage de 90 degrés l'un avec l'autre.

- Lorsque la fréquence AC de l'alimentation de puissance correspond exactement à la fréquence des oscillations naturelles d'un circuit établie par les composants L et C, une condition de *résonance* aura été atteinte.

6.2 Résonance parallèlele simple (circuit réservoir)

Une condition de résonance sera expérimentée dans un circuit réservoir lorsque les réactances du condensateur et de l'inductance sont égales. Comme la réactance inductive augmente avec la fréquence et la réactance capacitive diminue avec l'augmentation de fréquence, il n'y aura qu'une fréquence où les deux réactances seront égales.



Dans le circuit ci-dessus, nous avons un condensateur de 10 μF et une inductance de 100 mH. Comme nous connaissons les équations pour déterminer la réactance à chaque fréquence donnée et nous cherchons pour quel point les deux réactances sont égales et résoudre la fréquence algébraïquement :

$$X_L = 2\pi fL \qquad X_C = \frac{1}{2\pi fC}$$

. . . setting the two equal to each other, representing a condition of equal reactance (resonance) . . .

$$2\pi fL = \frac{1}{2\pi fC}$$

Multiplying both sides by f eliminates the f term in the denominator of the fraction . . .

$$2\pi f^2L = \frac{1}{2\pi C}$$

Dividing both sides by 2πL leaves f^2 by itself on the left-hand side of the equation . . .

$$f^2 = \frac{1}{2\pi 2\pi LC}$$

Taking the square root of both sides of the equation leaves f by itself on the left side . . .

$$f = \frac{\sqrt{1}}{\sqrt{2\pi 2\pi LC}}$$

. . . simplifying . . .

$$\boxed{f = \frac{1}{2\pi\sqrt{LC}}}$$

Nous l'avons donc : une formule qui nous indique la fréquence de résonance d'un circuit réservoir, pour une valeur donnée d'inductance (L) en Henrys et le condensateur (C) en Farads. En cherchant dans les valeurs de L et C dans notre circuit d'exemple, nous arrivons à une fréquence de résonance de 159.155 Hz.

Ce qui se produit lors de la résonance est assez intéressant. Avec des réactances capacitives et inductives égales, l'impédance totale augmente vers l'infini, signifiant que le circuit réservoir de tire pas de courant de la source d'alimentation AC! Nous pouvons calculer les impédances individuelles du condensateur de 10 μF et de l'inductance de 100 mH et travailler avec la formule d'impédance parallèle pour démontrer ceci mathématiquement :

X_L=2ΠfL

X_L=2Π(159.155 Hz)(100 mH)

X_L=100Ω

$$X_C = \frac{1}{2\pi fC}$$

$$X_C = \frac{1}{(2)(\pi)(159.155\ \text{Hz})(10\ \mu\text{F})}$$

X_L=100Ω

Comme vous pouvez l'avoir deviné, j'ai choisi ces valeurs de composants pour qu'il soit facile d'élaborer une impédance de résonance (même avec 100 Ω). Nous utilisons maintenant la formule de l'impédance parallèle pour voir ce qui se passe au Z total :

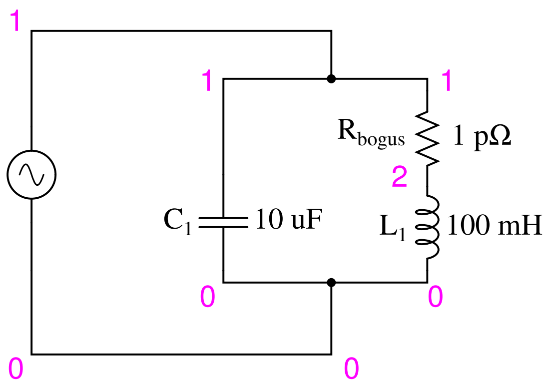
$$Z_{\text{parallel}} = \frac{1}{\frac{1}{Z_L} + \frac{1}{Z_C}}$$

$$Z_{\text{parallel}} = \frac{1}{\frac{1}{100\ \Omega \angle 90^o} + \frac{1}{100\ \Omega \angle -90^o}}$$

$$Z_{\text{parallel}} = \frac{1}{0.01 \angle -90^o + 0.01 \angle 90^o}$$

$$Z_{\text{parallel}} = \frac{1}{0} \quad \textit{Undefined!}$$

Nous ne pouvons pas diviser un nombre par zéro et obtenir un résultat significatif mais nous pouvons dire que ce résultat approche une valeur *infinie* lorsque les deux impédances parallèles s'approchent l'une de l'autre. Cela signifie en termes pratiques que l'impédance totale du circuit réservoir est infinie (elle se comporte comme un *circuit ouvert*) à la résonance. Nous pouvons tracer les conséquences de ceci en prenant un large éventail de fréquences avec une courte simulation SPICE :




```
tank circuit frequency sweep
v1 1 0 ac 1 sin
c1 1 0 10u
* rbogus is necessary to eliminate a direct loop
```

```
* between v1 and l1, which SPICE can't handle
rbogus 1 2 1e-12
l1 2 0 100m
.ac lin 20 100 200
.plot ac i(v1)
.end

freq       i(v1)       3.162E-04   1.000E-03   3.162E-03  1.0E-02
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
1.000E+02  9.632E-03 .       .           .           .         *
1.053E+02  8.506E-03 .       .           .           .       * .
1.105E+02  7.455E-03 .       .           .           .     *   .
1.158E+02  6.470E-03 .       .           .           .    *    .
1.211E+02  5.542E-03 .       .           .           .  *      .
1.263E+02  4.663E-03 .       .           .           . *       .
1.316E+02  3.828E-03 .       .           .           .*        .
1.368E+02  3.033E-03 .       .           .          *.         .
1.421E+02  2.271E-03 .       .           .       *   .         .
1.474E+02  1.540E-03 .       .           .    *      .         .
1.526E+02  8.373E-04 .       .         * .           .         .
1.579E+02  1.590E-04 . *     .           .           .         .
1.632E+02  4.969E-04 .       .    *      .           .         .
1.684E+02  1.132E-03 .       .           . *         .         .
1.737E+02  1.749E-03 .       .           .    *      .         .
1.789E+02  2.350E-03 .       .           .       *   .         .
1.842E+02  2.934E-03 .       .           .          *.         .
1.895E+02  3.505E-03 .       .           .           .*        .
1.947E+02  4.063E-03 .       .           .           .  *      .
2.000E+02  4.609E-03 .       .           .           .    *    .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

La résistance de 1 pico-ohm (1 pΩ) est placée dans l'analyse SPICE pour dépasser une de ses limitations : à savoir qu'il ne peut pas analiser un circuit contanant une boucle de source de tension directement sur une inductance. Une valeur de résistance très basse a été choisie pour avoir un effet minimum sur le comportement du circuit.

Cette simulation SPICE trace le courant du circuit pour un champ de fréquence de 100 à 200 Hz en vingt pas pair (100 et 200 Hz inclus). La magnitude du courant sur le graphe augmente de la gauche vers la droite, alors que les fréquences augmentent du haut vers le bas. Le courant, dans ce circuit, a un trou autour du point d'analyse de 157.9 Hz, qui est le point analyse le plus proche de notre prédiction de la fréquence du point de résonance de 159.155 Hz. C'est à ce point que le courant total de la source de tension tombe à zéro.

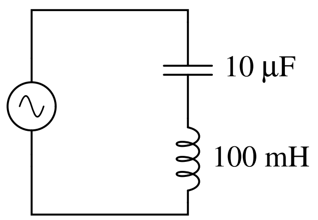
Incidement, la sortie du graphe produite par cette analyse de SPICE est plus généralemenr connue comme un *tracé de Bode*. De tels graphiques tracent l'amplitude ou le décalage de phase sur un axe et la fréquence sur l'autre. L'étalonnage d'une courbe de tracé de Bode caractérise la "réponse en fréquence" du circuit ou sa sensibilité aux changements de fréquence.

- **RÉSUMÉ :**

- La résonance se produit lorsque les réactances capacitives et inductives sont égales.
- Pour un circuit réservoir sans résistance (R), la fréquence de résonance peut être facilement calculée avec la formule suivante :
- $$f_{resonant} = \frac{1}{2\pi \sqrt{LC}}$$
- L'impédance totale d'un circuit parallèle LC s'approche de l'infini alors que la fréquence de l'alimentation approche de la résonance.
- Un *tracé de Bode* est un graphe traçant une amplitude ou une phase de forme d'onde sur un axe et la fréquence sur l'autre.

6.3 Résonance série simple

Un effet similaire se produit dans les circuits inductifs/capacitifs série. Lorsqu'un état de résonance est atteint (réactances capacitives ou inductives égales), les deux impédances s'annulent les unes les autres et l'impédance totale tombe à zéro!



At 159.155 Hz:

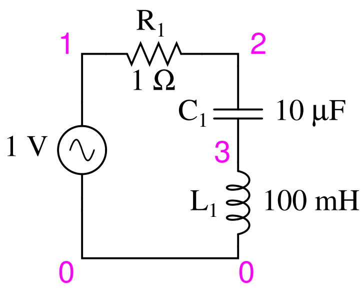
$Z_L = 0 + j100\ \Omega$ $\qquad Z_C = 0 - j100\ \Omega$

$$Z_{series} = Z_L + Z_C$$

$$Z_{series} = (0 + j100\ \Omega) + (0 - j100\ \Omega)$$

$$Z_{series} = 0\ \Omega$$

Avec une impédance série totale égale à 0 Ω à la fréquence de résonance de 159.155 Hz, le résultat est un *court circuit* sur la source d'alimentation AC à la résonnace. Dans le circuit dessiné ci-dessus, ce ne serait pas bon. J'ajoute une petite résistance en série avec le condensateur et l'inductance pour maintenir le courant maximum du circuit limité dans une certaine mesure et effectue une autre analyse SPICE sur la même gamme de fréquences :




```
series lc circuit
v1 1 0 ac 1 sin
r1 1 2 1
c1 2 3 10u
l1 3 0 100m
.ac lin 20 100 200
.plot ac i(v1)
.end
```

```
freq        i(v1)          3.162E-02     1.000E-01  3.162E-01  1.0
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
1.000E+02  1.038E-02 *              .             .          .    .
1.053E+02  1.176E-02 . *            .             .          .    .
1.105E+02  1.341E-02 .   *          .             .          .    .
1.158E+02  1.545E-02 .    *         .             .          .    .
1.211E+02  1.804E-02 .      *       .             .          .    .
1.263E+02  2.144E-02 .        *     .             .          .    .
1.316E+02  2.611E-02 .           *  .             .          .    .
1.368E+02  3.296E-02 .              .*            .          .    .
1.421E+02  4.399E-02 .              .   *         .          .    .
1.474E+02  6.478E-02 .              .        *    .          .    .
1.526E+02  1.186E-01 .              .             . *        .    .
1.579E+02  5.324E-01 .              .             .          . *  .
1.632E+02  1.973E-01 .              .             .    *     .    .
1.684E+02  8.797E-02 .              .           * .          .    .
1.737E+02  5.707E-02 .              .      *      .          .    .
1.789E+02  4.252E-02 .              .   *         .          .    .
1.842E+02  3.406E-02 .              .*            .          .    .
1.895E+02  2.852E-02 .             *.             .          .    .
1.947E+02  2.461E-02 .           *  .             .          .    .
2.000E+02  2.169E-02 .         *    .             .          .    .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

Comme précédement, l'amplitude du courant dans le circuit augmente de la gauche vers la droite, alors que la fréquence augmente du haut vers le bas. Le pic est encore vu au point de fréquence tracé de 157.9 Hz, le point analysé le plus proche de notre point de résonance de 159.155 Hz. Cela suggérerait que notre formule de fréquence de résonance reste vrai pour les circuits LC série simples, comme il l'est pour les circuits LC parallèles simples, ce qui est le cas :

$$f_{resonant} = \frac{1}{2\pi\sqrt{LC}}$$

Un petit mot d'avertissement avec les circuits résonants LC série : comme des courant très importants peuvent être présents dans un circuit LC série à la résonance, il est possible qu'il y ait une production de haute tension dangereuse aux bornes du condensateur et de l'inductance car chacun des composants possède une impédance significative. Nous pouvons éditer les netlists SPICE dans l'exemple ci-dessus pour inclure un tracé de la tension aux bornes du condensateur et de l'inductance pour démontrer ce qui se produit :

```
series lc circuit
v1 1 0 ac 1 sin
r1 1 2 1
c1 2 3 10u
l1 3 0 100m
.ac lin 20 100 200
.plot ac i(v1) v(2,3) v(3)
.end
```

```
legend :
* : i(v1)
+ : v(2,3)
= : v(3)
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
( *)----------- 1.000E-02     3.162E-02     1.000E-01    0.3162  1
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
(+)----------- 1.000E+00     3.162E+00     1.000E+01    31.62 100
```

```
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
(=)----------- 1.000E-01     1.000E+00     1.000E+01     100 1000
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
freq        i(v1)
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
1.000E+02  1.038E-02 *     +    = .          .           .     .
1.053E+02  1.176E-02 . *    +    =.          .           .     .
1.105E+02  1.341E-02 .   *   +    =          .           .     .
1.158E+02  1.545E-02 .    *   +   .=         .           .     .
1.211E+02  1.804E-02 .      *   + . =        .           .     .
1.263E+02  2.144E-02 .        *  +.  =       .           .     .
1.316E+02  2.611E-02 .           *+    =     .           .     .
1.368E+02  3.296E-02 .            .*+   =    .           .     .
1.421E+02  4.399E-02 .            .   *+  =  .           .     .
1.474E+02  6.478E-02 .            .        *+=           .     .
1.526E+02  1.186E-01 .            .          .=*+        .     .
1.579E+02  5.324E-01 .            .          .       =   .   x .
1.632E+02  1.973E-01 .            .          .   = x     .     .
1.684E+02  8.797E-02 .            .        x =           .     .
1.737E+02  5.707E-02 .            .     +* = .           .     .
1.789E+02  4.252E-02 .            . + *  =   .           .     .
1.842E+02  3.406E-02 .           +.*   =     .           .     .
1.895E+02  2.852E-02 .         + *.   =      .           .     .
1.947E+02  2.461E-02 .      +  *  .   =      .           .     .
2.000E+02  2.169E-02 .     + *    .  =       .           .     .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

Selon SPICE, la tension aux bornes du condensateur et de l'inductance (tracé avec les symboles "+" et "=", respectivement) atteignent un pic entre 100 et 1000 volts (marqué par le "x" où les tracés se superposent)! C'est assez impressionnant pour une alimentation qui ne génère que 1 volt. Il est sans besoin de dire qu'il faut faire attention avec les expérimentations sur de tels circuits.

- **RÉSUMÉ :**

- L'impédance totale d'un circuit LC série approche zéro car la fréquence de l'alimentation approche la résonance.

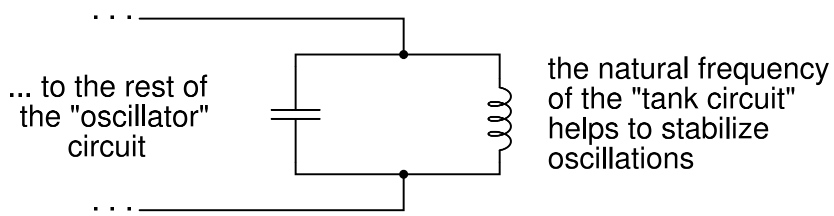
- La même formule pour déterminer la fréquence de résonance dans un circuit réservoir simple s'applique aussi aux circuits série simple.

- Des tension extrèmement hautes peuvent être formées aux bornes des composants individuels des circuits LC série à la résonance, à cause des flux importants de courant et des des impédances individuelles des composants essentiels.

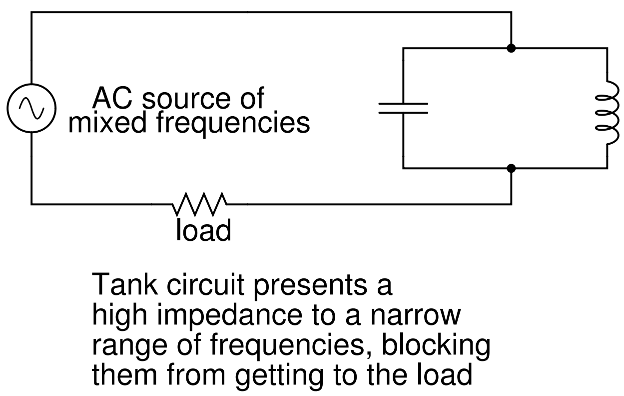
6.4 Applications de la résonance

Néanmoins, ce phénomène de résonance semble être une curiosité sans intérêt, ou, au pire, une nuisance devant être évitée (spécialement si la résonance série fait un court circuit sur la source de tension AC!). Ce n'est pourtant pas le cas. La résonance est une propriété précieuse des circuits réactifs AC, employés dans un éventail d'applications.

Une des utilisations de la résonance est d'établir une condition de fréquence stable dans les circuits conçus pour produire des signaux AC. Habituellement, un circuit parallèle (réservoir) est utilisé dans ce but, avec le condensateur et l'inductance directement connectés ensemble, échangeant de l'énergie l'un avec l'autre. De la même manière qu'un pendule peut être utilisé pour stabiliser la fréquence des oscillations d'un mécanisme d'horloge, un circuit réservoir peut être utilisé pour stabiliser la fréquence électrique d'un circuit *oscillant* AC. Comme nous l'avons noté plutôt, la fréquence initialisée par le circuit réservoir est exclusivement dépendante des valeurs de L et C et pas sur la magnitude de tension ou de courant présent dans les oscillations :



Une autre utilisation de la résonance est dans les applications qui ont besoin d'une grande augmentation ou diminution d'impédance à une fréquence particulière. Un circuit résonant peut être utilisé pour "bloquer" (passage par une haute impédance) une fréquence ou un champ de fréquences, et donc agir comme une sorte de fréquence "filtre" pour enlever certaines fréquences parmi un mélange d'autres. En fait, ces circuits particuliers sont appelés *filtres* et leur conception constitue une discipline d'étude en elle-même :



En essence, c'est la manière dont les circuits de récepteurs radio analogiques fonctionnent pour filtrer ou sélectionner la fréquence d'une station parmi le choix des différentes fréquences interceptées par l'antenne.

- **RÉSUMÉ :**

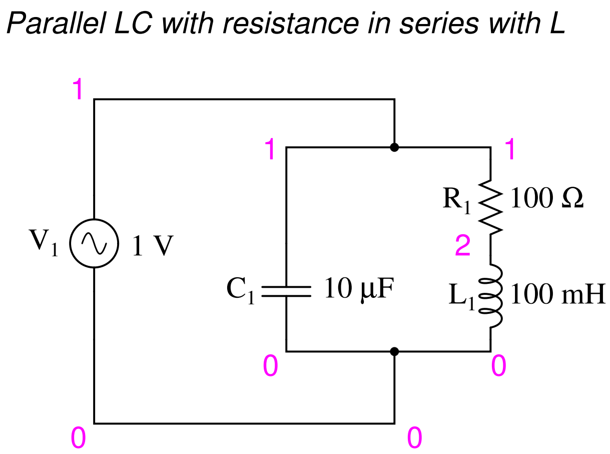
- La résonance peut être employée pour maintenir les oscillations AC du circuit à une fréquence constante, juste comme un pendule peut être utilisé pour maintenir une vitesse d'oscillation constante dans un mécanisme de maintient du chronométrage.
- La résonance peut être exploitée pour ses propriétés d'impédance : soit une augmentation ou une diminution dramatique de l'impédance pour certaines fréquences. Les circuits conçus pour chercher certaines fréquences parmi un mélange sont appelés *filtres*.

6.5 Résonance dans les circuits série-parallèle

Dans les circuits réactifs simples avec peu ou pas de résistance, les effets de l'impédance radicalement altérée se manifesteront à la fréquence de résonance prédite par l'équation donnée plutôt. Dans un circuit LC parallèle (réservoir), cela signifie une impédance infinie à la résonance. Dans un circuit LC, cela signifie que une imédance nulle à la résonance :

$$f_{resonant} = \frac{1}{2\pi\sqrt{LC}}$$

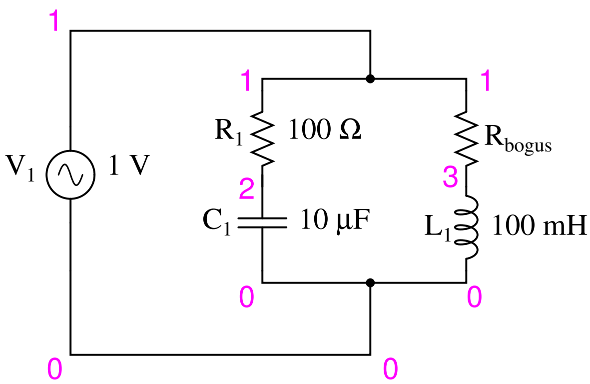
Néanmoins, dès que des résistances significatives sont introduites dans la plupart des circuits LC, ce simple calcul de résonance devient invalide. Jetons un coup d'oeil aux divers circuits LC avec une résistance ajoutée, en utilisant les mêmes valeurs pour les condensateurs et inductances comme avant : 10 μF and 100 mH, respectivement. Selon notre équation simple, la fréquence de résonance devrait être de 159.155 Hz. Regardons cependant, où le courant atteint un maximum ou un minimum dans les analyses SPICE suivantes :




```
resonant circuit
v1 1 0 ac 1 sin
c1 1 0 10u
r1 1 2 100
l1 2 0 100m
.ac lin 20 100 200
```

```
.plot ac i(v1)
.end

freq       i(v1)           7.079E-03     7.943E-03     8.913E-03
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
1.000E+02  7.387E-03 .             .    *        .             .
1.053E+02  7.242E-03 .             .  *          .             .
1.105E+02  7.115E-03 .             .*            .             .
1.158E+02  7.007E-03 .            *.             .             .
1.211E+02  6.921E-03 .          *  .             .             .
1.263E+02  6.859E-03 .         *   .             .             .
1.316E+02  6.823E-03 .         *   .             .             .
1.368E+02  6.813E-03 .        *    .             .             .
1.421E+02  6.830E-03 .         *   .             .             .
1.474E+02  6.874E-03 .         *   .             .             .
1.526E+02  6.946E-03 .           * .             .             .
1.579E+02  7.044E-03 .            *.             .             .
1.632E+02  7.167E-03 .             .*            .             .
1.684E+02  7.315E-03 .             .   *         .             .
1.737E+02  7.485E-03 .             .      *      .             .
1.789E+02  7.676E-03 .             .         *   .             .
1.842E+02  7.886E-03 .             .            *.             .
1.895E+02  8.114E-03 .             .             .  *          .
1.947E+02  8.358E-03 .             .             .     *       .
2.000E+02  8.616E-03 .             .             .         *   .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
Le minimum de courant est à 136.8 Hz au lieu de 159.2 Hz!
```

Parallel LC with resistance in series with C



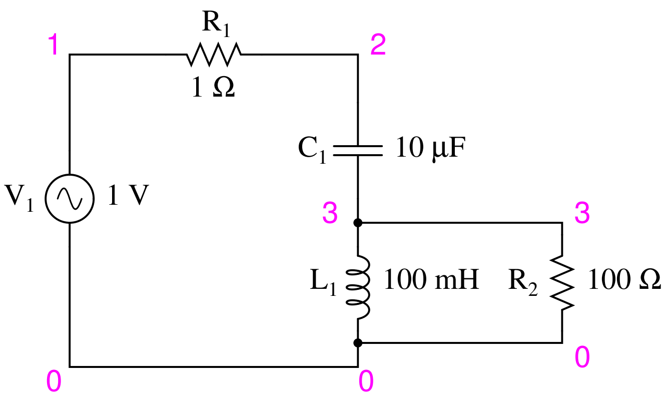
Ici, une résistance supplémentaire (R_{bogus}) est nécessair pour éviter à SPICE de rencontrer des problèmes dans l'analyse. SPICE ne peut pas traiter une inductance connectée directement en parallèle avec toute source de tension ou toute autre inductance, donc l'addition d'une résistance en série est nécessaire pour "casser" la boucle de source de tension/inductance qui se formerait dans le cas contraire. Cette résistance est choisie pour être une *très* faible valeur pour un impact minimum sur le comportement du circuit.

```
resonant circuit
v1 1 0 ac 1 sin
r1 1 2 100
c1 2 0 10u
rbogus 1 3 1e-12
l1 3 0 100m
.ac lin 20 100 400
.plot ac i(v1)
.end
```

```
freq       i(v1)           7.943E-03     1.000E-02     1.259E-02
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
1.000E+02  1.176E-02 .             .             .         *   .
1.158E+02  9.635E-03 .             .           * .             .
1.316E+02  8.257E-03 .             . *           .             .
1.474E+02  7.430E-03 .         *   .             .             .
1.632E+02  6.998E-03 .     *       .             .             .
1.789E+02  6.835E-03 .    *        .             .             .
1.947E+02  6.839E-03 .    *        .             .             .
2.105E+02  6.941E-03 .     *       .             .             .
2.263E+02  7.093E-03 .      *      .             .             .
2.421E+02  7.268E-03 .        *    .             .             .
2.579E+02  7.449E-03 .         *   .             .             .
2.737E+02  7.626E-03 .           * .             .             .
2.895E+02  7.794E-03 .            *.             .             .
3.053E+02  7.951E-03 .             *             .             .
3.211E+02  8.096E-03 .             .*            .             .
3.368E+02  8.230E-03 .             . *           .             .
3.526E+02  8.352E-03 .             .  *          .             .
3.684E+02  8.464E-03 .             .   *         .             .
3.842E+02  8.567E-03 .             .    *        .             .
4.000E+02  8.660E-03 .             .    *        .             .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
Le courant minimum est à presque 180 Hz au lieu de 159.2 Hz!
```

Plaçons notre attention sur les circuits LC série, nous expérimentons le placement de résistances significatives en parallèle avec soit L, soit C. Dans les exmples de circuits série suivants, une résistance de 1 Ω (R_1) est placé en série avec ybe inductance et un condensateur pour limiter le courant total à la résonance. La résistance "supplémentaire" insérée pour influencer les effets de la fréquence de résonance est de 100 Ω, R_2 :

Series LC with resistance in parallel with L

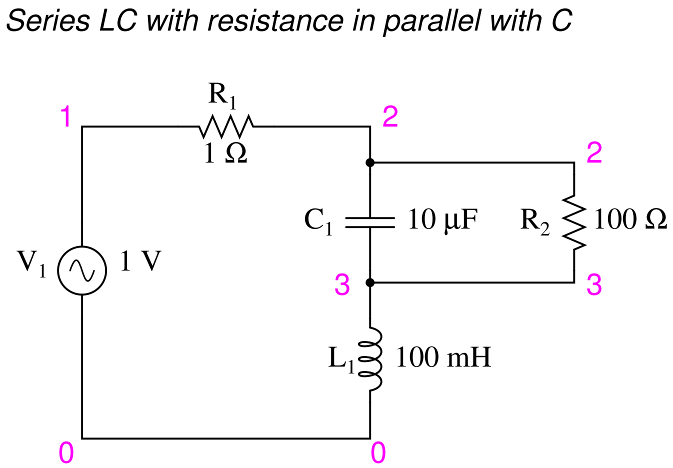



```
resonant circuit
v1 1 0 ac 1 sin
r1 1 2 1
c1 2 3 10u
l1 3 0 100m
r2 3 0 100
.ac lin 20 100 400
.plot ac i(v1)
.end
```

```
freq       i(v1)            1.000E-02     1.259E-02     1.585E-02
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
1.000E+02  8.488E-03 .   *         .             .             .
1.158E+02  1.034E-02 .             . *           .             .
1.316E+02  1.204E-02 .             .          *  .             .
1.474E+02  1.336E-02 .             .             .   *         .
1.632E+02  1.415E-02 .             .             .      *      .
1.789E+02  1.447E-02 .             .             .       *     .
1.947E+02  1.445E-02 .             .             .       *     .
2.105E+02  1.424E-02 .             .             .      *      .
2.263E+02  1.393E-02 .             .             .     *       .
2.421E+02  1.360E-02 .             .             .    *        .
2.579E+02  1.327E-02 .             .             .  *          .
2.737E+02  1.296E-02 .             .             . *           .
2.895E+02  1.269E-02 .             .             *             .
3.053E+02  1.244E-02 .             .            *.             .
3.211E+02  1.222E-02 .             .           * .             .
3.368E+02  1.202E-02 .             .          *  .             .
3.526E+02  1.185E-02 .             .         *   .             .
3.684E+02  1.169E-02 .             .        *    .             .
```

```
3.842E+02  1.155E-02 .             .        *    .             .
4.000E+02  1.143E-02 .             .       *     .             .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
Le courant maximum proche de 178.9 Hz au lieu de 159.2 Hz!
```

Et finalement, un circuit LC série avec une résistance significative en parallèle avec le condensateur :




```
resonant circuit
v1 1 0 ac 1 sin
r1 1 2 1
c1 2 3 10u
r2 2 3 100
l1 3 0 100m
.ac lin 20 100 200
.plot ac i(v1)
.end
```

```
freq      i(v1)
freq       i(v1)           1.259E-02     1.413E-02     1.585E-02
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
1.000E+02  1.336E-02 .             .      *      .             .
1.053E+02  1.363E-02 .             .         *   .             .
1.105E+02  1.387E-02 .             .           * .             .
1.158E+02  1.408E-02 .             .             *             .
1.211E+02  1.426E-02 .             .             .*            .
1.263E+02  1.439E-02 .             .             . *           .
1.316E+02  1.447E-02 .             .             .  *          .
1.368E+02  1.450E-02 .             .             .  *          .
1.421E+02  1.447E-02 .             .             .  *          .
1.474E+02  1.438E-02 .             .             . *           .
```

```
1.526E+02  1.424E-02 .             .             .*            .
1.579E+02  1.405E-02 .             .            *.             .
1.632E+02  1.382E-02 .             .          *  .             .
1.684E+02  1.355E-02 .             .        *    .             .
1.737E+02  1.325E-02 .             .     *       .             .
1.789E+02  1.293E-02 .             .  *          .             .
1.842E+02  1.259E-02 .             *             .             .
1.895E+02  1.225E-02 .          *  .             .             .
1.947E+02  1.190E-02 .      *      .             .             .
2.000E+02  1.155E-02 .  *          .             .             .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
Courant maximum à 136.8 Hz au lieu de 159.2 Hz!
```

La tendance d'une résistance ajoutée à décaler le point pour lequel l'impédance atteint un maximum ou un minimum dans un circuit LC est appelé *anti-résonance*. Un observateur astucieux notera un motif entre les quatres exemples SPICE fournis ci-dessus, en termes dont la résistance affecte le pic de résonance d'un circuit :

- **Circuit LC parallèle ("réservoir") :**
- R en série avec L : fréquence de résonance décalée vers le *bas*
- R en série avec C : fréquence de résonance décalée vers le *haut*

- **Circuits LC série :**
- R en parallèle avec L : fréquence de résonance décalée vers le *haut*
- R en parallèle avec C : fréquence de résonance décalée vers le *bas*

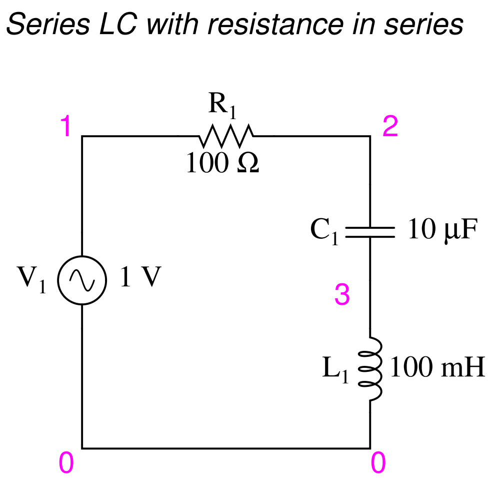
Cela illustre encore une fois la nature complémentaire des condensateurs et inductances : comment les résistances en série avec une autre créent un effet d'anti-résonance équivalent à une résistance en parallèle avec une autre. Si vous regardez encore plus près les quatre exemples SPICE donnés, vous verrez que les fréquences sont décalées de la *même valeur* et que la forme des graphes complémentaires sont l'image miroir l'un de l'autre!

L'anti-résonance est un effet auquel les concepteurs de circuits résonants doivent faire attention. Les équations pour déterminer le "décalage" de l'anti-résonance sont complexes et ne seront pas couvertes dans cette brève leçon. Il devrait suffire aux étudiants débutants en électronique que les effets existent et quelles sont les tendances générales.

L'ajoût de résistance dans une circuit LC n'est pas seulement académique. Alors qu'il est possible de fabriquer des condensateurs avec des résistances non désirées négligeables, les inductances sont typiquement poluées avec des valeurs substantielles de résistances dues à la longueur des fils utilisés dans le construction. De plus, la résistance du fil tend à augmenter avec la fréquence, à cause d'un phénomène étrange connu comme *l'effet de peau* où le courant AC tend à être exclu du centre d'un fil, réduisant d'autant la section effective d'un fil. Les inductances n'ont donc pas seulement une résistance fixe mais qui *change en fonction de la fréquence*.

Comme si la résistance du fil d'une inductance ne posait pas suffisamment de problèmes, nous devons aussi faire avec les "pertes de noyau" (core loss) d'une inductance à noyau en fer, qui se manifestent elles-mêmes comme des résistances additionnelles dans le circuit. Comme le fer est conducteur d'électricité de même qu'un conducteur de flux magnétique, le changement de flux produit par le courant alternatif dans la bobine (coil) tendra à induire des courants électriques dans le noyau lui-même (*courants eddy*). Cet effet peut être considéré comme étant malgré tout dans le le coeur de fer du transformateur où une sorte de bobine de transformateur secondaire alimente une charge résistive : la conductivité moins-que-parfaite du fer. Cet effet peut être minimisé avec des feuilles laminées, une bonne conception et des matériaux à haute permitivité mais jamais complètement éliminés.

Une exception notable de la règle de la résistance du circuit causant un décalage de fréquence de résonance est le cas des circuits résistance-inductance-condensateurs série ("RLC"). Tant que *tous* les composants sont connecté en série les uns avec les autres, la fréquence de résonance du circuit ne sera pas affectée par la résistance :




```
series rlc circuit
v1 1 0 ac 1 sin
r1 1 2 100
c1 2 3 10u
l1 3 0 100m
.ac lin 20 100 200
.plot ac i(v1)
.end
```

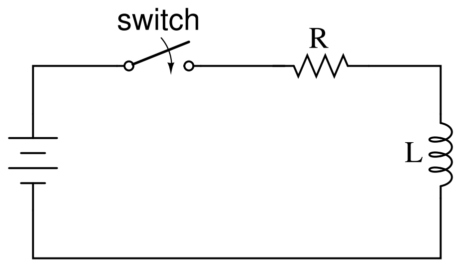
```
freq          i(v1)          7.943E-03     8.913E-03     1.000E-02
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
1.000E+02   7.202E-03 *         .             .             .
1.053E+02   7.617E-03 .     *   .             .             .
1.105E+02   8.017E-03 .         .*            .             .
1.158E+02   8.396E-03 .         .      *      .             .
```

```
1.211E+02  8.747E-03 .          .           * .             .
1.263E+02  9.063E-03 .          .             . *           .
1.316E+02  9.339E-03 .          .             .     *       .
1.368E+02  9.570E-03 .          .             .        *    .
1.421E+02  9.752E-03 .          .             .          *  .
1.474E+02  9.883E-03 .          .             .            *.
1.526E+02  9.965E-03 .          .             .             .
1.579E+02  9.999E-03 .          .             .             *
1.632E+02  9.988E-03 .          .             .             *
1.684E+02  9.936E-03 .          .             .            *.
1.737E+02  9.850E-03 .          .             .           * .
1.789E+02  9.735E-03 .          .             .          *  .
1.842E+02  9.595E-03 .          .             .        *    .
1.895E+02  9.437E-03 .          .             .      *      .
1.947E+02  9.265E-03 .          .             .    *        .
2.000E+02  9.082E-03 .          .             . *           .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
Courant maximum à 159.2 Hz encore une fois!
```

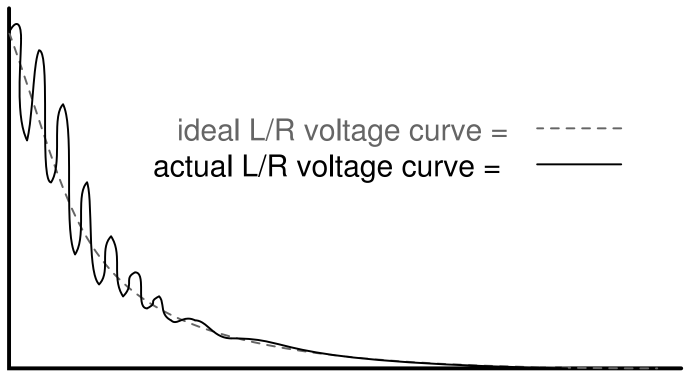
Notez que le graphe du courant n'a pas changé depuis le circuit LC série précédent (celui avec la résistance de 1 Ω), alors même que la résistance est maintenant 100 fois plus grande. La seule chose qui a changé est la "raideur" de la courbe. Ce circuit ne résonne manifestement pas aussi fortement qu'un avec une résistance série moindre (il est dit "moins sélectif") mais au moins, il possède la même fréquence naturelle!

Il est notable que l'anti-résonance a l'effet d'amortir les oscillations d'un circuit LC libre tel qu'un circuit réservoir. Dans le début de ce chapitre, nous avons vu comment un condensateur et une inductance connectés directement agiront ensemble comme quelque chose comme un pendule, échangeant de la tension et du courant comme un pendule échangent de l'énergie cinétique et potentielle. Dans un circuit réservoir parfait (pas de résistance), cette oscillation continuera à vie, de la même manière qu'un pendule sans friction continuerait d'osciller à sa fréquence de résonance à vie. Mais les machines sans friction sont difficiles à trouver dans le monde réel et c'est le cas des circuits réservoirs sans perte. L'énergie perdue dans la résistance (ou les pertes de noyau de l'inductance ou les onde électromagnétiques irradiées ou ...) d'un circuit réservoir fera diminuer les oscillations en amplitude jusqu'à ce qu'elles disparaissent. S'il y a suffisamment de pertes d'énergie dans un circuit réservoir, il ne pourra pas du tout résonner.

L'effet d'amortissement de l'anti-résonance est plus qu'une simple curiosité : il peut être utilisé assez efficacement pour éliminer les oscillations *indésirables* dans un circuit contenant des inductances parasites et/ou des condensateurs, comme tous les circuits le font. regardons le circuit retardateur L/R suivant :



L'idée de ce circuit est simple : pour "charger" l'inductance lorsque l'interrupteur est fermé. La vitesse de chargement de l'inductance sera initialisée par le ratio L/R, qui est la constante de temps d'un circuit en secondes. Néanmoins, si vous créez un tel circuit, vous pourriez trouver des oscillations de tension inattendues (AC) dans l'inductance lorsque l'interrupteur est fermé. Que se passe-t-il? Il n'y a pas de condensateur dans le circuit, donc, comment pouvons-nous avoir des oscillations résonantes avec juste une inductance, une résistance et une batterie?



Toutes les inductances contiennent des capacités parasites dues aux espacements isolés des bobinages. De même, le placement de conducteurs sur un circuit peut créer des capacités parasites. Alors que le placement sur un circuit est important pour éliminer beaucoup de ces capacités parasites, il en restera toujours qui ne peuvent pas être éliminées. Si cela crée des problèmes de résonance (oscillations AC indésirables), l'ajout de résistance peut être une manière pour le combattre. Si la résistance R est suffisamment grande, elle créera une condition d'anti-résonance, dissipant suffisamment d'énergie pour éviter à un condensateur ou une inductance de maintenir les oscillations très longtemps.

D'une manière suffisamment intéressante, le principe d'emploi des résistances pour éliminer la résonance indésirée est utilisée dans la conception de systèmes mécaniques, où tout objet en déplacement avec une masse est un résonateur en puissance. Une application habituelle est l'utilisation d'absorbeurs de chocs dans les automobiles. Without shock absorbers, cars would bounce wildly at their resonant frequency after hitting any bump in the road. The shock absorber's job is to introduce a strong antiresonant effect by dissipating energy hydraulically (in the same way that a resistor dissipates energy electrically).

- **RÉSUMÉ :**
- L'ajout d'une résistance à un circuit LC peut créer une condition connue comme *anti-résonance*,

où les effets du pic d'impédance se produisent à des fréquences autres que celles données par l'égalité entre les réactances capacitives et inductives.

- Les résistances indésirables, inhérentes dans les inductances du monde réel peut grandement contribuer aux conditions de l'anti-résonance. Une source d'une telle résistance est *l'effet de peau*, causé par l'exclusion du courant AC au centre du conducteur. Une autre source est constitué par les *pertes de noyau* dans un inducteur en fer.

- Dans un circuit LC série simple contenant une résistance (un circuit "RLC"), cette dernière *ne produit pas* l'anti-résonance. La résonance se produit encore lorsque les réactances capacitives et inductives sont égales.

6.6 Contributeurs

Les contributeurs de ce chapitre sont listés dans l'ordre chronologique de leurs contributions, depuis le plus récent jusqu'au premier. Voyez l'Annexe 2 (Liste des contributeur) pour les dates et les informations de contact.

Jason Starck (June 2000) : HTML document formatting, which led to a much better-looking second edition.

Chapter 7

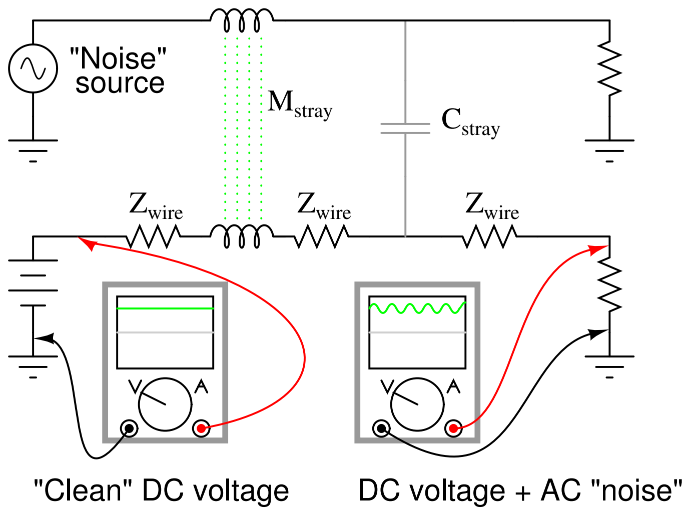
MIXED-FREQUENCY AC SIGNALS

7.1 Introduction

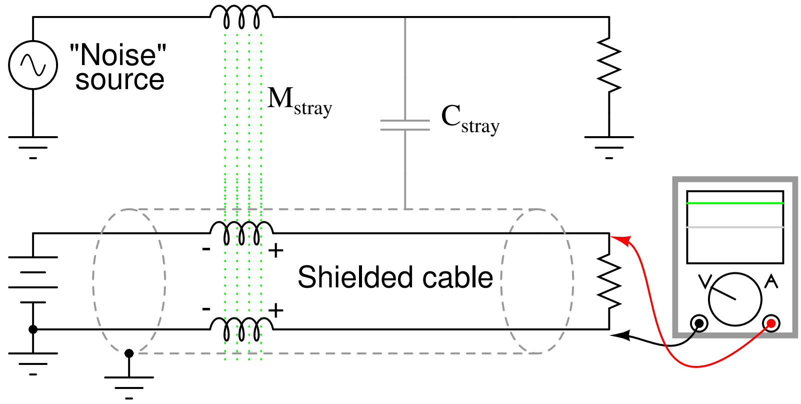
In our study of AC circuits thus far, we've explored circuits powered by a single-frequency sine voltage waveform. In many applications of electronics, though, single-frequency signals are the exception rather than the rule. Quite often we may encounter circuits where multiple frequencies of voltage coexist simultaneously. Also, circuit waveforms may be something other than sine-wave shaped, in which case we call them *non-sinusoidal waveforms.*

Additionally, we may encounter situations where DC is mixed with AC : where a waveform is superimposed on a steady (DC) signal. The result of such a mix is a signal varying in intensity, but never changing polarity, or changing polarity asymmetrically (spending more time positive than negative, for example). Since DC does not alternate as AC does, its "frequency" is said to be zero, and any signal containing DC along with a signal of varying intensity (AC) may be rightly called a mixed-frequency signal as well. In any of these cases where there is a mix of frequencies in the same circuit, analysis is more complex than what we've seen up to this point.

Sometimes mixed-frequency voltage and current signals are created accidentally. This may be the result of unintended connections between circuits – called *coupling* – made possible by stray capacitance and/or inductance between the conductors of those circuits. A classic example of coupling phenomenon is seen frequently in industry where DC signal wiring is placed in close proximity to AC power wiring. The nearby presence of high AC voltages and currents may cause "foreign" voltages to be impressed upon the length of the signal wiring. Stray capacitance formed by the electrical insulation separating power conductors from signal conductors may cause voltage (with respect to earth ground) from the power conductors to be impressed upon the signal conductors, while stray inductance formed by parallel runs of wire in conduit may cause current from the power conductors to electromagnetically induce voltage along the signal conductors. The result is a mix of DC and AC at the signal load. The following schematic shows how an AC "noise" source may "couple" to a DC circuit through mutual inductance (M_{stray}) and capacitance (C_{stray}) along the length of the conductors.

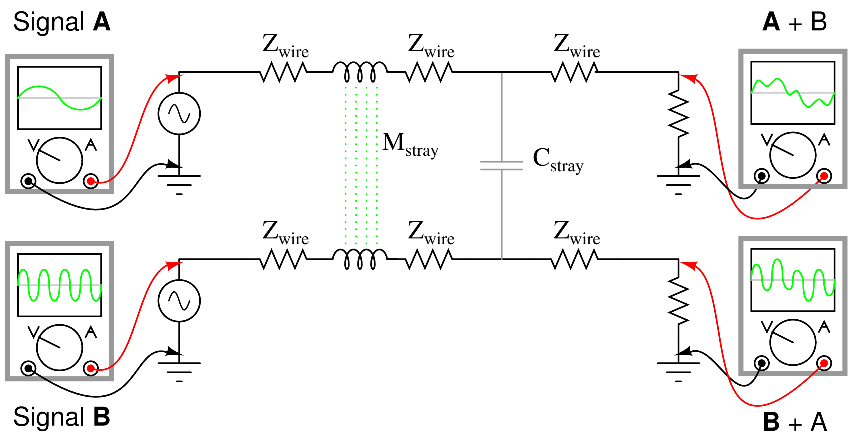


When stray AC voltages from a "noise" source mix with DC signals conducted along signal wiring, the results are usually undesirable. For this reason, power wiring and low-level signal wiring should *always* be routed through separated, dedicated metal conduit, and signals should be conducted via 2-conductor "twisted pair" cable rather than through a single wire and ground connection :

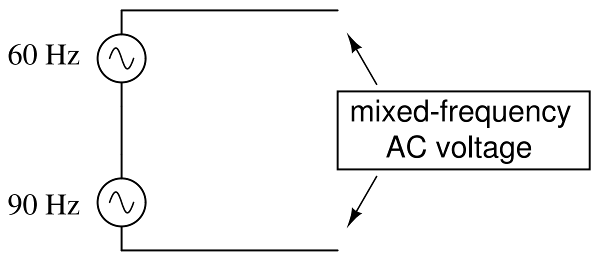


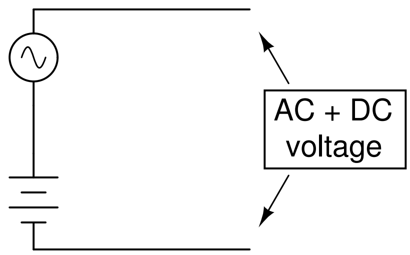
The grounded cable shield – a wire braid or metal foil wrapped around the two insulated conductors – isolates both conductors from electrostatic (capacitive) coupling by blocking any external electric fields, while the parallal proximity of the two conductors effectively cancels any electromagnetic (mutually inductive) coupling because any induced noise voltage will be approximately equal in magnitude and opposite in phase along both conductors, canceling each other at the receiving end for a net (differential) noise voltage of almost zero. Polarity marks placed near each inductive portion of signal conductor length shows how the induced voltages are phased in such a way as to cancel one another.

Coupling may also occur between two sets of conductors carrying AC signals, in which case both signals may become "mixed" with each other :



Coupling is but one example of how signals of different frequencies may become mixed. Whether it be AC mixed with DC, or two AC signals mixing with each other, signal coupling via stray inductance and capacitance is usually accidental and undesired. In other cases, mixed-frequency signals are the result of intentional design or they may be an intrinsic quality of a signal. It is generally quite easy to create mixed-frequency signal sources. Perhaps the easiest way is to simply connect voltage sources in series :





Some computer communications networks operate on the principle of superimposing high-frequency voltage signals along 60 Hz power-line conductors, so as to convey computer data along existing lengths of power cabling. This technique has been used for years in electric power distribution networks to communicate load data along high-voltage power lines. Certainly these are examples of mixed-frequency AC voltages, under conditions that are deliberately established.

In some cases, mixed-frequency signals may be produced by a single voltage source. Such is the case with microphones, which convert audio-frequency air pressure waves into corresponding voltage waveforms. The particular mix of frequencies in the voltage signal output by the microphone is dependent on the sound being reproduced. If the sound waves consist of a single, pure note or tone, the voltage waveform will likewise be a sine wave at a single frequency. If the sound wave is a chord or other harmony of several notes, the resulting voltage waveform produced by the microphone will consist of those frequencies mixed together. Very few natural sounds consist of single, pure sine wave vibrations but rather are a mix of different frequency vibrations at different amplitudes.

Musical *chords* are produced by blending one frequency with other frequencies of particular fractional multiples of the first. However, investigating a little further, we find that even a single piano note (produced by a plucked string) consists of one predominant frequency mixed with several other frequencies, each frequency a whole-number multiple of the first (called *harmonics*, while the first frequency is called the *fundamental*). An illustration of these terms is shown below with a fundamental frequency of 1000 Hz (an arbitrary figure chosen for this example), each of the frequency multiples appropriately labeled :

```
FOR A "BASE" FREQUENCY OF 1000 Hz :
Frequency (Hz)          Term
-------------------------------------------
1000 --------- 1st harmonic, or fundamental
2000 --------- 2nd harmonic
3000 --------- 3rd harmonic
4000 --------- 4th harmonic
5000 --------- 5th harmonic
6000 --------- 6th harmonic
7000 --------- 7th harmonic
ad infinitum
```

Sometimes the term "overtone" is used to describe the a harmonic frequency produced by a musical instrument. The "first" overtone is the first harmonic frequency *greater than* the fundamental. If we had an instrument producing the entire range of harmonic frequencies shown in the table above, the first overtone would be 2000 Hz (the 2nd harmonic), while the second overtone would be 3000 Hz (the 3rd harmonic), etc. However, this application of the term "overtone" is specific to particular instruments.

It so happens that certain instruments are incapable of producing certain types of harmonic frequencies. For example, an instrument made from a tube that is open on one end and closed on the other (such as a bottle, which produces sound when air is blown across the opening) is incapable of producing even-numbered harmonics. Such an instrument set up to produce a fundamental frequency of 1000 Hz would also produce frequencies of 3000 Hz, 5000 Hz, 7000 Hz, etc, but would *not* produce 2000 Hz, 4000 Hz, 6000 Hz, or any other even-multiple frequencies of the fundamental. As such, we would say that the first overtone (the first frequency greater than the fundamental) in such an instrument would be 3000 Hz (the 3rd harmonic), while the second overtone would be 5000 Hz (the 5th harmonic), and so on.

A pure sine wave (single frequency), being entirely devoid of any harmonics, sounds very "flat" and "featureless" to the human ear. Most musical instruments are incapable of producing sounds this simple. What gives each instrument its distinctive tone is the same phenomenon that gives each person a distinctive voice : the unique blending of harmonic waveforms with each fundamental note, described by the physics of motion for each unique object producing the sound.

Brass instruments do not possess the same "harmonic content" as woodwind instruments, and neither produce the same harmonic content as stringed instruments. A distinctive blend of frequencies is what gives a musical instrument its characteristic tone. As anyone who has played guitar can tell you, steel strings have a different sound than nylon strings. Also, the tone produced by a guitar string changes depending on where along its length it is plucked. These differences in tone, as well, are a result of different harmonic content produced by differences in the mechanical vibrations of an instrument's parts. All these instruments produce harmonic frequencies (whole-number multiples of the fundamental frequency) when a single note is played, but the relative amplitudes of those harmonic frequencies are different for different instruments. In musical terms, the measure of a tone's harmonic content is called *timbre* or *color*.

Musical tones become even more complex when the resonating element of an instrument is a two-dimensional surface rather than a one-dimensional string. Instruments based on the vibration of a string (guitar, piano, banjo, lute, dulcimer, etc.) or of a column of air in a tube (trumpet, flute, clarinet, tuba, pipe organ, etc.) tend to produce sounds composed of a single frequency (the "fundamental") and a mix of harmonics. Instruments based on the vibration of a flat plate (steel drums, and some types of bells), however, produce a much broader range of frequencies, not limited to whole-number multiples of the fundamental. The result is a distinctive tone that some people find acoustically offensive.

As you can see, music provides a rich field of study for mixed frequencies and their effects. Later sections of this chapter will refer to musical instruments as sources of waveforms for analysis in more detail.

- **REVIEW :**

- A *sinusoidal* waveform is one shaped exactly like a sine wave.

- A *non-sinusoidal* waveform can be anything from a distorted sine-wave shape to something completely different like a square wave.

- Mixed-frequency waveforms can be accidently created, purposely created, or simply exist out of necessity. Most musical tones, for instance, are not composed of a single frequency sine-wave, but are rich blends of different frequencies.

- When multiple sine waveforms are mixed together (as is often the case in music), the lowest frequency sine-wave is called the *fundamental*, and the other sine-waves whose frequencies are whole-number multiples of the fundamental wave are called *harmonics*.

- An *overtone* is a harmonic produced by a particular device. The "first" overtone is the first frequency greater than the fundamental, while the "second" overtone is the next greater frequency produced. Successive overtones may or may not correspond to incremental harmonics, depending on the device producing the mixed frequencies. Some devices and systems do not permit the establishment of certain harmonics, and so their overtones would only include some (not all) harmonic frequencies.

7.2 Square wave signals

It has been found that *any* repeating, non-sinusoidal waveform can be equated to a combination of DC voltage, sine waves, and/or cosine waves (sine waves with a 90 degree phase shift) at various amplitudes and frequencies. This is true no matter how strange or convoluted the waveform in question may be. So long as it repeats itself regularly over time, it is reducible to this series of sinusoidal waves. In particular, it has been found that square waves are mathematically equivalent to the sum of a sine wave at that same frequency, plus an infinite series of odd-multiple frequency sine waves at diminishing amplitude :

1 V (peak) repeating square wave at 50 Hz is equivalent to:

$$\left(\frac{4}{\pi}\right)\text{(1 V peak sine wave at 50 Hz)}$$

$$+ \left(\frac{4}{\pi}\right)\text{(1/3 V peak sine wave at 150 Hz)}$$

$$+ \left(\frac{4}{\pi}\right)\text{(1/5 V peak sine wave at 250 Hz)}$$

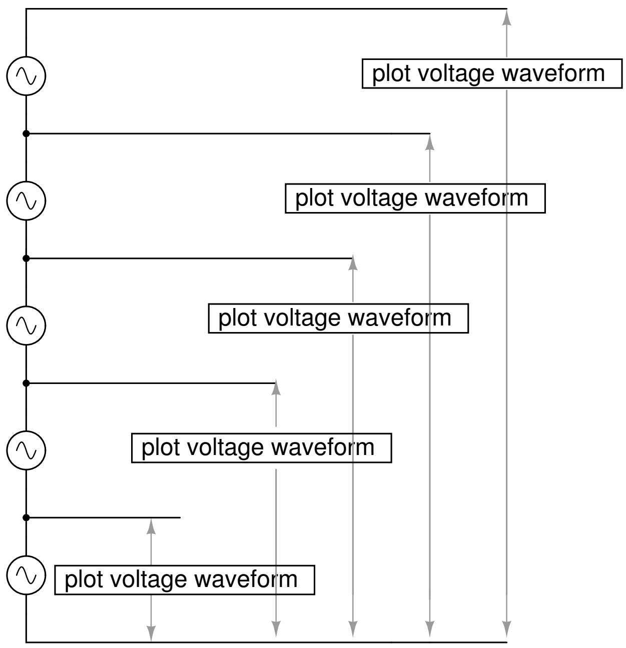
$$+ \left(\frac{4}{\pi}\right)\text{(1/7 V peak sine wave at 350 Hz)}$$

$$+ \left(\frac{4}{\pi}\right)\text{(1/9 V peak sine wave at 450 Hz)}$$

+ . . . *ad infinitum* . . .

This truth about waveforms at first may seem too strange to believe. However, if a square wave is actually an infinite series of sine wave harmonics added together, it stands to reason that we should be able to prove this by adding together several sine wave harmonics to produce a close approximation of a square wave. This reasoning is not only sound, but easily demonstrated with SPICE.

The circuit we'll be simulating is nothing more than several sine wave AC voltage sources of the proper amplitudes and frequencies connected together in series. We'll use SPICE to plot the voltage waveforms across successive additions of voltage sources, like this :



In this particular SPICE simulation, I've summed the 1st, 3rd, 5th, 7th, and 9th harmonic voltage sources in series for a total of five AC voltage sources. The fundamental frequency is 50 Hz and each harmonic is, of course, an integer multiple of that frequency. The amplitude (voltage) figures are not random numbers; rather, they have been arrived at through the equations shown in the frequency series (the fraction $4/\pi$ multiplied by 1, 1/3, 1/5, 1/7, etc. for each of the increasing odd harmonics).

```
building a squarewave
v1 1 0 sin (0 1.27324 50 0 0)       1st harmonic (50 Hz)
v3 2 1 sin (0 424.413m 150 0 0)     3rd harmonic
v5 3 2 sin (0 254.648m 250 0 0)     5th harmonic
v7 4 3 sin (0 181.891m 350 0 0)     7th harmonic
v9 5 4 sin (0 141.471m 450 0 0)     9th harmonic
r1 5 0 10k
.tran 1m 20m
.plot tran v(1,0)    Plot 1st harmonic
.plot tran v(2,0)    Plot 1st + 3rd harmonics
.plot tran v(3,0)    Plot 1st + 3rd + 5th harmonics
.plot tran v(4,0)    Plot 1st + 3rd + 5th + 7th harmonics
.plot tran v(5,0)    Plot 1st + . . . + 9th harmonics
.end
```

I'll narrate the analysis step by step from here, explaining what it is we're looking at. In this first plot, we see the fundamental-frequency sine-wave of 50 Hz by itself. It is nothing but a pure sine shape, with no additional harmonic content. This is the kind of waveform produced by an ideal AC power source :

```
time       v(1)          -1.000E+00    0.000E+00    1.000E+00
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
0.000E+00  0.000E+00 .       .             *             .       .
1.000E-03  3.915E-01 .       .             .    *        .       .
2.000E-03  7.414E-01 .       .             .         *   .       .
3.000E-03  1.020E+00 .       .             .             *       .
4.000E-03  1.199E+00 .       .             .             .  *    .
5.000E-03  1.261E+00 .       .             .             .   *   .
6.000E-03  1.199E+00 .       .             .             .  *    .
7.000E-03  1.020E+00 .       .             .             *       .
8.000E-03  7.405E-01 .       .             .         *   .       .
9.000E-03  3.890E-01 .       .             .    *        .       .
1.000E-02 -5.819E-04 .       .             *             .       .
1.100E-02 -3.901E-01 .       .        *    .             .       .
1.200E-02 -7.414E-01 .       .   *         .             .       .
1.300E-02 -1.020E+00 .       *             .             .       .
1.400E-02 -1.199E+00 .    *  .             .             .       .
1.500E-02 -1.261E+00 .   *   .             .             .       .
1.600E-02 -1.199E+00 .    *  .             .             .       .
1.700E-02 -1.020E+00 .       *             .             .       .
1.800E-02 -7.405E-01 .       .   *         .             .       .
1.900E-02 -3.890E-01 .       .        *    .             .       .
2.000E-02  5.819E-04 .       .             *             .       .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

Next, we see what happens when this clean and simple waveform is combined with the third harmonic (three times 50 Hz, or 150 Hz). Suddenly, it doesn't look like a clean sine wave any more :

```
time       v(2)          -1.000E+00    0.000E+00    1.000E+00
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
0.000E+00  0.000E+00 .       .             *             .       .
1.000E-03  7.199E-01 .       .             .         *   .       .
2.000E-03  1.108E+00 .       .             .             . *     .
3.000E-03  1.135E+00 .       .             .             . *     .
4.000E-03  9.672E-01 .       .             .             *       .
5.000E-03  8.731E-01 .       .             .           * .       .
6.000E-03  9.751E-01 .       .             .             *       .
7.000E-03  1.144E+00 .       .             .             . *     .
8.000E-03  1.111E+00 .       .             .             . *     .
9.000E-03  6.995E-01 .       .             .         *   .       .
1.000E-02 -5.697E-03 .       .             *             .       .
1.100E-02 -7.066E-01 .       .   *         .             .       .
1.200E-02 -1.108E+00 .     * .             .             .       .
1.300E-02 -1.135E+00 .     * .             .             .       .
1.400E-02 -9.672E-01 .       *             .             .       .
1.500E-02 -8.731E-01 .       . *           .             .       .
```

```
1.600E-02 -9.751E-01 .       *             .             .       .
1.700E-02 -1.144E+00 .     * .             .             .       .
1.800E-02 -1.111E+00 .     * .             .             .       .
1.900E-02 -6.995E-01 .       .   *         .             .       .
2.000E-02  5.697E-03 .       .             *             .       .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

The rise and fall times between positive and negative cycles are much steeper now, and the crests of the wave are closer to becoming flat like a squarewave. Watch what happens as we add the next odd harmonic frequency :

```
time      v(3)
time       v(3)         -1.000E+00    0.000E+00     1.000E+00
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
0.000E+00  0.000E+00 .       .             *             .       .
1.000E-03  9.436E-01 .       .             .            *.       .
2.000E-03  1.095E+00 .       .             .             .*      .
3.000E-03  9.388E-01 .       .             .            *.       .
4.000E-03  9.807E-01 .       .             .             *       .
5.000E-03  1.069E+00 .       .             .             .*      .
6.000E-03  9.616E-01 .       .             .            *.       .
7.000E-03  9.479E-01 .       .             .            *.       .
8.000E-03  1.124E+00 .       .             .             . *     .
9.000E-03  8.957E-01 .       .             .            *.       .
1.000E-02 -1.925E-02 .       .             *             .       .
1.100E-02 -9.029E-01 .       .*            .             .       .
1.200E-02 -1.095E+00 .      *.             .             .       .
1.300E-02 -9.388E-01 .       .*            .             .       .
1.400E-02 -9.807E-01 .       *             .             .       .
1.500E-02 -1.069E+00 .      *.             .             .       .
1.600E-02 -9.616E-01 .       .*            .             .       .
1.700E-02 -9.479E-01 .       .*            .             .       .
1.800E-02 -1.124E+00 .     * .             .             .       .
1.900E-02 -8.957E-01 .       .*            .             .       .
2.000E-02  1.925E-02 .       .             *             .       .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

The most noticeable change here is how the crests of the wave have flattened even more. There are more several dips and crests at each end of the wave, but those dips and crests are smaller in amplitude than they were before. Watch again as we add the next odd harmonic waveform to the mix :

```
time       v(4)           -1.000E+00    0.000E+00    1.000E+00
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
0.000E+00  0.000E+00 .       .             *             .       .
1.000E-03  1.055E+00 .       .             .             .*      .
```

```
2.000E-03  9.861E-01 .       .             .            *       .
3.000E-03  9.952E-01 .       .             .            *       .
4.000E-03  1.023E+00 .       .             .            *       .
5.000E-03  9.631E-01 .       .             .           *.       .
6.000E-03  1.044E+00 .       .             .            .*      .
7.000E-03  9.572E-01 .       .             .           *.       .
8.000E-03  1.031E+00 .       .             .            *       .
9.000E-03  9.962E-01 .       .             .            *       .
1.000E-02 -4.396E-02 .       .            *.            .       .
1.100E-02 -9.743E-01 .       *             .            .       .
1.200E-02 -9.861E-01 .       *             .            .       .
1.300E-02 -9.952E-01 .       *             .            .       .
1.400E-02 -1.023E+00 .       *             .            .       .
1.500E-02 -9.631E-01 .       .*            .            .       .
1.600E-02 -1.044E+00 .      *.             .            .       .
1.700E-02 -9.572E-01 .       .*            .            .       .
1.800E-02 -1.031E+00 .       *             .            .       .
1.900E-02 -9.962E-01 .       *             .            .       .
2.000E-02  4.396E-02 .       .             .*           .       .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

Here we can see the wave becoming flatter at each peak. Finally, adding the 9th harmonic, the fifth sine wave voltage source in our circuit, we obtain this result :

```
time       v(5)          -1.000E+00    0.000E+00    1.000E+00
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
0.000E+00  0.000E+00 .       .             *            .       .
1.000E-03  1.079E+00 .       .             .            .*      .
2.000E-03  9.845E-01 .       .             .            *       .
3.000E-03  1.017E+00 .       .             .            *       .
4.000E-03  9.835E-01 .       .             .            *       .
5.000E-03  1.017E+00 .       .             .            *       .
6.000E-03  9.814E-01 .       .             .            *       .
7.000E-03  1.023E+00 .       .             .            *       .
8.000E-03  9.691E-01 .       .             .            *       .
9.000E-03  1.048E+00 .       .             .            .*      .
1.000E-02 -8.103E-02 .       .            *.            .       .
1.100E-02 -9.557E-01 .       .*            .            .       .
1.200E-02 -9.845E-01 .       *             .            .       .
1.300E-02 -1.017E+00 .       *             .            .       .
1.400E-02 -9.835E-01 .       *             .            .       .
1.500E-02 -1.017E+00 .       *             .            .       .
1.600E-02 -9.814E-01 .       *             .            .       .
1.700E-02 -1.023E+00 .       *             .            .       .
1.800E-02 -9.691E-01 .       *             .            .       .
1.900E-02 -1.048E+00 .      *.             .            .       .
```

```
2.000E-02  8.103E-02 .       .             .*           .       .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

The end result of adding the first five odd harmonic waveforms together (all at the proper amplitudes, of course) is a close approximation of a square wave. The point in doing this is to illustrate how we can build a square wave up from multiple sine waves at different frequencies, to prove that a pure square wave is actually equivalent to a *series* of sine waves. When a square wave AC voltage is applied to a circuit with reactive components (capacitors and inductors), those components react as if they were being exposed to several sine wave voltages of different frequencies, which in fact they are.

The fact that repeating, non-sinusoidal waves are equivalent to a definite series of additive DC voltage, sine waves, and/or cosine waves is a consequence of how waves work : a fundamental property of all wave-related phenomena, electrical or otherwise. The mathematical process of reducing a non-sinusoidal wave into these constituent frequencies is called *Fourier analysis*, the details of which are well beyond the scope of this text. However, computer algorithms have been created to perform this analysis at high speeds on real waveforms, and its application in AC power quality and signal analysis is widespread.

SPICE has the ability to sample a waveform and reduce it into its constituent sine wave harmonics by way of a *Fourier Transform* algorithm, outputting the frequency analysis as a table of numbers. Let's try this on a square wave, which we already know is composed of odd-harmonic sine waves :

```
squarewave analysis netlist
v1 1 0 pulse (-1 1 0 .1m .1m 10m 20m)
r1 1 0 10k
.tran 1m 40m
.plot tran v(1,0)
.four 50 v(1,0)
.end
```

The *pulse* option in the netlist line describing voltage source `v1` instructs SPICE to simulate a square-shaped "pulse" waveform, in this case one that is symmetrical (equal time for each half-cycle) and has a peak amplitude of 1 volt. First we'll plot the square wave to be analyzed :

```
time        v(1)     -1          -0.5          0          0.5      1
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
0.000E+00 -1.000E+00 *           .            .           .        .
1.000E-03  1.000E+00 .           .            .           .        *
2.000E-03  1.000E+00 .           .            .           .        *
3.000E-03  1.000E+00 .           .            .           .        *
4.000E-03  1.000E+00 .           .            .           .        *
5.000E-03  1.000E+00 .           .            .           .        *
6.000E-03  1.000E+00 .           .            .           .        *
7.000E-03  1.000E+00 .           .            .           .        *
8.000E-03  1.000E+00 .           .            .           .        *
9.000E-03  1.000E+00 .           .            .           .        *
1.000E-02  1.000E+00 .           .            .           .        *
```

```
1.100E-02 -1.000E+00 *         .           .           .         .
1.200E-02 -1.000E+00 *         .           .           .         .
1.300E-02 -1.000E+00 *         .           .           .         .
1.400E-02 -1.000E+00 *         .           .           .         .
1.500E-02 -1.000E+00 *         .           .           .         .
1.600E-02 -1.000E+00 *         .           .           .         .
1.700E-02 -1.000E+00 *         .           .           .         .
1.800E-02 -1.000E+00 *         .           .           .         .
1.900E-02 -1.000E+00 *         .           .           .         .
2.000E-02 -1.000E+00 *         .           .           .         .
2.100E-02  1.000E+00 .         .           .           .         *
2.200E-02  1.000E+00 .         .           .           .         *
2.300E-02  1.000E+00 .         .           .           .         *
2.400E-02  1.000E+00 .         .           .           .         *
2.500E-02  1.000E+00 .         .           .           .         *
2.600E-02  1.000E+00 .         .           .           .         *
2.700E-02  1.000E+00 .         .           .           .         *
2.800E-02  1.000E+00 .         .           .           .         *
2.900E-02  1.000E+00 .         .           .           .         *
3.000E-02  1.000E+00 .         .           .           .         *
3.100E-02 -1.000E+00 *         .           .           .         .
3.200E-02 -1.000E+00 *         .           .           .         .
3.300E-02 -1.000E+00 *         .           .           .         .
3.400E-02 -1.000E+00 *         .           .           .         .
3.500E-02 -1.000E+00 *         .           .           .         .
3.600E-02 -1.000E+00 *         .           .           .         .
3.700E-02 -1.000E+00 *         .           .           .         .
3.800E-02 -1.000E+00 *         .           .           .         .
3.900E-02 -1.000E+00 *         .           .           .         .
4.000E-02 -1.000E+00 *         .           .           .         .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

Next, we'll print the Fourier analysis generated by SPICE for this square wave :

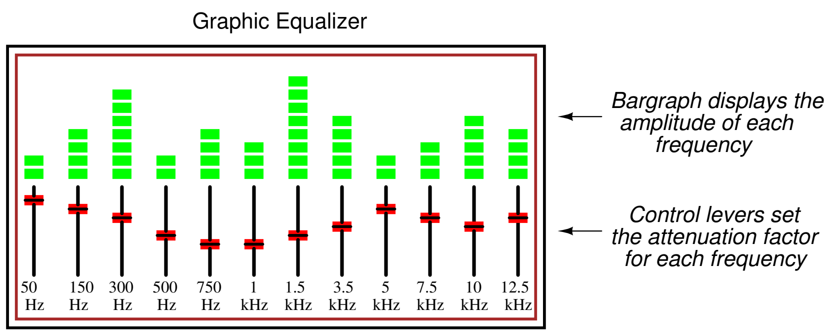
```
fourier components of transient response v(1)
dc component =  -2.439E-02
harmonic  frequency   fourier     normalized   phase   normalized
no          (hz)     component    component    (deg)   phase (deg)
1         5.000E+01   1.274E+00    1.000000    -2.195       0.000
2         1.000E+02   4.892E-02    0.038415   -94.390     -92.195
3         1.500E+02   4.253E-01    0.333987    -6.585      -4.390
4         2.000E+02   4.936E-02    0.038757   -98.780     -96.585
5         2.500E+02   2.562E-01    0.201179   -10.976      -8.780
6         3.000E+02   5.010E-02    0.039337  -103.171    -100.976
7         3.500E+02   1.841E-01    0.144549   -15.366     -13.171
8         4.000E+02   5.116E-02    0.040175  -107.561    -105.366
```

```
9         4.500E+02   1.443E-01     0.113316   -19.756     -17.561
total harmonic distortion =     43.805747  percent
```

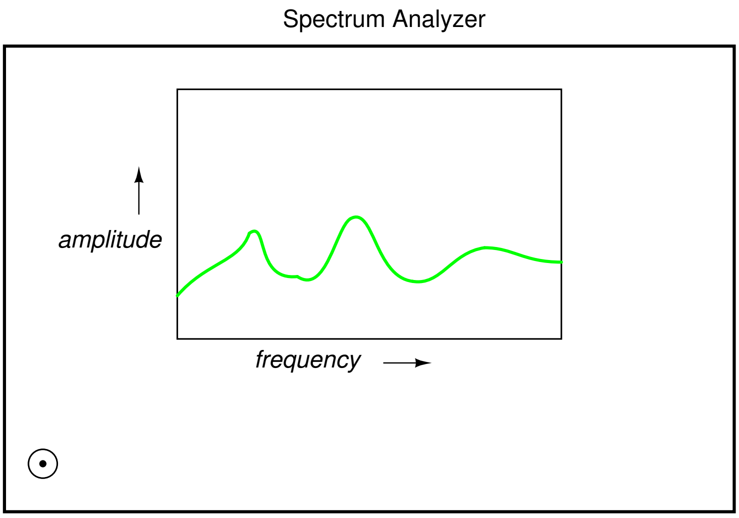
Here, SPICE has broken the waveform down into a spectrum of sinusoidal frequencies up to the ninth harmonic, plus a small DC voltage labelled `DC component`. I had to inform SPICE of the fundamental frequency (for a square wave with a 20 millisecond period, this frequency is 50 Hz), so it knew how to classify the harmonics. Note how small the figures are for all the even harmonics (2nd, 4th, 6th, 8th), and how the amplitudes of the odd harmonics diminish (1st is largest, 9th is smallest).

This same technique of "Fourier Transformation" is often used in computerized power instrumentation, sampling the AC waveform(s) and determining the harmonic content thereof. A common computer algorithm (sequence of program steps to perform a task) for this is the *Fast Fourier Transform* or *FFT* function. You need not be concerned with exactly how these computer routines work, but be aware of their existence and application.

This same mathematical technique used in SPICE to analyze the harmonic content of waves can be applied to the technical analysis of music : breaking up any particular sound into its constituent sine-wave frequencies. In fact, you may have already seen a device designed to do just that without realizing what it was! A *graphic equalizer* is a piece of high-fidelity stereo equipment that controls (and sometimes displays) the nature of music's harmonic content. Equipped with several knobs or slide levers, the equalizer is able to selectively attenuate (reduce) the amplitude of certain frequencies present in music, to "customize" the sound for the listener's benefit. Typically, there will be a "bar graph" display next to each control lever, displaying the amplitude of each particular frequency.



A device built strictly to display – not control – the amplitudes of each frequency range for a mixed-frequency signal is typically called a *spectrum analyzer*. The design of spectrum analyzers may be as simple as a set of "filter" circuits (see the next chapter for details) designed to separate the different frequencies from each other, or as complex as a special-purpose digital computer running an FFT algorithm to mathematically split the signal into its harmonic components. Spectrum analyzers are often designed to analyze extremely high-frequency signals, such as those produced by radio transmitters and computer network hardware. In that form, they often have an appearance like that of an oscilloscope :



Like an oscilloscope, the spectrum analyzer uses a CRT (or a computer display mimicking a CRT) to display a plot of the signal. Unlike an oscilloscope, this plot is amplitude over *frequency* rather than amplitude over *time*. In essence, a frequency analyzer gives the operator a Bode plot of the signal : something an engineer might call a *frequency-domain* rather than a *time-domain* analysis.

The term "domain" is mathematical : a sophisticated word to describe the horizontal axis of a graph. Thus, an oscilloscope's plot of amplitude (vertical) over time (horizontal) is a "time-domain" analysis, whereas a spectrum analyzer's plot of amplitude (vertical) over frequency (horizontal) is a "frequency-domain" analysis. When we use SPICE to plot signal amplitude (either voltage or current amplitude) over a range of frequencies, we are performing *frequency-domain* analysis.

Please take note of how the Fourier analysis from the last SPICE simulation isn't "perfect." Ideally, the amplitudes of all the even harmonics should be absolutely zero, and so should the DC component. Again, this is not so much a quirk of SPICE as it is a property of waveforms in general. A waveform of infinite duration (infinite number of cycles) can be analyzed with absolute precision, but the less cycles available to the computer for analysis, the less precise the analysis. It is only when we have an equation describing a waveform in its entirety that Fourier analysis can reduce it to a definite series of sinusoidal waveforms. The fewer times that a wave cycles, the less certain its frequency is. Taking this concept to its logical extreme, a short pulse – a waveform that doesn't even complete a cycle – actually *has no frequency*, but rather acts as an infinite range of frequencies. This principle is common to *all* wave-based phenomena, not just AC voltages and currents.

Suffice it to say that the number of cycles and the certainty of a waveform's frequency component(s) are directly related. We could improve the precision of our analysis here by letting the wave oscillate on and on for many cycles, and the result would be a spectrum analysis more consistent with the ideal. In the following analysis, I've omitted the waveform plot for brevity's sake – it's just a really long square wave :

```
squarewave
v1 1 0 pulse (-1 1 0 .1m .1m 10m 20m)
r1 1 0 10k
```

```
.option limpts=1001
.tran 1m 1
.plot tran v(1,0)
.four 50 v(1,0)
.end

fourier components of transient response v(1)
dc component =   9.999E-03
harmonic  frequency    fourier    normalized   phase   normalized
no           (hz)      component   component    (deg)   phase (deg)
1         5.000E+01   1.273E+00     1.000000    -1.800        0.000
2         1.000E+02   1.999E-02     0.015704    86.382       88.182
3         1.500E+02   4.238E-01     0.332897    -5.400       -3.600
4         2.000E+02   1.997E-02     0.015688    82.764       84.564
5         2.500E+02   2.536E-01     0.199215    -9.000       -7.200
6         3.000E+02   1.994E-02     0.015663    79.146       80.946
7         3.500E+02   1.804E-01     0.141737   -12.600      -10.800
8         4.000E+02   1.989E-02     0.015627    75.529       77.329
9         4.500E+02   1.396E-01     0.109662   -16.199      -14.399
```

Notice how this analysis shows less of a DC component voltage and lower amplitudes for each of the even harmonic frequency sine waves, all because we let the computer sample more cycles of the wave. Again, the imprecision of the first analysis is not so much a flaw in SPICE as it is a fundamental property of waves and of signal analysis.

- **REVIEW :**
- Square waves are equivalent to a sine wave at the same (fundamental) frequency added to an infinite series of odd-multiple sine-wave harmonics at decreasing amplitudes.
- Computer algorithms exist which are able to sample waveshapes and determine their constituent sinusoidal components. The *Fourier Transform* algorithm (particularly the *Fast Fourier Transform*, or *FFT*) is commonly used in computer circuit simulation programs such as SPICE and in electronic metering equipment for determining power quality.

7.3 Other waveshapes

As strange as it may seem, *any* repeating, non-sinusoidal waveform is actually equivalent to a series of sinusoidal waveforms of different amplitudes and frequencies added together. Square waves are a very common and well-understood case, but not the only one.

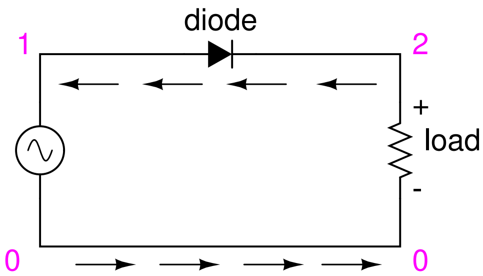
Electronic power control devices such as transistors and silicon-controlled rectifiers (*SCRs*) often produce voltage and current waveforms that are essentially chopped-up versions of the otherwise "clean" (pure) sine-wave AC from the power supply. These devices have the ability to suddenly *change* their resistance with the application of a control signal voltage or current, thus "turning on" or "turning off" almost instantaneously, producing current waveforms bearing little resemblance to the source voltage waveform powering the circuit. These current waveforms then produce changes in the voltage waveform to other circuit components, due to voltage drops created by the non-sinusoidal current through circuit impedances.

Circuit components that distort the normal sine-wave shape of AC voltage or current are called *nonlinear*. Nonlinear components such as SCRs find popular use in power electronics due to their ability to regulate large amounts of electrical power without dissipating much heat. While this is an advantage from the perspective of energy efficiency, the waveshape distortions they introduce can cause problems.

These non-sinusoidal waveforms, regardless of their actual shape, are equivalent to a series of sinusoidal waveforms of higher (harmonic) frequencies. If not taken into consideration by the circuit designer, these harmonic waveforms created by electronic switching components may cause erratic circuit behavior. It is becoming increasingly common in the electric power industry to observe overheating of transformers and motors due to distortions in the sine-wave shape of the AC power line voltage stemming from "switching" loads such as computers and high-efficiency lights. This is no theoretical exercise : it is very real and potentially very troublesome.

In this section, I will investigate a few of the more common waveshapes and show their harmonic components by way of Fourier analysis using SPICE.

One very common way harmonics are generated in an AC power system is when AC is converted, or "rectified" into DC. This is generally done with components called *diodes*, which only allow passage current in one direction. The simplest type of AC/DC rectification is *half-wave*, where a single diode blocks half of the AC current (over time) from passing through the load. Oddly enough, the conventional diode schematic symbol is drawn such that electrons flow *against* the direction of the symbol's arrowhead :



The diode only allows electron flow in a counter-clockwise direction.

```
halfwave rectifier
v1 1 0 sin(0 15 60 0 0)
rload 2 0 10k
d1 1 2 mod1
.model mod1 d
.tran .5m 17m
.plot tran v(1,0) v(2,0)
.four 60 v(1,0) v(2,0)
.end
```

```
legend :
* : v(1)
+ : v(2)
time      v(1)
(*)----------      -20     -10           0          10          20
(+)----------       -5       0           5          10          15
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
0.000E+00  0.000E+00 .       +           *           .           .
5.000E-04  2.806E+00 .       .   +       .   *       .           .
1.000E-03  5.483E+00 .       .           +       *   .           .
1.500E-03  7.929E+00 .       .           .     +    *.           .
2.000E-03  1.013E+01 .       .           .           +*          .
2.500E-03  1.198E+01 .       .           .           .  *+       .
3.000E-03  1.338E+01 .       .           .           .    *  +   .
3.500E-03  1.435E+01 .       .           .           .     *   + .
4.000E-03  1.476E+01 .       .           .           .      *   +.
4.500E-03  1.470E+01 .       .           .           .      *   +.
5.000E-03  1.406E+01 .       .           .           .     *   + .
5.500E-03  1.299E+01 .       .           .           .   *  +    .
6.000E-03  1.139E+01 .       .           .           . x         .
6.500E-03  9.455E+00 .       .           .        + *.           .
7.000E-03  7.113E+00 .       .           .   +    *  .           .
7.500E-03  4.591E+00 .       .        +  .     *     .           .
8.000E-03  1.841E+00 .       . +         .  *        .           .
8.500E-03 -9.177E-01 .       +          *.           .           .
9.000E-03 -3.689E+00 .       +      *    .           .           .
9.500E-03 -6.298E+00 .       +  *        .           .           .
1.000E-02 -8.701E+00 .       +*          .           .           .
1.050E-02 -1.079E+01 .      *+           .           .           .
1.100E-02 -1.249E+01 .    *  +           .           .           .
1.150E-02 -1.377E+01 .  *    +           .           .           .
1.200E-02 -1.453E+01 . *     +           .           .           .
1.250E-02 -1.482E+01 .*      +           .           .           .
1.300E-02 -1.452E+01 . *     +           .           .           .
1.350E-02 -1.378E+01 .  *    +           .           .           .
1.400E-02 -1.248E+01 .    *  +           .           .           .
1.450E-02 -1.081E+01 .      *+           .           .           .
1.500E-02 -8.681E+00 .       +*          .           .           .
1.550E-02 -6.321E+00 .       +  *        .           .           .
1.600E-02 -3.666E+00 .       +      *    .           .           .
1.650E-02 -9.432E-01 .       .   +      *.           .           .
1.700E-02  1.865E+00 .       .      +    .  *        .           .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

First, we'll see how SPICE analyzes the source waveform, a pure sine wave voltage :

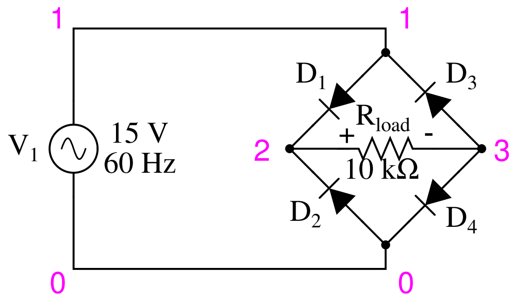
```
fourier components of transient response v(1)
dc component =   8.016E-04
harmonic   frequency  fourier    normalized   phase   normalized
no           (hz)     component  component    (deg)   phase (deg)
1          6.000E+01  1.482E+01    1.000000    -0.005       0.000
2          1.200E+02  2.492E-03    0.000168  -104.347    -104.342
3          1.800E+02  6.465E-04    0.000044   -86.663     -86.658
4          2.400E+02  1.132E-03    0.000076   -61.324     -61.319
5          3.000E+02  1.185E-03    0.000080   -70.091     -70.086
6          3.600E+02  1.092E-03    0.000074   -63.607     -63.602
7          4.200E+02  1.220E-03    0.000082   -56.288     -56.283
8          4.800E+02  1.354E-03    0.000091   -54.669     -54.664
9          5.400E+02  1.467E-03    0.000099   -52.660     -52.655
```

Notice the extremely small harmonic and DC components of this sinusoidal waveform. Ideally, there would be nothing but the fundamental frequency showing (being a perfect sine wave), but our Fourier analysis figures aren't perfect because SPICE doesn't have the luxury of sampling a waveform of infinite duration. Next, we'll compare this with the Fourier analysis of the half-wave "rectified" voltage across the load resistor :

```
fourier components of transient response v(2)
dc component =   4.456E+00
harmonic  frequency   fourier    normalized   phase    normalized
no          (hz)      component  component    (deg)    phase (deg)
1         6.000E+01   7.000E+00    1.000000    -0.195       0.000
2         1.200E+02   3.016E+00    0.430849   -89.765     -89.570
3         1.800E+02   1.206E-01    0.017223  -168.005    -167.810
4         2.400E+02   5.149E-01    0.073556   -87.295     -87.100
5         3.000E+02   6.382E-02    0.009117  -152.790    -152.595
6         3.600E+02   1.727E-01    0.024676   -79.362     -79.167
7         4.200E+02   4.492E-02    0.006417  -132.420    -132.224
8         4.800E+02   7.493E-02    0.010703   -61.479     -61.284
9         5.400E+02   4.051E-02    0.005787  -115.085    -114.889
```

Notice the relatively large even-multiple harmonics in this analysis. By cutting out half of our AC wave, we've introduced the equivalent of several higher-frequency sinusoidal (actually, cosine) waveforms into our circuit from the original, pure sine-wave. Also take note of the large DC component : 4.456 volts. Because our AC voltage waveform has been "rectified" (only allowed to push in one direction across the load rather than back-and-forth), it behaves a lot more like DC.

Another method of AC/DC conversion is called *full-wave*, which as you may have guessed utilizes the full cycle of AC power from the source, reversing the polarity of half the AC cycle to get electrons to flow through the load the same direction all the time. I won't bore you with details of exactly how this is done, but we can examine the waveform and its harmonic analysis through SPICE :




```
fullwave bridge rectifier
v1 1 0 sin(0 15 60 0 0)
rload 2 3 10k
d1 1 2 mod1
d2 0 2 mod1
d3 3 1 mod1
d4 3 0 mod1
.model mod1 d
.tran .5m 17m
.plot tran v(1,0) v(2,3)
.four 60 v(2,3)
.end
```

```
legend :
* : v(1)
+ : v(2,3)
time       v(1)
(*)----------      -20         -10     0.000E+00     1.000E+01
(+)----------  0.000E+00     5.000E+00     1.000E+01     1.500E+01
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
0.000E+00  0.000E+00 +          .            *            .      .
5.000E-04  2.806E+00 .   +      .            .   *        .      .
1.000E-03  5.483E+00 .         +.            .      *     .      .
1.500E-03  7.929E+00 .          .    +       .         *  .      .
2.000E-03  1.013E+01 .          .         +  .            *      .
2.500E-03  1.198E+01 .          .            . +          . *    .
3.000E-03  1.338E+01 .          .            .     +      .   *  .
3.500E-03  1.435E+01 .          .            .       +    .     *.
4.000E-03  1.476E+01 .          .            .        +   .      *
4.500E-03  1.470E+01 .          .            .        +   .     *.
5.000E-03  1.406E+01 .          .            .       +    .    * .
5.500E-03  1.299E+01 .          .            .    +       .  *   .
6.000E-03  1.139E+01 .          .            +            .*     .
6.500E-03  9.455E+00 .          .       +    .           *.      .
7.000E-03  7.113E+00 .          . +          .        *   .      .
```

```
7.500E-03  4.591E+00 .      +   .            .     *      .      .
8.000E-03  1.841E+00 .   +      .            .  *         .      .
8.500E-03 -9.177E-01 . +        .           *.            .      .
9.000E-03 -3.689E+00 .    +     .       *    .            .      .
9.500E-03 -6.298E+00 .          +    *       .            .      .
1.000E-02 -8.701E+00 .          . *    +     .            .      .
1.050E-02 -1.079E+01 .         *.           +.            .      .
1.100E-02 -1.249E+01 .       *  .            .  +         .      .
1.150E-02 -1.377E+01 .     *    .            .      +     .      .
1.200E-02 -1.453E+01 .    *     .            .        +   .      .
1.250E-02 -1.482E+01 .   *      .            .         +  .      .
1.300E-02 -1.452E+01 .    *     .            .        +   .      .
1.350E-02 -1.378E+01 .     *    .            .      +     .      .
1.400E-02 -1.248E+01 .       *  .            .  +         .      .
1.450E-02 -1.081E+01 .         *.           +.            .      .
1.500E-02 -8.681E+00 .          . *    +     .            .      .
1.550E-02 -6.321E+00 .          +    *       .            .      .
1.600E-02 -3.666E+00 .     +    .       *    .            .      .
1.650E-02 -9.432E-01 .   +      .           *.            .      .
1.700E-02  1.865E+00 .    +     .            .  *         .      .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

fourier components of transient response v(2,3)
dc component =   8.273E+00
harmonic   frequency  fourier    normalized   phase   normalized
no           (hz)    component   component    (deg)   phase (deg)
1        6.000E+01   7.000E-02    1.000000   -93.519       0.000
2        1.200E+02   5.997E+00   85.669415   -90.230       3.289
3        1.800E+02   7.241E-02    1.034465   -93.787      -0.267
4        2.400E+02   1.013E+00   14.465161   -92.492       1.027
5        3.000E+02   7.364E-02    1.052023   -95.026      -1.507
6        3.600E+02   3.337E-01    4.767350  -100.271      -6.752
7        4.200E+02   7.496E-02    1.070827   -94.023      -0.504
8        4.800E+02   1.404E-01    2.006043  -118.839     -25.319
9        5.400E+02   7.457E-02    1.065240   -90.907       2.612
```

What a difference! According to SPICE's Fourier transform, we have a 2nd harmonic component to this waveform that's over 85 times the amplitude of the original AC source frequency! The DC component of this wave shows up as being 8.273 volts (almost twice what is was for the half-wave rectifier circuit) while the second harmonic is almost 6 volts in amplitude. Notice all the other harmonics further on down the table. The odd harmonics are actually stronger at some of the higher frequencies than they are at the lower frequencies, which is interesting.

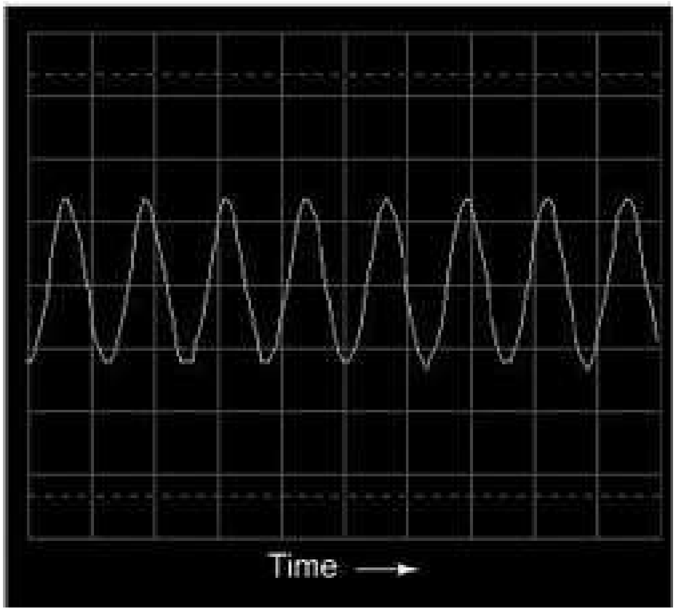
As you can see, what may begin as a neat, simple AC sine-wave may end up as a complex mess of harmonics after passing through just a few electronic components. While the complex mathematics behind all this Fourier transformation is not necessary for the beginning student of electric circuits to understand, it is of the utmost importance to realize the principles at work and to grasp the practical effects that harmonic signals may have on circuits. The practical effects of harmonic frequencies in circuits will be explored in the last section of this chapter, but before we do that we'll take a closer look at waveforms and their respective harmonics.

- **REVIEW :**
- *Any* waveform at all, so long as it is repetitive, can be reduced to a series of sinusoidal waveforms added together. Different waveshapes consist of different blends of sine-wave harmonics.
- Rectification of AC to DC is a very common source of harmonics within industrial power systems.

7.4 More on spectrum analysis

Computerized Fourier analysis, particularly in the form of the *FFT* algorithm, is a powerful tool for furthering our understanding of waveforms and their related spectral components. This same mathematical routine programmed into the SPICE simulator as the `.fourier` option is also programmed into a variety of electronic test instruments to perform real-time Fourier analysis on measured signals. This section is devoted to the use of such tools and the analysis of several different waveforms.

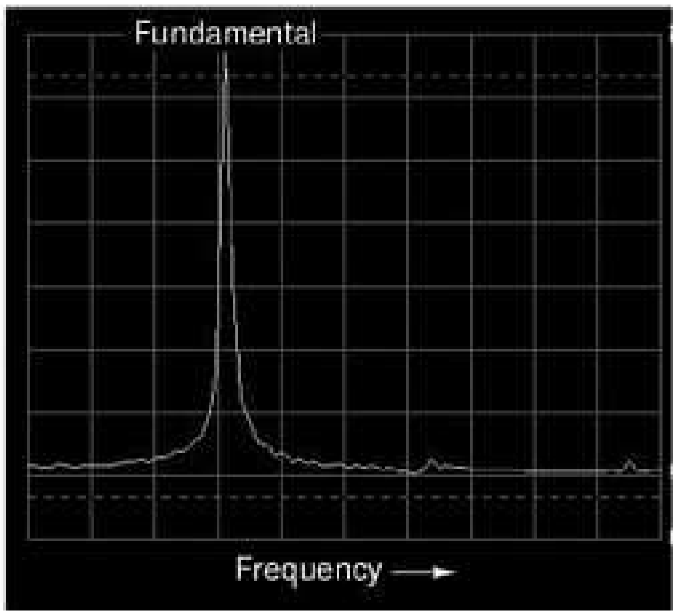
First we have a simple sine wave at a frequency of 523.25 Hz. This particular frequency value is a "C" pitch on a piano keyboard, one octave above "middle C". Actually, the signal measured for this demonstration was created by an electronic keyboard set to produce the tone of a panflute, the closest instrument "voice" I could find resembling a perfect sine wave. The plot below was taken from an oscilloscope display, showing signal amplitude (voltage) over time :



Viewed with an oscilloscope, a sine wave looks like a wavy curve traced horizontally on the screen. The horizontal axis of this oscilloscope display is marked with the word "Time" and an arrow pointing in the direction of time's progression. The curve itself, of course, represents the cyclic increase and decrease of voltage over time.

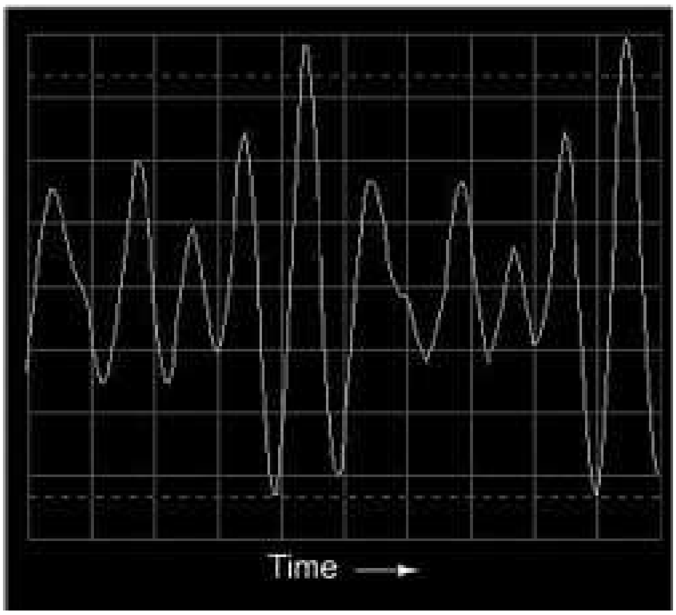
Close observation reveals imperfections in the sine-wave shape. This, unfortunately, is a result of the specific equipment used to analyze the waveform. Characteristics like these due to quirks of the test equipment are technically known as *artifacts* : phenomena existing solely because of a peculiarity in the equipment used to perform the experiment.

If we view this same AC voltage on a spectrum analyzer, the result is quite different :

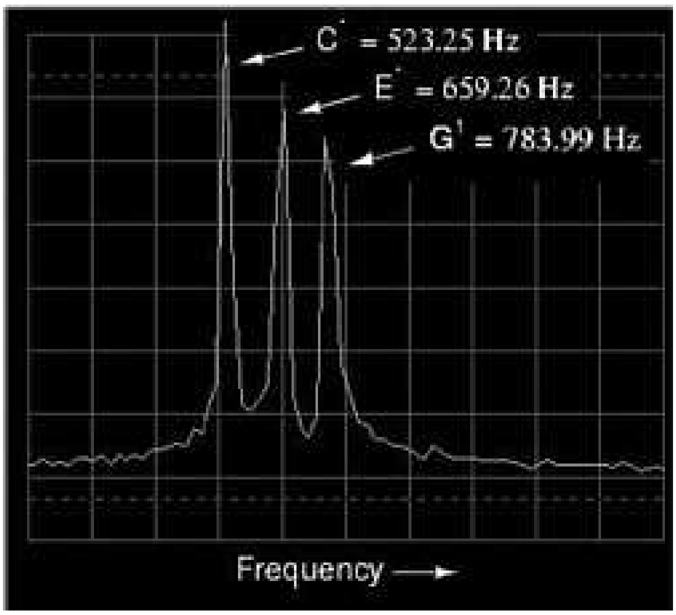


As you can see, the horizontal axis of the display is marked with the word "Frequency," denoting the domain of this measurement. The single peak on the curve represents the predominance of a single frequency within the range of frequencies covered by the width of the display. If the scale of this analyzer instrument were marked with numbers, you would see that this peak occurs at 523.25 Hz. The height of the peak represents the signal amplitude (voltage).

If we mix three different sine-wave tones together on the electronic keyboard (C-E-G, a C-major chord) and measure the result, both the oscilloscope display and the spectrum analyzer display reflect this increased complexity :



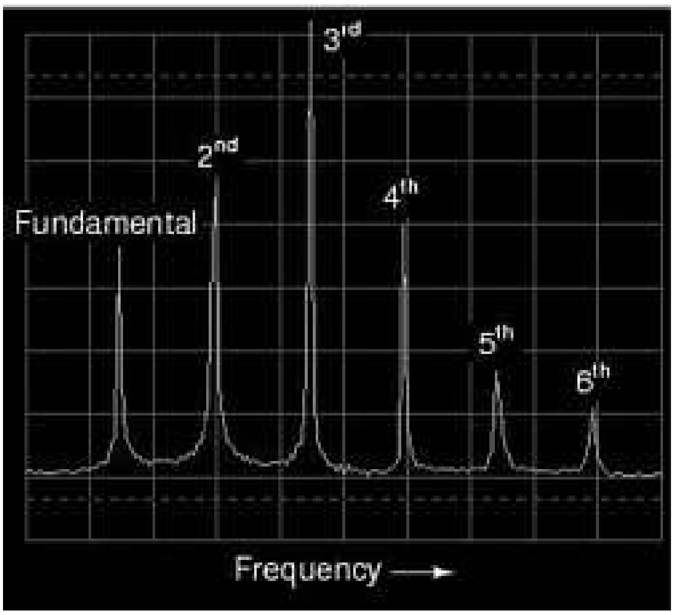
The oscilloscope display (time-domain) shows a waveform with many more peaks and valleys than before, a direct result of the mixing of these three frequencies. As you will notice, some of these peaks are higher than the peaks of the original single-pitch waveform, while others are lower. This is a result of the three different waveforms alternately reinforcing and canceling each other as their respective phase shifts change in time.



The spectrum display (frequency-domain) is much easier to interpret : each pitch is represented by its own peak on the curve. The difference in height between these three peaks is another artifact of the test equipment : a consequence of limitations within the equipment used to generate and analyze these waveforms, and not a necessary characteristic of the musical chord itself.

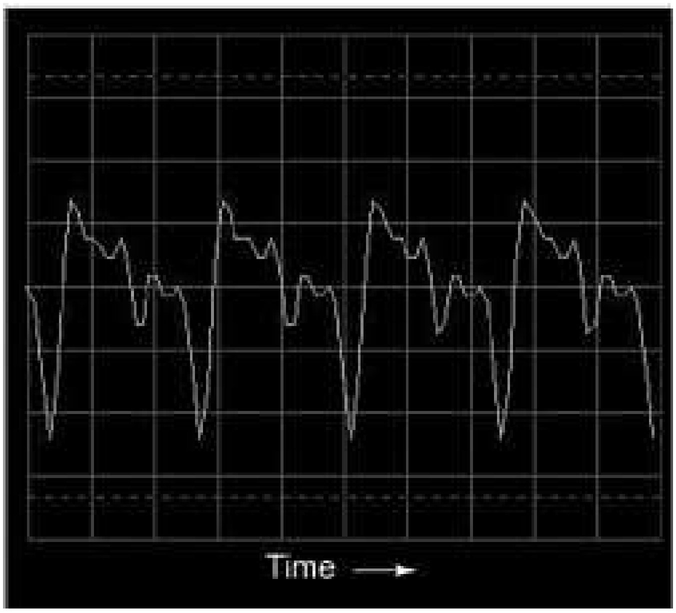
As was stated before, the device used to generate these waveforms is an electronic keyboard : a musical instrument designed to mimic the tones of many different instruments. The panflute "voice" was chosen for the first demonstrations because it most closely resembled a pure sine wave (a single frequency on the spectrum analyzer display). Other musical instrument "voices" are not as simple as this one, though. In fact, the unique tone produced by *any* instrument is a function of its waveshape (or spectrum of frequencies). For example, let's view the signal for a trumpet tone :

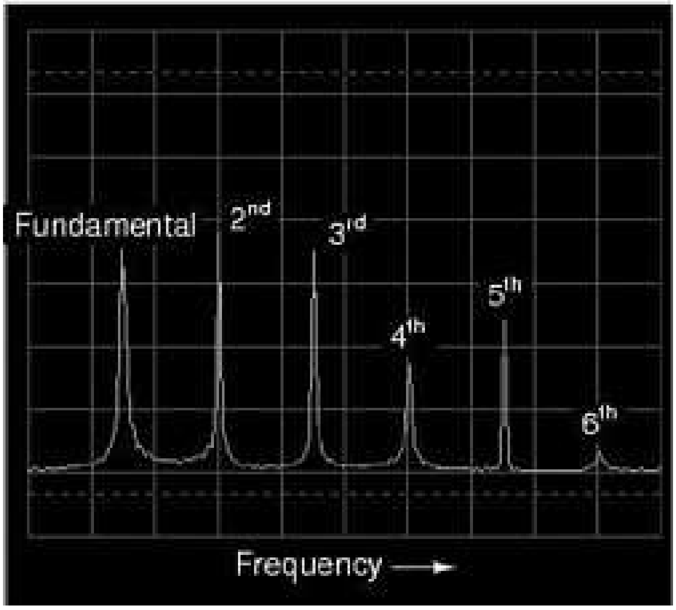
The fundamental frequency of this tone is the same as in the first panflute example : 523.25 Hz, one octave above "middle C." The waveform itself is far from a pure and simple sine-wave form. Knowing that any repeating, non-sinusoidal waveform is equivalent to a series of sinusoidal waveforms at different amplitudes and frequencies, we should expect to see multiple peaks on the spectrum analyzer display :



Indeed we do! The fundamental frequency component of 523.25 Hz is represented by the left-most peak, with each successive harmonic represented as its own peak along the width of the analyzer screen. The second harmonic is twice the frequency of the fundamental (1046.5 Hz), the third harmonic three times the fundamental (1569.75 Hz), and so on. This display only shows the first six harmonics, but there are many more comprising this complex tone.

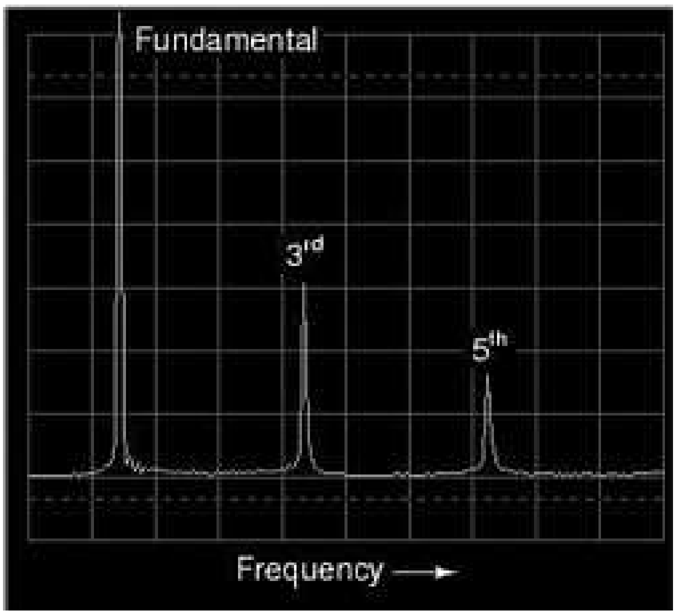
Trying a different instrument voice (the accordion) on the keyboard, we obtain a similarly complex oscilloscope (time-domain) plot and spectrum analyzer (frequency-domain) display :



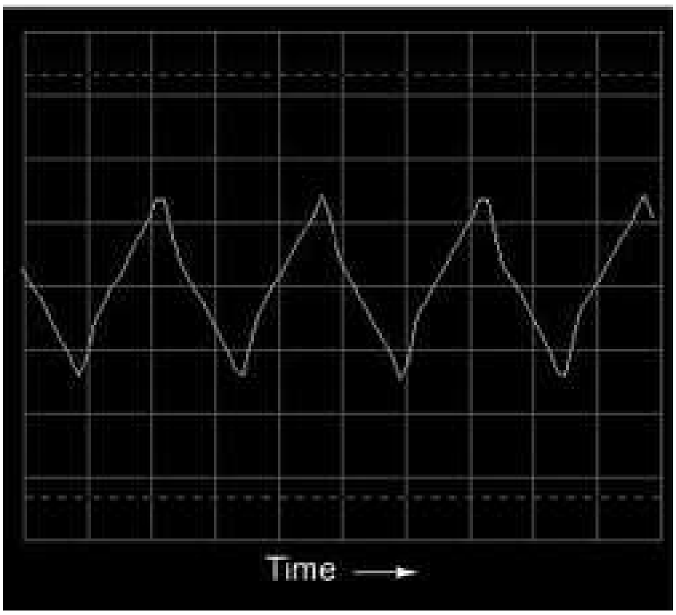


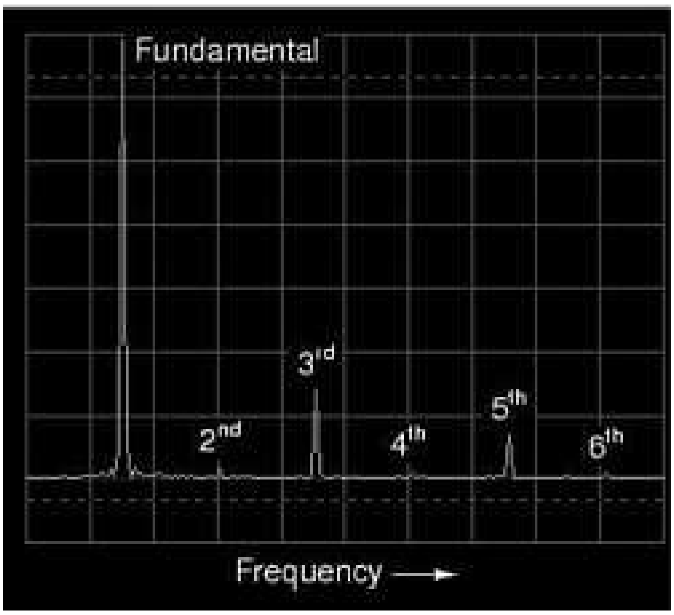
Note the differences in relative harmonic amplitudes (peak heights) on the spectrum displays for trumpet and accordion. Both instrument tones contain harmonics all the way from 1st (fundamental) to 6th (and beyond!), but the proportions aren't the same. Each instrument has a unique harmonic "signature" to its tone. Bear in mind that all this complexity is in reference to *a single note* played with these two instrument "voices." Multiple notes played on an accordion, for example, would create a much more complex mixture of frequencies than what is seen here.

The analytical power of the oscilloscope and spectrum analyzer permit us to derive general rules about waveforms and their harmonic spectra from real waveform examples. We already know that any deviation from a pure sine-wave results in the equivalent of a mixture of multiple sine-wave waveforms at different amplitudes and frequencies. However, close observation allows us to be more specific than this. Note, for example, the time- and frequency-domain plots for a waveform approximating a square wave :



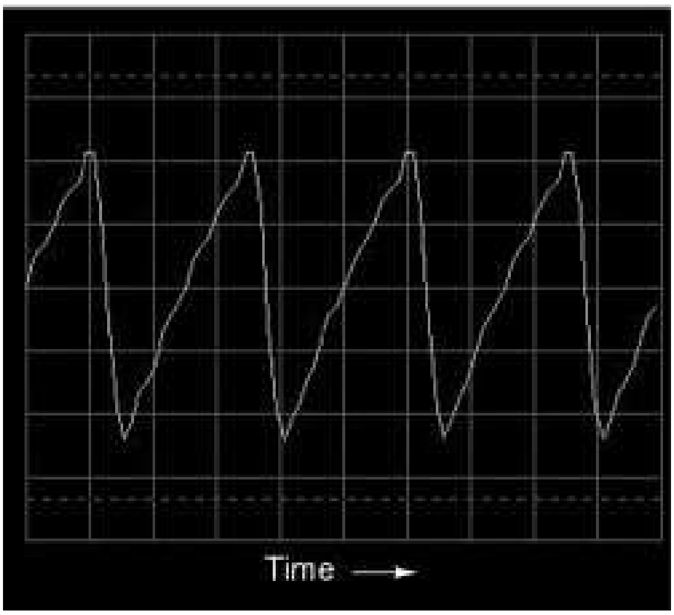
According to the spectrum analysis, this waveform contains *no* even harmonics, only odd. Although this display doesn't show frequencies past the sixth harmonic, the pattern of odd-only harmonics in descending amplitude continues indefinitely. This should come as no surprise, as we've already seen with SPICE that a square wave is comprised of an infinitude of odd harmonics. The trumpet and accordion tones, however, contained *both* even and odd harmonics. This difference in harmonic content is noteworthy. Let's continue our investigation with an analysis of a triangle wave :



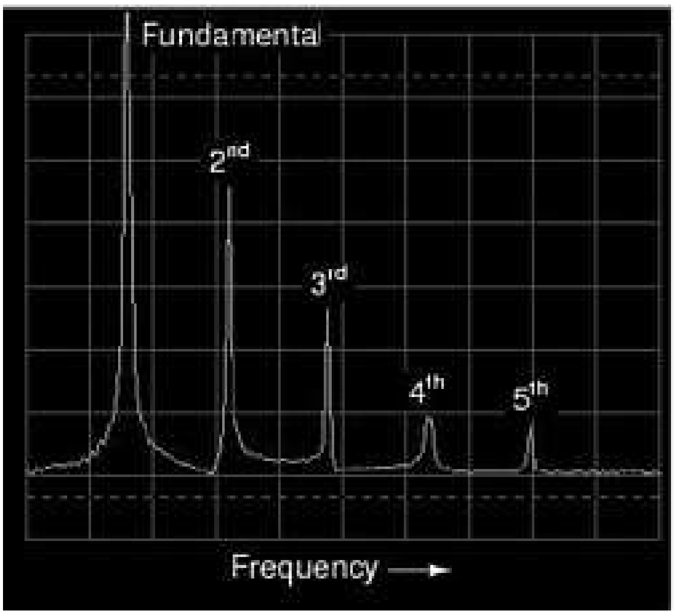


In this waveform there are practically no even harmonics : the only significant frequency peaks on the spectrum analyzer display belong to odd-numbered multiples of the fundamental frequency. Tiny peaks can be seen for the second, fourth, and sixth harmonics, but this is due to imperfections in this particular triangle waveshape (once again, artifacts of the test equipment used in this analysis). A perfect triangle waveshape produces no even harmonics, just like a perfect square wave. It should be obvious from inspection that the harmonic spectrum of the triangle wave is not identical to the spectrum of the square wave : the respective harmonic peaks are of different heights. However, the two different waveforms are common in their lack of even harmonics.

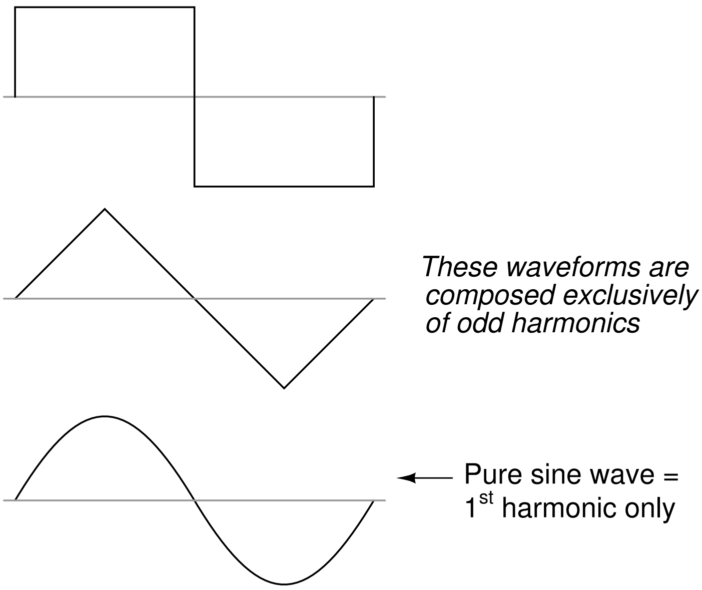
Let's examine another waveform, this one very similar to the triangle wave, except that its rise-time is not the same as its fall-time. Known as a *sawtooth wave*, its oscilloscope plot reveals it to be aptly named :



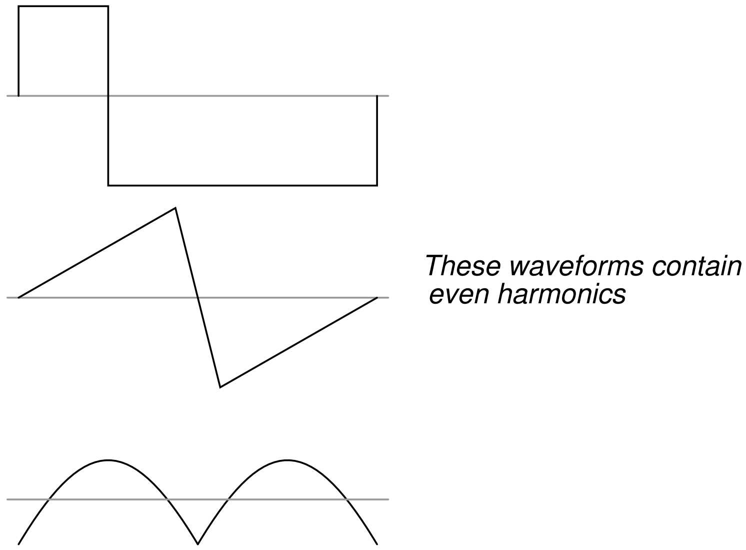
When the spectrum analysis of this waveform is plotted, we see a result that is quite different from that of the regular triangle wave, for this analysis shows the strong presence of even-numbered harmonics (second and fourth) :



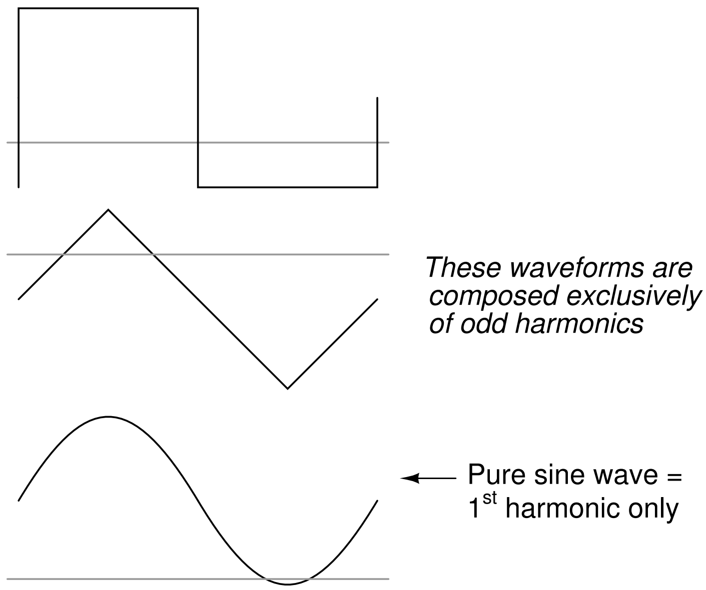
The distinction between a waveform having even harmonics versus no even harmonics resides in the difference between a triangle waveshape and a sawtooth waveshape. That difference is *symmetry* above and below the horizontal centerline of the wave. A waveform that is symmetrical above and below its centerline (the shape on both sides mirror each other precisely) will contain *no* even-numbered harmonics.



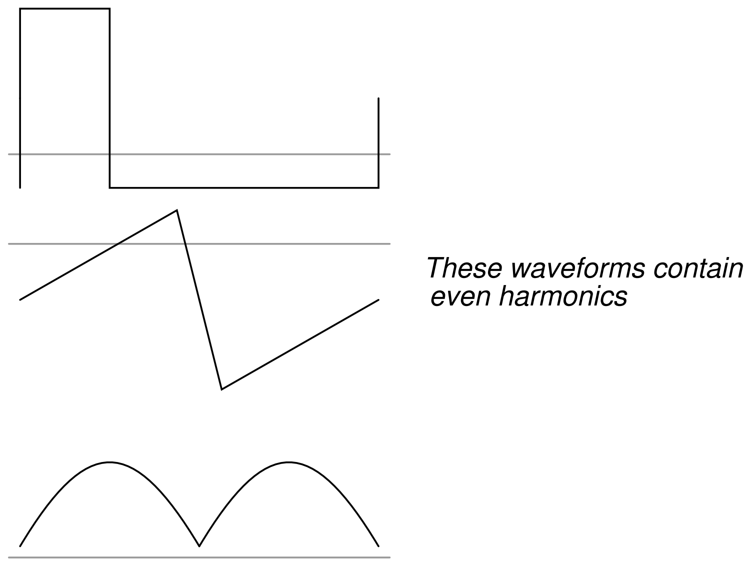
Square waves, triangle waves, and pure sine waves all exhibit this symmetry, and all are devoid of even harmonics. Waveforms like the trumpet tone, the accordion tone, and the sawtooth wave are unsymmetrical around their centerlines and therefore *do* contain even harmonics.



This principle of centerline symmetry should not be confused with symmetry around the *zero* line. In the examples shown, the horizontal centerline of the waveform happens to be zero volts on the time-domain graph, but this has nothing to do with harmonic content. This rule of harmonic content (even harmonics only with unsymmetrical waveforms) applies whether or not the waveform is shifted above or below zero volts with a "DC component." For further clarification, I will show the same sets of waveforms, shifted with DC voltage, and note that their harmonic contents are unchanged.



Again, the amount of DC voltage present in a waveform has nothing to do with that waveform's harmonic frequency content.



Why is this harmonic rule-of-thumb an important rule to know? It can help us comprehend the relationship between harmonics in AC circuits and specific circuit components. Since most sources of sine-wave distortion in AC power circuits tend to be symmetrical, even-numbered harmonics are rarely seen in those applications. This is good to know if you're a power system designer and are planning ahead for harmonic reduction : you only have to concern yourself with mitigating the odd harmonic frequencies, even harmonics being practically nonexistent. Also, if you happen to measure even harmonics in an AC circuit with a spectrum analyzer or frequency meter, you know that something in that circuit must be *unsymmetrically* distorting the sine-wave voltage or current, and that clue may be helpful in locating the source of a problem (look for components or conditions more likely to distort one half-cycle of the AC waveform more than the other).

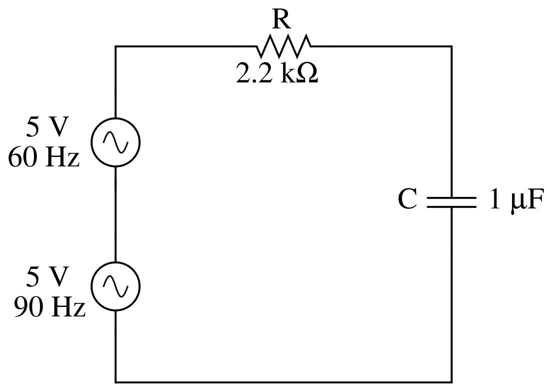
Now that we have this rule to guide our interpretation of nonsinusoidal waveforms, it makes more sense that a waveform like that produced by a rectifier circuit should contain such strong even harmonics, there being no symmetry at all above and below center.

- **REVIEW :**
- Waveforms that are symmetrical above and below their horizontal centerlines contain no even-numbered harmonics.
- The amount of DC "bias" voltage present (a waveform's "DC component") has no impact on that wave's harmonic frequency content.

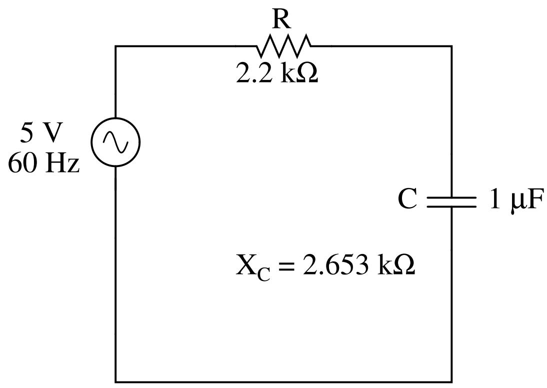
7.5 Circuit effects

The principle of non-sinusoidal, repeating waveforms being equivalent to a series of sine waves at different frequencies is a fundamental property of waves in general and it has great practical import in the study of AC circuits. It means that any time we have a waveform that isn't perfectly sine-wave-shaped, the circuit in question will react as though it's having an array of different frequency voltages imposed on it at once.

When an AC circuit is subjected to a source voltage consisting of a mixture of frequencies, the components in that circuit respond to each constituent frequency in a different way. Any reactive component such as a capacitor or an inductor will simultaneously present a unique amount of impedance to each and every frequency present in a circuit. Thankfully, the analysis of such circuits is made relatively easy by applying the *Superposition Theorem*, regarding the multiple-frequency source as a set of single-frequency voltage sources connected in series, and analyzing the circuit for one source at a time, summing the results at the end to determine the aggregate total :

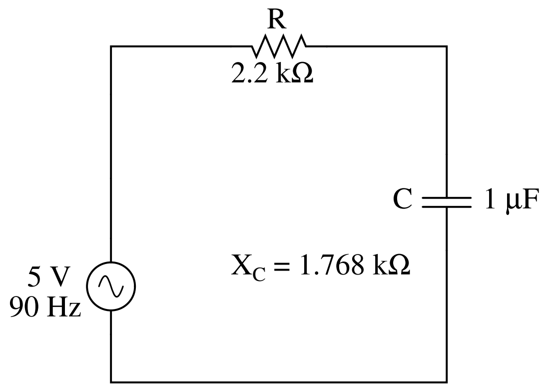


Analyzing circuit for 60 Hz source alone :



	R	C	Total	
E	2.0377 + j2.4569 3.1919 ∠ 50.328°	2.9623 - j2.4569 3.8486 ∠ -39.6716°	5 + j0 5 ∠ 0°	Volts
I	926.22µ + j1.1168m 1.4509m ∠ 50.328°	926.22µ + j1.1168m 1.4509m ∠ 50.328°	926.22µ + j1.1168m 1.4509m ∠ 50.328°	Amps
Z	2.2k + j0 2.2k ∠ 0°	0 - j2.653k 2.653k ∠ -90°	2.2k - j2.653k 3.446k ∠ -50.328°	Ohms

Analyzing the circuit for 90 Hz source alone :



	R	C	Total	
E	3.0375 + j2.4415 3.8971 ∠ 38.793°	1.9625 - j2.4415 3.1325 ∠ -51.207°	5 + j0 5 ∠ 0°	Volts
I	1.3807m + j1.1098m 1.7714m ∠ 38.793°	1.3807m + j1.1098m 1.7714m ∠ 38.793°	1.3807m + j1.1098m 1.7714m ∠ 38.793°	Amps
Z	2.2k + j0 2.2k ∠ 0°	0 - j1.768k 1.768k ∠ -90°	2.2k - j1.768k 2.823k ∠ -38.793°	Ohms

Superimposing the voltage drops across R and C, we get :

$$E_R = [3.1919\text{ V} \angle 50.328^\circ\ (60\text{ Hz})] + [3.8971\text{ V} \angle 38.793^\circ\ (90\text{ Hz})]$$

$$E_C = [3.8486\text{ V} \angle -39.6716^\circ\ (60\text{ Hz})] + [3.1325\text{ V} \angle -51.207^\circ\ (90\text{ Hz})]$$

Because the two voltages across each component are at different frequencies, we cannot consolidate them into a single voltage figure as we could if we were adding together two voltages of different amplitude and/or phase angle at the same frequency. Complex number notation give us the ability to represent waveform amplitude (polar magnitude) and phase angle (polar angle), but not frequency.

What we can tell from this application of the superposition theorem is that there will be a greater 60 Hz voltage dropped across the capacitor than a 90 Hz voltage. Just the opposite is true for the resistor's voltage drop. This is worthy to note, especially in light of the fact that the two source voltages are equal. It is this kind of unequal circuit response to signals of differing frequency that will be our specific focus in the next chapter.

We can also apply the superposition theorem to the analysis of a circuit powered by a non-sinusoidal voltage, such as a square wave. If we know the Fourier series (multiple sine/cosine wave equivalent) of that wave, we can regard it as originating from a series-connected string of multiple sinusoidal voltage sources at the appropriate amplitudes, frequencies, and phase shifts. Needless to say, this can be a laborious task for some waveforms (an accurate square-wave Fourier Series is considered to be expressed out to the ninth harmonic, or five sine waves in all!), but it is possible. I mention this not to scare you, but to inform you of the potential complexity lurking behind seemingly simple waveforms. A real-life circuit will respond just the same to being powered by a square wave as being powered by an *infinite* series of sine waves of odd-multiple frequencies and diminishing amplitudes. This has been known to translate into unexpected circuit resonances, transformer and inductor core overheating due to eddy currents, electromagnetic noise over broad ranges of the frequency spectrum, and the like. Technicians and engineers need to be made aware of the potential effects of non-sinusoidal waveforms in reactive circuits.

Harmonics are known to manifest their effects in the form of electromagnetic radiation as well. Studies have been performed on the potential hazards of using portable computers aboard passenger aircraft, citing the fact that computers' high frequency square-wave "clock" voltage signals are capable of generating radio waves that could interfere with the operation of the aircraft's electronic navigation equipment. It's bad enough that typical microprocessor clock signal frequencies are within the range of aircraft radio frequency bands, but worse yet is the fact that the harmonic multiples of those fundamental frequencies span an even larger range, due to the fact that clock signal voltages are square-wave in shape and not sine-wave.

Electromagnetic "emissions" of this nature can be a problem in industrial applications, too, with harmonics abounding in very large quantities due to (nonlinear) electronic control of motor and electric furnace power. The fundamental power line frequency may only be 60 Hz, but those harmonic frequency multiples theoretically extend into infinitely high frequency ranges. Low frequency power line voltage and current doesn't radiate into space very well as electromagnetic energy, but high frequencies do.

Also, capacitive and inductive "coupling" caused by close-proximity conductors is usually more severe at high frequencies. Signal wiring nearby power wiring will tend to "pick up" harmonic interference from the power wiring to a far greater extent than pure sine-wave interference. This problem can manifest itself in industry when old motor controls are replaced with new, solid-state electronic motor controls providing greater energy efficiency. Suddenly there may be weird electrical noise being impressed upon signal wiring that never used to be there, because the old controls never generated harmonics, and those high-frequency harmonic voltages and currents tend to inductively and capacitively "couple" better to nearby conductors than any 60 Hz signals from the old controls used to.

- **REVIEW :**

- Any regular (repeating), non-sinusoidal waveform is equivalent to a particular series of sine/cosine waves of different frequencies, phases, and amplitudes, plus a DC offset voltage if necessary. The mathematical process for determining the sinusoidal waveform equivalent for any waveform is called *Fourier analysis*.

- Multiple-frequency voltage sources can be simulated for analysis by connecting several single-frequency voltage sources in series. Analysis of voltages and currents is accomplished by using the superposition theorem. NOTE : superimposed voltages and currents of different frequencies *cannot* be added together in complex number form, since complex numbers only account for amplitude and phase shift, not frequency!

- Harmonics can cause problems by impressing unwanted (”noise”) voltage signals upon nearby circuits. These unwanted signals may come by way of capacitive coupling, inductive coupling, electromagnetic radiation, or a combination thereof.

7.6 Contributors

Les contributeurs de ce chapitre sont listés dans l'ordre chronologique de leurs contributions, depuis le plus récent jusqu'au premier. Voyez l'Annexe 2 (Liste des contributeur) pour les dates et les informations de contact.

Jason Starck (June 2000) : HTML document formatting, which led to a much better-looking second edition.

Chapter 8

FILTERS

8.1 What is a filter?

It is sometimes desirable to have circuits capable of selectively filtering one frequency or range of frequencies out of a mix of different frequencies in a circuit. A circuit designed to perform this frequency selection is called a *filter circuit*, or simply a *filter*. A common need for filter circuits is in high-performance stereo systems, where certain ranges of audio frequencies need to be amplified or suppressed for best sound quality and power efficiency. You may be familiar with *equalizers*, which allow the amplitudes of several frequency ranges to be adjusted to suit the listener's taste and acoustic properties of the listening area. You may also be familiar with *crossover networks*, which block certain ranges of frequencies from reaching speakers. A tweeter (high-frequency speaker) is inefficient at reproducing low-frequency signals such as drum beats, so a crossover circuit is connected between the tweeter and the stereo's output terminals to block low-frequency signals, only passing high-frequency signals to the speaker's connection terminals. This gives better audio system efficiency and thus better performance. Both equalizers and crossover networks are examples of filters, designed to accomplish filtering of certain frequencies.

Another practical application of filter circuits is in the "conditioning" of non-sinusoidal voltage waveforms in power circuits. Some electronic devices are sensitive to the presence of harmonics in the power supply voltage, and so require power conditioning for proper operation. If a distorted sine-wave voltage behaves like a series of harmonic waveforms added to the fundamental frequency, then it should be possible to construct a filter circuit that only allows the fundamental waveform frequency to pass through, blocking all (higher-frequency) harmonics.

We will be studying the design of several elementary filter circuits in this lesson. To reduce the load of math on the reader, I will make extensive use of SPICE as an analysis tool, displaying Bode plots (amplitude versus frequency) for the various kinds of filters. Bear in mind, though, that these circuits can be analyzed over several points of frequency by repeated series-parallel analysis, much like the previous example with two sources (60 and 90 Hz), if the student is willing to invest a lot of time working and re-working circuit calculations for each frequency.

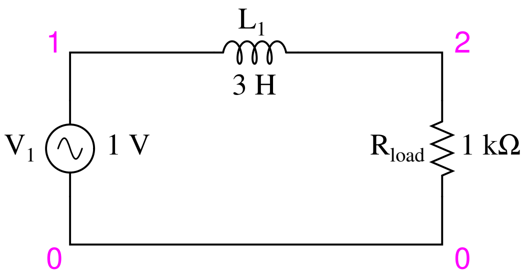
- **REVIEW :**
- A *filter* is an AC circuit that separates some frequencies from others in within mixed-frequency signals.

- Audio *equalizers* and *crossover networks* are two well-known applications of filter circuits.
- A *Bode plot* is a graph plotting waveform amplitude or phase on one axis and frequency on the other.

8.2 Low-pass filters

By definition, a low-pass filter is a circuit offering easy passage to low-frequency signals and difficult passage to high-frequency signals. There are two basic kinds of circuits capable of accomplishing this objective, and many variations of each one :

Inductive low-pass filter

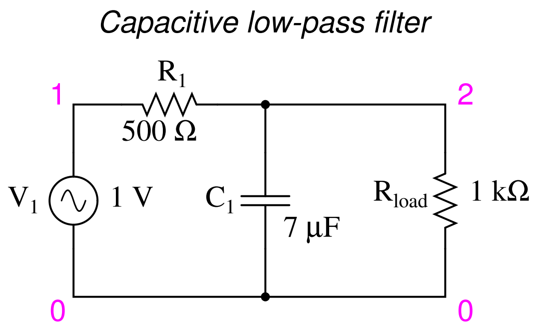


The inductor's impedance increases with increasing frequency. This high impedance in series tends to block high-frequency signals from getting to the load. This can be demonstrated with a SPICE analysis :

```
inductive lowpass filter
v1 1 0 ac 1 sin
l1 1 2 3
rload 2 0 1k
.ac lin 20 1 200
.plot ac v(2)
.end
```

```
freq       v(2)    0.2512      0.3981        0.631          1
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
1.000E+00  9.998E-01 .            .            .            *
1.147E+01  9.774E-01 .            .            .           *.
2.195E+01  9.240E-01 .            .            .          * .
3.242E+01  8.533E-01 .            .            .       *    .
4.289E+01  7.776E-01 .            .            .     *      .
5.337E+01  7.050E-01 .            .            .  *         .
6.384E+01  6.391E-01 .            .            *            .
7.432E+01  5.810E-01 .            .         *  .            .
8.479E+01  5.304E-01 .            .        *   .            .
9.526E+01  4.865E-01 .            .     *      .            .
```

```
1.057E+02  4.485E-01 .            .   *        .            .
1.162E+02  4.153E-01 .            .*           .            .
1.267E+02  3.863E-01 .           *.            .            .
1.372E+02  3.607E-01 .         *  .            .            .
1.476E+02  3.382E-01 .        *   .            .            .
1.581E+02  3.181E-01 .      *     .            .            .
1.686E+02  3.002E-01 .    *       .            .            .
1.791E+02  2.841E-01 .   *        .            .            .
1.895E+02  2.696E-01 . *          .            .            .
2.000E+02  2.564E-01 .*           .            .            .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
Load voltage decreases with increasing frequency
```



The capacitor's impedance decreases with increasing frequency. This low impedance in parallel with the load resistance tends to short out high-frequency signals, dropping most of the voltage gets across series resistor R_1.

```
capacitive lowpass filter
v1 1 0 ac 1 sin
r1 1 2 500
c1 2 0 7u
rload 2 0 1k
.ac lin 20 30 150
.plot ac v(2)
.end
```

```
freq       v(2)            0.3162     0.3981     0.5012  0.631
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
3.000E+01  6.102E-01 .        .         .         .        *.
3.632E+01  5.885E-01 .        .         .         .      *  .
4.263E+01  5.653E-01 .        .         .         .   *     .
4.895E+01  5.416E-01 .        .         .         .  *      .
5.526E+01  5.180E-01 .        .         .         .*        .
6.158E+01  4.948E-01 .        .         .        *.         .
6.789E+01  4.725E-01 .        .         .     *   .         .
```

```
7.421E+01  4.511E-01 .         .         .   *     .         .
8.053E+01  4.309E-01 .         .         . *       .         .
8.684E+01  4.118E-01 .         .         .*        .         .
9.316E+01  3.938E-01 .         .        *.         .         .
9.947E+01  3.770E-01 .         .      *  .         .         .
1.058E+02  3.613E-01 .         .    *    .         .         .
1.121E+02  3.465E-01 .         .  *      .         .         .
1.184E+02  3.327E-01 .         .*        .         .         .
1.247E+02  3.199E-01 .         *         .         .         .
1.311E+02  3.078E-01 .       * .         .         .         .
1.374E+02  2.965E-01 .     *   .         .         .         .
1.437E+02  2.859E-01 .   *     .         .         .         .
1.500E+02  2.760E-01 .*        .         .         .         .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
Load voltage decreases with increasing frequency
```

The inductive low-pass filter is the pinnacle of simplicity, with only one component comprising the filter. The capacitive version of this filter is not that much more complex, with only a resistor and capacitor needed for operation. However, despite their increased complexity, capacitive filter designs are generally preferred over inductive because capacitors tend to be "purer" reactive components than inductors and therefore are more predictable in their behavior. By "pure" I mean that capacitors exhibit little resistive effects than inductors, making them almost 100% reactive. Inductors, on the other hand, typically exhibit significant dissipative (resistor-like) effects, both in the long lengths of wire used to make them, and in the magnetic losses of the core material. Capacitors also tend to participate less in "coupling" effects with other components (generate and/or receive interference from other components via mutual electric or magnetic fields) than inductors, and are less expensive.

However, the inductive low-pass filter is often preferred in AC-DC power supplies to filter out the AC "ripple" waveform created when AC is converted (rectified) into DC, passing only the pure DC component. The primary reason for this is the requirement of low filter resistance for the output of such a power supply. A capacitive low-pass filter requires an extra resistance in series with the source, whereas the inductive low-pass filter does not. In the design of a high-current circuit like a DC power supply where additional series resistance is undesirable, the inductive low-pass filter is the better design choice. On the other hand, if low weight and compact size are higher priorities than low internal supply resistance in a power supply design, the capacitive low-pass filter might make more sense.

All low-pass filters are rated at a certain *cutoff frequency*. That is, the frequency above which the output voltage falls below 70.7% of the input voltage. This cutoff percentage of 70.7 is not really arbitrary, all though it may seem so at first glance. In a simple capacitive/resistive low-pass filter, it is the frequency at which capacitive reactance in ohms equals resistance in ohms. In a simple capacitive low-pass filter (one resistor, one capacitor), the cutoff frequency is given as :

$$f_{cutoff} = \frac{1}{2\pi RC}$$

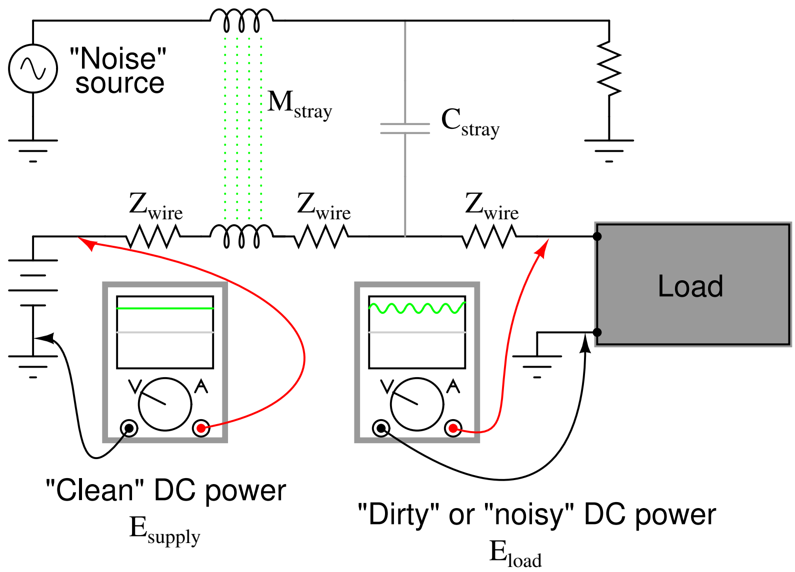
Inserting the values of R and C from the last SPICE simulation into this formula, we arrive at a cutoff frequency of 45.473 Hz. However, when we look at the plot generated by the SPICE simulation, we see the load voltage well below 70.7% of the source voltage (1 volt) even at a frequency as low as 30 Hz, below the calculated cutoff point. What's wrong? The problem here is that the load resistance of 1 kΩ affects the frequency response of the filter, skewing it down from what the formula told us it would be. Without that load resistance in place, SPICE produces a Bode plot whose numbers make more sense :

```
capacitive lowpass filter
v1 1 0 ac 1 sin
r1 1 2 500
c1 2 0 7u
* note : no load resistor!
.ac lin 20 40 50
.plot ac v(2)
.end
```

```
freq       v(2)    0.6607      0.6918       0.7244       0.7586
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
4.000E+01  7.508E-01 .           .             .          *  .
4.053E+01  7.465E-01 .           .             .        *    .
4.105E+01  7.423E-01 .           .             .      *      .
4.158E+01  7.380E-01 .           .             .     *       .
4.211E+01  7.338E-01 .           .             .   *         .
4.263E+01  7.295E-01 .           .             . *           .
4.316E+01  7.253E-01 .           .             *             .
4.368E+01  7.211E-01 .           .            *.             .
4.421E+01  7.170E-01 .           .          *  .             .
4.474E+01  7.129E-01 .           .        *    .             .
4.526E+01  7.087E-01 .           .      *      .             .
4.579E+01  7.046E-01 .           .     *       .             .
4.632E+01  7.006E-01 .           .   *         .             .
4.684E+01  6.965E-01 .           . *           .             .
4.737E+01  6.925E-01 .           *             .             .
4.789E+01  6.885E-01 .          *.             .             .
4.842E+01  6.846E-01 .        *  .             .             .
4.895E+01  6.806E-01 .      *    .             .             .
4.947E+01  6.767E-01 .    *      .             .             .
5.000E+01  6.728E-01 .   *       .             .             .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
At 45.26 Hz, the output voltage is above 70.7 percent;
At 45.79 Hz, the output voltage is below 70.7 percent;
It should be exactly 70.7% at 45.473 Hz!
```

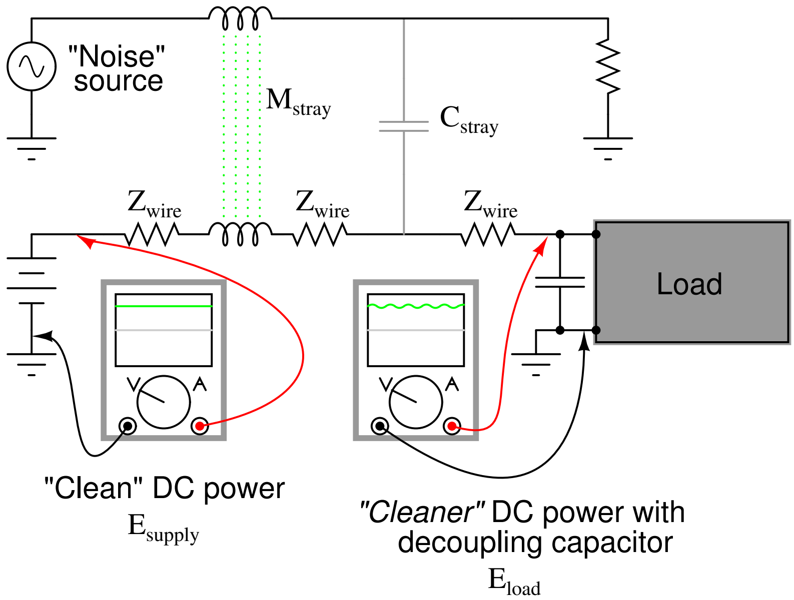
When dealing with filter circuits, it is always important to note that the response of the filter depends on the filter's component values *and* the impedance of the load. If a cutoff frequency equation fails to give consideration to load impedance, it assumes no load and will fail to give accurate results for a real-life filter conducting power to a load.

One frequent application of the capacitive low-pass filter principle is in the design of circuits having components or sections sensitive to electrical "noise." As mentioned at the beginning of the last chapter, sometimes AC signals can "couple" from one circuit to another via capacitance (C_{stray}) and/or mutual inductance (M_{stray}) between the two sets of conductors. A prime example of this is unwanted AC signals ("noise") becoming impressed on DC power lines supplying sensitive circuits :



The oscilloscope-meter on the left shows the "clean" power from the DC voltage source. After coupling with the AC noise source via stray mutual inductance and stray capacitance, though, the voltage as measured at the load terminals is now a mix of AC and DC, the AC being unwanted. Normally, one would expect E_{load} to be precisely identical to E_{source}, because the uninterrupted conductors connecting them should make the two sets of points electrically common. However, power conductor impedance allows the two voltages to differ, which means the noise magnitude can vary at different points in the DC system.

If we wish to prevent such "noise" from reaching the DC load, all we need to do is connect a low-pass filter near the load to block any coupled signals. In its simplest form, this is nothing more than a capacitor connected directly across the power terminals of the load, the capacitor behaving as a very low impedance to any AC noise, and shorting it out. Such a capacitor is called a *decoupling capacitor* :

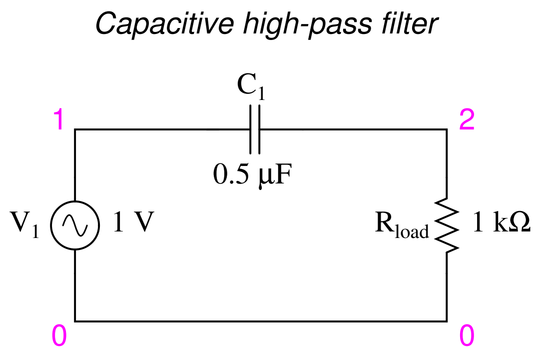


A cursory glance at a crowded printed-circuit board (PCB) will typically reveal decoupling capacitors scattered throughout, usually located as close as possible to the sensitive DC loads. Capacitor size is usually 0.1 μF or more, a minimum amount of capacitance needed to produce a low enough impedance to short out any noise. Greater capacitance will do a better job at filtering noise, but size and economics limit decoupling capacitors to meager values.

- **REVIEW :**
- A low-pass filter allows for easy passage of low-frequency signals from source to load, and difficult passage of high-frequency signals.
- Inductive low-pass filters insert an inductor in series with the load; capacitive low-pass filters insert a resistor in series and a capacitor in parallel with the load. The former filter design tries to "block" the unwanted frequency signal while the latter tries to short it out.
- The *cutoff frequency* for a low-pass filter is that frequency at which the output (load) voltage equals 70.7% of the input (source) voltage. Above the cutoff frequency, the output voltage is lower than 70.7% of the input, and visa-versa.

8.3 High-pass filters

A high-pass filter's task is just the opposite of a low-pass filter : to offer easy passage of a high-frequency signal and difficult passage to a low-frequency signal. As one might expect, the inductive and capacitive versions of the high-pass filter are just the opposite of their respective low-pass filter designs :

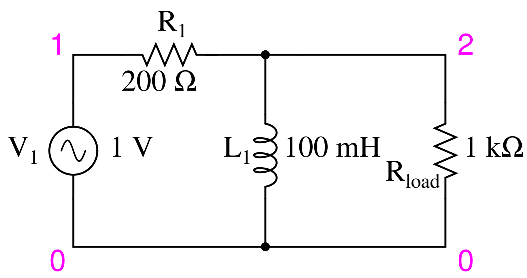


The capacitor's impedance increases with decreasing frequency. This high impedance in series tends to block low-frequency signals from getting to load.

```
capacitive highpass filter
v1 1 0 ac 1 sin
c1 1 2 0.5u
rload 2 0 1k
.ac lin 20 1 200
.plot ac v(2)
.end
```

```
freq        v(2)  1.000E-03     1.000E-02     1.000E-01  1.000E+00
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
1.000E+00  3.142E-03 .      *      .             .             .
1.147E+01  3.602E-02 .             .      *      .             .
2.195E+01  6.879E-02 .             .          * .             .
3.242E+01  1.013E-01 .             .            *             .
4.289E+01  1.336E-01 .             .             . *           .
5.337E+01  1.654E-01 .             .             .  *          .
6.384E+01  1.966E-01 .             .             .   *         .
7.432E+01  2.274E-01 .             .             .    *        .
8.479E+01  2.574E-01 .             .             .     *       .
9.526E+01  2.867E-01 .             .             .     *       .
1.057E+02  3.152E-01 .             .             .      *      .
1.162E+02  3.429E-01 .             .             .      *      .
1.267E+02  3.698E-01 .             .             .       *     .
1.372E+02  3.957E-01 .             .             .       *     .
1.476E+02  4.207E-01 .             .             .        *    .
1.581E+02  4.448E-01 .             .             .        *    .
1.686E+02  4.680E-01 .             .             .        *    .
1.791E+02  4.903E-01 .             .             .         *   .
1.895E+02  5.116E-01 .             .             .         *   .
2.000E+02  5.320E-01 .             .             .         *   .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
Load voltage increases with increasing frequency
```

Inductive high-pass filter



The inductor's impedance decreases with decreasing frequency. This low impedance in parallel tends to short out low-frequency signals from getting to the load resistor. As a consequence, most of the voltage gets dropped across series resistor R_1.

```
inductive highpass filter
v1 1 0 ac 1 sin
r1 1 2 200
l1 2 0 100m
rload 2 0 1k
.ac lin 20 1 200
.plot ac v(2)
.end
```

```
freq       v(2)   1.000E-03  1.000E-02   1.000E-01    1.000E+00
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - 
1.000E+00  3.142E-03 .  *      .             .             .
1.147E+01  3.601E-02 .         .       *     .             .
2.195E+01  6.871E-02 .         .           * .             .
3.242E+01  1.011E-01 .         .             *             .
4.289E+01  1.330E-01 .         .             . *           .
5.337E+01  1.644E-01 .         .             .  *          .
6.384E+01  1.950E-01 .         .             .   *         .
7.432E+01  2.248E-01 .         .             .    *        .
8.479E+01  2.537E-01 .         .             .     *       .
9.526E+01  2.817E-01 .         .             .     *       .
1.057E+02  3.086E-01 .         .             .      *      .
1.162E+02  3.344E-01 .         .             .      *      .
1.267E+02  3.591E-01 .         .             .       *     .
1.372E+02  3.828E-01 .         .             .       *     .
1.476E+02  4.053E-01 .         .             .        *    .
1.581E+02  4.267E-01 .         .             .        *    .
1.686E+02  4.470E-01 .         .             .        *    .
1.791E+02  4.662E-01 .         .             .        *    .
1.895E+02  4.845E-01 .         .             .         *   .
2.000E+02  5.017E-01 .         .             .         *   .
```

- -

`Load voltage increases with increasing frequency`

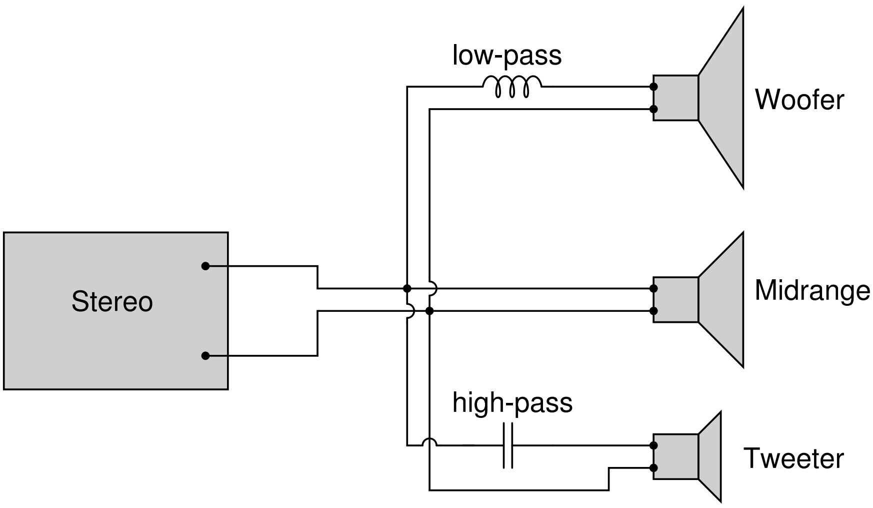
This time, the capacitive design is the simplest, requiring only one component above and beyond the load. And, again, the reactive purity of capacitors over inductors tends to favor their use in filter design, especially with high-pass filters where high frequencies commonly cause inductors to behave strangely due to the skin effect and electromagnetic core losses.

As with low-pass filters, high-pass filters have a rated *cutoff frequency*, above which the output voltage increases above 70.7% of the input voltage. Just as in the case of the capacitive low-pass filter circuit, the capacitive high-pass filter's cutoff frequency can be found with the same formula :

$$f_{cutoff} = \frac{1}{2\pi RC}$$

In the example circuit, there is no resistance other than the load resistor, so that is the value for R in the formula.

Using a stereo system as a practical example, a capacitor connected in series with the tweeter (treble) speaker will serve as a high-pass filter, imposing a high impedance to low-frequency bass signals, thereby preventing that power from being wasted on a speaker inefficient for reproducing such sounds. In like fashion, an inductor connected in series with the woofer (bass) speaker will serve as a low-pass filter for the low frequencies that particular speaker is designed to reproduce. In this simple example circuit, the midrange speaker is subjected to the full spectrum of frequencies from the stereo's output. More elaborate filter networks are sometimes used, but this should give you the general idea. Also bear in mind that I'm only showing you one channel (either left or right) on this stereo system. A real stereo would have six speakers : 2 woofers, 2 midranges, and 2 tweeters.



For better performance yet, we might like to have some kind of filter circuit capable of passing frequencies that are between low (bass) and high (treble) to the midrange speaker so that none of the low- or high-frequency signal power is wasted on a speaker incapable of efficiently reproducing those sounds. What we would be looking for is called a *band-pass* filter, which is the topic of the next section.

- **REVIEW :**

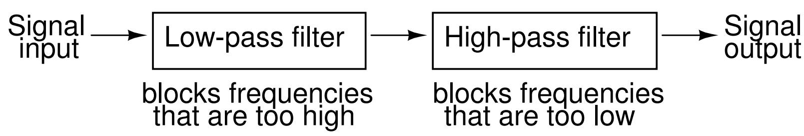
- A high-pass filter allows for easy passage of high-frequency signals from source to load, and difficult passage of low-frequency signals.

- Capacitive high-pass filters insert a capacitor in series with the load; inductive high-pass filters insert a resistor in series and an inductor in parallel with the load. The former filter design tries to "block" the unwanted frequency signal while the latter tries to short it out.

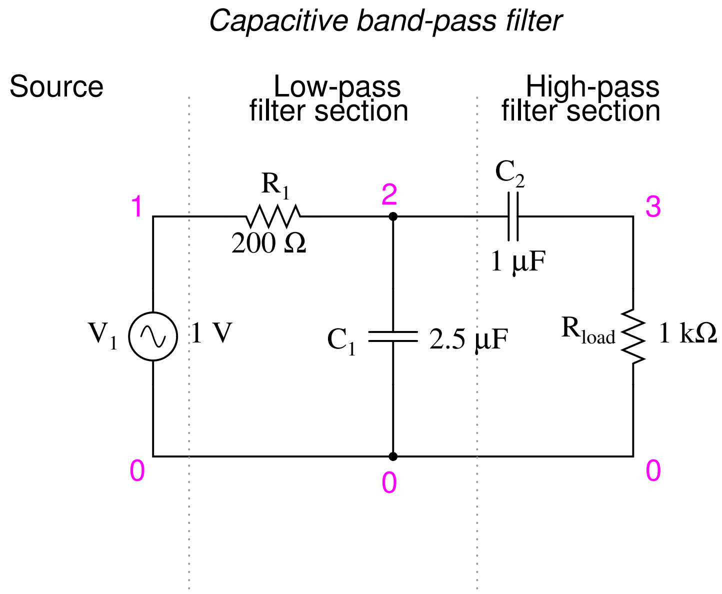
- The *cutoff frequency* for a high-pass filter is that frequency at which the output (load) voltage equals 70.7% of the input (source) voltage. Above the cutoff frequency, the output voltage is greater than 70.7% of the input, and visa-versa.

8.4 Band-pass filters

There are applications where a particular band, or spread, or frequencies need to be filtered from a wider range of mixed signals. Filter circuits can be designed to accomplish this task by combining the properties of low-pass and high-pass into a single filter. The result is called a *band-pass* filter. Creating a bandpass filter from a low-pass and high-pass filter can be illustrated using block diagrams :



What emerges from the series combination of these two filter circuits is a circuit that will only allow passage of those frequencies that are neither too high nor too low. Using real components, here is what a typical schematic might look like :

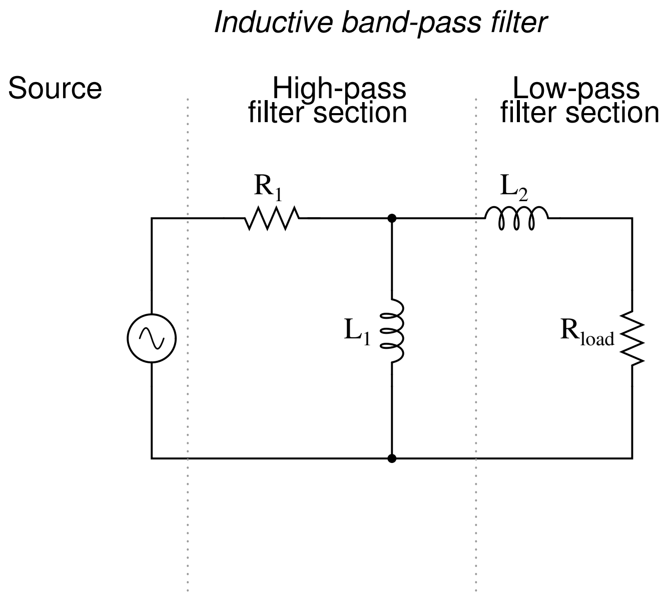



```
capacitive bandpass filter
v1 1 0 ac 1 sin
r1 1 2 200
c1 2 0 2.5u
c2 2 3 1u
rload 3 0 1k
.ac lin 20 100 500
.plot ac v(3)
.end
```

```
freq          v(3)    4.467E-01      5.012E-01      5.623E-01      6.310E-01
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
1.000E+02   4.703E-01 .     *        .             .             .
1.211E+02   5.155E-01 .              .  *          .             .
1.421E+02   5.469E-01 .              .          *  .             .
1.632E+02   5.676E-01 .              .             .*            .
1.842E+02   5.801E-01 .              .             .   *         .
2.053E+02   5.865E-01 .              .             .    *        .
2.263E+02   5.882E-01 .              .             .    *        .
2.474E+02   5.864E-01 .              .             .    *        .
2.684E+02   5.820E-01 .              .             .   *         .
2.895E+02   5.755E-01 .              .             .  *          .
3.105E+02   5.676E-01 .              .             .*            .
3.316E+02   5.585E-01 .              .            *.             .
3.526E+02   5.487E-01 .              .          *  .             .
3.737E+02   5.384E-01 .              .        *    .             .
```

```
3.947E+02  5.277E-01 .             .     *       .            .
4.158E+02  5.169E-01 .             .   *         .            .
4.368E+02  5.060E-01 .             .*            .            .
4.579E+02  4.951E-01 .            *.             .            .
4.789E+02  4.843E-01 .         *   .             .            .
5.000E+02  4.736E-01 .      *      .             .            .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
Load voltage peaks within narrow frequency range
```

Band-pass filters can also be constructed using inductors, but as mentioned before, the reactive "purity" of capacitors gives them a design advantage. If we were to design a bandpass filter using inductors, it might look something like this :



The fact that the high-pass section comes "first" in this design instead of the low-pass section makes no difference in its overall operation. It will still filter out all frequencies too high or too low.

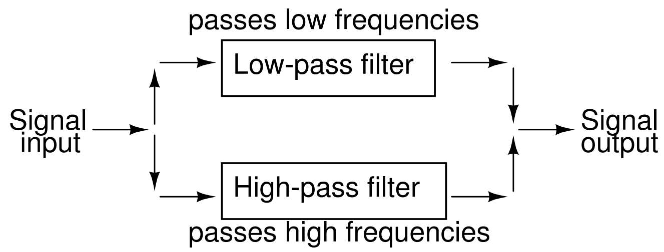
While the general idea of combining low-pass and high-pass filters together to make a bandpass filter is sound, it is not without certain limitations. Because this type of band-pass filter works by relying on either section to *block* unwanted frequencies, it can be difficult to design such a filter to allow unhindered passage within the desired frequency range. Both the low-pass and high-pass sections will always be blocking signals to some extent, and their combined effort makes for an attenuated (reduced amplitude) signal at best, even at the peak of the "pass-band" frequency range. Notice the curve peak on the previous SPICE analysis : the load voltage of this filter never rises above 0.59 volts, although the source voltage is a full volt. This signal attenuation becomes more pronounced if the filter is designed to be more selective (steeper curve, narrower band of passable frequencies).

There are other methods to achieve band-pass operation without sacrificing signal strength within the pass-band. We will discuss those methods a little later in this chapter.

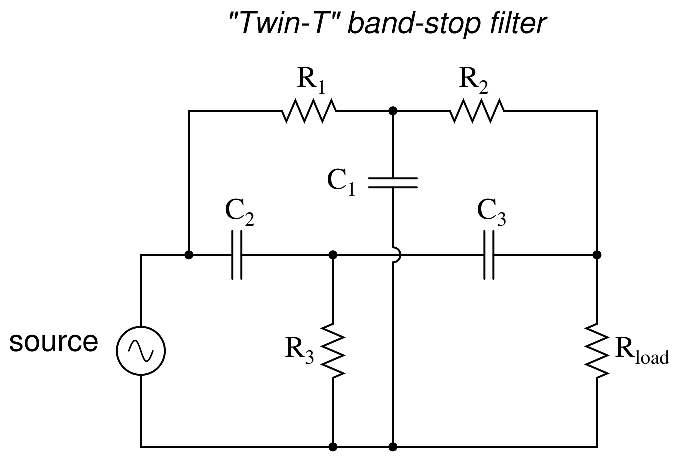
- **REVIEW :**
- A *band-pass* filter works to screen out frequencies that are too low or too high, giving easy passage only to frequencies within a certain range.
- Band-pass filters can be made by stacking a low-pass filter on the end of a high-pass filter, or visa-versa.
- "Attenuate" means to reduce or diminish in amplitude. When you turn down the volume control on your stereo, you are "attenuating" the signal being sent to the speakers.

8.5 Band-stop filters

Also called *band-elimination*, *band-reject*, or *notch* filters, this kind of filter passes all frequencies above and below a particular range set by the component values. Not surprisingly, it can be made out of a low-pass and a high-pass filter, just like the band-pass design, except that this time we connect the two filter sections in parallel with each other instead of in series.



Constructed using two capacitive filter sections, it looks something like this :



The low-pass filter section is comprised of R_1, R_2, and C_1 in a "T" configuration. The high-pass filter section is comprised of C_2, C_3, and R_3 in a "T" configuration as well. Together, this arrangement is commonly known as a "Twin-T" filter, giving sharp response when the component values are chosen in the following ratios :

Component value ratios for the "Twin-T" band-stop filter

$$R_1 = R_2 = 2(R_3)$$

$$C_2 = C_3 = (0.5)C_1$$

Given these component ratios, the frequency of maximum rejection (the "notch frequency") can be calculated as follows :

$$f_{notch} = \frac{1}{4\pi R_3 C_3}$$

The impressive band-stopping ability of this filter is illustrated by the following SPICE analysis :

```
twin-t bandstop filter
v1 1 0 ac 1 sin
r1 1 2 200
c1 2 0 2u
r2 2 3 200
c2 1 4 1u
r3 4 0 100
c3 4 3 1u
rload 3 0 1k
.ac lin 20 200 1.5k
.plot ac v(3)
.end
```

```
freq       v(3)  1.000E-02  3.162E-02  1.000E-01    3.162E-01
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
2.000E+02  5.400E-01 .         .           .             .      *.
2.684E+02  4.512E-01 .         .           .             .   *   .
3.368E+02  3.686E-01 .         .           .             . *     .
4.053E+02  2.946E-01 .         .           .            *.       .
4.737E+02  2.290E-01 .         .           .         *   .       .
5.421E+02  1.707E-01 .         .           .      *      .       .
6.105E+02  1.185E-01 .         .           . *           .       .
6.789E+02  7.134E-02 .         .       *   .             .       .
7.474E+02  2.832E-02 .        *.           .             .       .
8.158E+02  1.126E-02 .*        .           .             .       .
8.842E+02  4.796E-02 .         .    *      .             .       .
9.526E+02  8.222E-02 .         .         * .             .       .
1.021E+03  1.144E-01 .         .           . *           .       .
1.089E+03  1.447E-01 .         .           .   *         .       .
1.158E+03  1.734E-01 .         .           .      *      .       .
1.226E+03  2.007E-01 .         .           .       *     .       .
1.295E+03  2.267E-01 .         .           .         *   .       .
```

```
1.363E+03  2.515E-01 .          .           .          *  .       .
1.432E+03  2.752E-01 .          .           .           * .       .
1.500E+03  2.980E-01 .          .           .            *.       .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

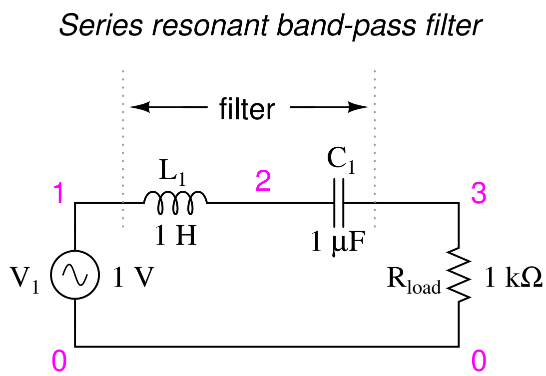
- **REVIEW :**
- A *band-stop* filter works to screen out frequencies that are within a certain range, giving easy passage only to frequencies outside of that range. Also known as *band-elimination*, *band-reject*, or *notch* filters.
- Band-stop filters can be made by placing a low-pass filter in parallel with a high-pass filter. Commonly, both the low-pass and high-pass filter sections are of the "T" configuration, giving the name "Twin-T" to the band-stop combination.
- The frequency of maximum attenuation is called the *notch* frequency.

8.6 Resonant filters

So far, the filter designs we've concentrated on have employed *either* capacitors *or* inductors, but never both at the same time. We should know by now that combinations of L and C will tend to resonate, and this property can be exploited in designing band-pass and band-stop filter circuits.

Series LC circuits give minimum impedance at resonance, while parallel LC ("tank") circuits give maximum impedance at their resonant frequency. Knowing this, we have two basic strategies for designing either band-pass or band-stop filters.

For band-pass filters, the two basic resonant strategies are this : series LC to pass a signal, or parallel LC to short a signal. The two schemes will be contrasted and simulated here :



Series LC components pass signal at resonance, and block signals of any other frequencies from getting to the load.

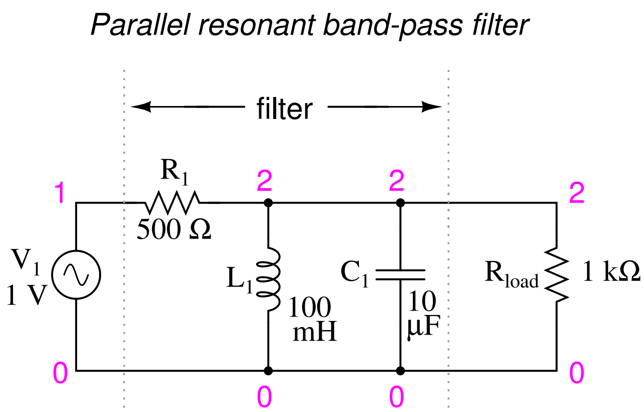
```
series resonant bandpass filter
v1 1 0 ac 1 sin
l1 1 2 1
```

```
c1 2 3 1u
rload 3 0 1k
.ac lin 20 50 250
.plot ac v(3)
.end
```

```
freq       v(3)   2.512E-01   3.981E-01    6.310E-01    1.000E+00
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
5.000E+01  3.291E-01 .     *     .             .             .
6.053E+01  4.063E-01 .           .*            .             .
7.105E+01  4.870E-01 .           .     *       .             .
8.158E+01  5.708E-01 .           .          *  .             .
9.211E+01  6.564E-01 .           .             .*            .
1.026E+02  7.411E-01 .           .             .    *        .
1.132E+02  8.210E-01 .           .             .       *     .
1.237E+02  8.910E-01 .           .             .         *   .
1.342E+02  9.460E-01 .           .             .           * .
1.447E+02  9.824E-01 .           .             .            *.
1.553E+02  9.988E-01 .           .             .             *
1.658E+02  9.967E-01 .           .             .             *
1.763E+02  9.796E-01 .           .             .            *.
1.868E+02  9.518E-01 .           .             .           * .
1.974E+02  9.174E-01 .           .             .          *  .
2.079E+02  8.797E-01 .           .             .         *   .
2.184E+02  8.408E-01 .           .             .        *    .
2.289E+02  8.026E-01 .           .             .      *      .
2.395E+02  7.657E-01 .           .             .     *       .
2.500E+02  7.307E-01 .           .             .   *         .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
Load voltage peaks at resonant frequency (159.15 Hz)
```

A couple of points to note : see how there is virtually no signal attenuation within the "pass band" (the range of frequencies near the load voltage peak), unlike the band-pass filters made from capacitors or inductors alone. Also, since this filter works on the principle of series LC resonance, the resonant frequency of which is unaffected by circuit resistance, the value of the load resistor will not skew the peak frequency. However, different values for the load resistor *will* change the "steepness" of the Bode plot (the "selectivity" of the filter).

The other basic style of resonant band-pass filters employs a tank circuit (parallel LC combination) to short out signals too high or too low in frequency from getting to the load :



The tank circuit will have a lot of impedance at resonance, allowing the signal to get to the load with minimal attenuation. Under or over resonant frequency, however, the tank circuit will have a low impedance, shorting out the signal and dropping most of it across series resistor R_1.

```
parallel resonant bandpass filter
v1 1 0 ac 1 sin
r1 1 2 500
l1 2 0 100m
c1 2 0 10u
rload 2 0 1k
.ac lin 20 50 250
.plot ac v(2)
.end
```

```
freq       v(2)   3.162E-02   1.000E-01    3.162E-01     1.000E+00
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
5.000E+01  6.933E-02 .       *   .             .             .
6.053E+01  8.814E-02 .         * .             .             .
7.105E+01  1.100E-01 .           .*            .             .
8.158E+01  1.361E-01 .           .   *         .             .
9.211E+01  1.684E-01 .           .     *       .             .
1.026E+02  2.096E-01 .           .        *    .             .
1.132E+02  2.640E-01 .           .           * .             .
1.237E+02  3.382E-01 .           .             .*            .
1.342E+02  4.392E-01 .           .             .   *         .
1.447E+02  5.630E-01 .           .             .      *      .
1.553E+02  6.578E-01 .           .             .        *    .
1.658E+02  6.432E-01 .           .             .        *    .
1.763E+02  5.503E-01 .           .             .      *      .
1.868E+02  4.543E-01 .           .             .   *         .
1.974E+02  3.792E-01 .           .             . *           .
2.079E+02  3.234E-01 .           .             *             .
2.184E+02  2.816E-01 .           .            *.             .
```

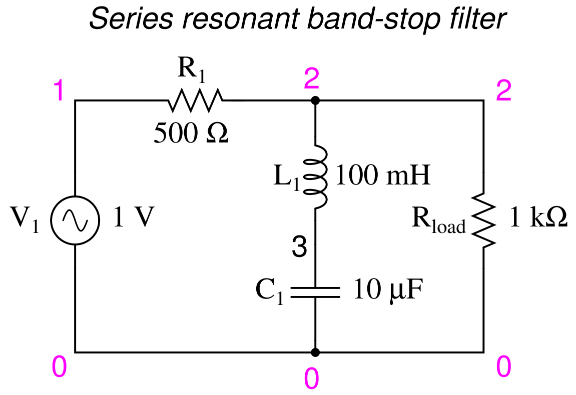
```
2.289E+02  2.495E-01 .           .          *  .            .
2.395E+02  2.242E-01 .           .         *   .            .
2.500E+02  2.038E-01 .           .        *    .            .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
Load voltage peaks at resonant frequency (159.15 Hz)
```

Just like the low-pass and high-pass filter designs relying on a series resistance and a parallel "shorting" component to attenuate unwanted frequencies, this resonant circuit can never provide full input (source) voltage to the load. That series resistance will always be dropping some amount of voltage so long as there is a load resistance connected to the output of the filter.

It should be noted that this form of band-pass filter circuit is very popular in analog radio tuning circuitry, for selecting a particular radio frequency from the multitudes of frequencies available from the antenna. In most analog radio tuner circuits, the rotating dial for station selection moves a variable capacitor in a tank circuit.

The variable capacitor and air-core inductor shown in the above photograph of a simple radio comprise the main elements in the tank circuit filter used to discriminate one radio station's signal from another.

Just as we can use series and parallel LC resonant circuits to pass only those frequencies within a certain range, we can also use them to block frequencies within a certain range, creating a band-stop filter. Again, we have two major strategies to follow in doing this, to use either series or parallel resonance. First, we'll look at the series variety :



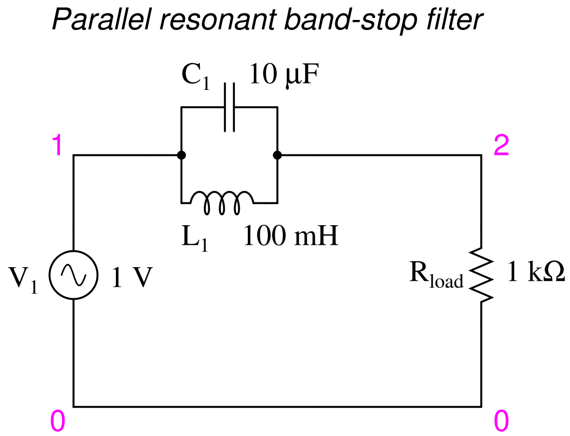
When the series LC combination reaches resonance, its very low impedance shorts out the signal, dropping it across resistor R_1 and preventing its passage on to the load.

```
series resonant bandstop filter
v1 1 0 ac 1 sin
r1 1 2 500
l1 2 3 100m
c1 3 0 10u
rload 2 0 1k
.ac lin 20 70 230
.plot ac v(2)
.end
```

```
freq        v(2)    1.000E-03  1.000E-02   1.000E-01     1.000E+00
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
7.000E+01  3.213E-01 .         .             .      *       .
7.842E+01  2.791E-01 .         .             .     *        .
8.684E+01  2.401E-01 .         .             .    *         .
9.526E+01  2.041E-01 .         .             .   *          .
1.037E+02  1.708E-01 .         .             .  *           .
1.121E+02  1.399E-01 .         .             . *            .
1.205E+02  1.111E-01 .         .             .*             .
1.289E+02  8.413E-02 .         .            *.              .
1.374E+02  5.887E-02 .         .          *  .              .
1.458E+02  3.508E-02 .         .       *     .              .
1.542E+02  1.262E-02 .         .*            .              .
1.626E+02  8.644E-03 .        *.             .              .
1.711E+02  2.884E-02 .         .     *       .              .
1.795E+02  4.805E-02 .         .         *   .              .
1.879E+02  6.638E-02 .         .           * .              .
1.963E+02  8.388E-02 .         .            *.              .
2.047E+02  1.006E-01 .         .             *              .
2.132E+02  1.167E-01 .         .             .*             .
```

```
2.216E+02  1.321E-01 .          .             . *           .
2.300E+02  1.469E-01 .          .             . *           .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
Notch frequency = LC resonant frequency (159.15 Hz)
```

Next, we will examine the parallel resonant band-stop filter :



The parallel LC components present a high impedance at resonant frequency, thereby blocking the signal from the load at that frequency. Conversely, it passes signals to the load at any other frequencies.

```
parallel resonant bandstop filter
v1 1 0 ac 1 sin
l1 1 2 100m
c1 1 2 10u
rload 2 0 1k
.ac lin 20 100 200
.plot ac v(2)
.end
```

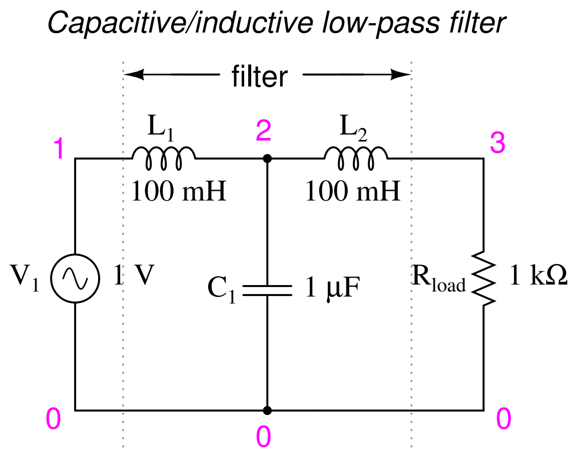
```
freq        v(2)  3.162E-02  1.000E-01    3.162E-01    1.000E+00
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
1.000E+02  9.947E-01 .          .             .             *     .
1.053E+02  9.932E-01 .          .             .             *     .
1.105E+02  9.911E-01 .          .             .             *     .
1.158E+02  9.883E-01 .          .             .             *     .
1.211E+02  9.841E-01 .          .             .             *     .
1.263E+02  9.778E-01 .          .             .             *     .
1.316E+02  9.675E-01 .          .             .             *     .
1.368E+02  9.497E-01 .          .             .            *.     .
1.421E+02  9.152E-01 .          .             .            *.     .
1.474E+02  8.388E-01 .          .             .           * .     .
```

```
1.526E+02  6.420E-01 .         .             .        *    .     .
1.579E+02  1.570E-01 .         .    *        .             .     .
1.632E+02  4.450E-01 .         .             .   *         .     .
1.684E+02  7.496E-01 .         .             .         *   .     .
1.737E+02  8.682E-01 .         .             .           * .     .
1.789E+02  9.201E-01 .         .             .            *.     .
1.842E+02  9.465E-01 .         .             .            *.     .
1.895E+02  9.616E-01 .         .             .             *     .
1.947E+02  9.710E-01 .         .             .             *     .
2.000E+02  9.773E-01 .         .             .             *     .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
Notch frequency = LC resonant frequency (159.15 Hz)
```

Once again, notice how the absence of a series resistor makes for minimum attenuation for all the desired (passed) signals. The amplitude at the notch frequency, on the other hand, is very low. In other words, this is a very "selective" filter.

In all these resonant filter designs, the selectivity depends greatly upon the "purity" of the inductance and capacitance used. If there is any stray resistance (especially likely in the inductor), this will diminish the filter's ability to finely discriminate frequencies, as well as introduce antiresonant effects that will skew the peak/notch frequency.

A word of caution to those designing low-pass and high-pass filters is in order at this point. After assessing the standard RC and LR low-pass and high-pass filter designs, it might occur to a student that a better, more effective design of low-pass or high-pass filter might be realized by combining capacitive and inductive elements together like this :



The inductors should block any high frequencies, while the capacitor should short out any high frequencies as well, both working together to allow only low frequency signals to reach the load.

At first, this seems to be a good strategy, and eliminates the need for a series resistance. However, the more insightful student will recognize that any combination of capacitors and inductors together in a circuit is likely to cause resonant effects to happen at a certain frequency. Resonance, as we have seen before, can cause strange things to happen. Let's plot a SPICE analysis and see what happens over a wide frequency range :

```
lc lowpass filter
v1 1 0 ac 1 sin
l1 1 2 100m
c1 2 0 1u
l2 2 3 100m
rload 3 0 1k
.ac lin 20 100 1k
.plot ac v(3)
.end
```

```
freq       v(3)  1.000E-01  3.162E-01    1.000E+00    3.162E+00
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
1.000E+02  1.033E+00 .         .            *             .
1.474E+02  1.074E+00 .         .            .*            .
1.947E+02  1.136E+00 .         .            . *           .
2.421E+02  1.228E+00 .         .            . *           .
2.895E+02  1.361E+00 .         .            .   *         .
3.368E+02  1.557E+00 .         .            .    *        .
3.842E+02  1.853E+00 .         .            .       *     .
4.316E+02  2.308E+00 .         .            .         *   .
4.789E+02  2.919E+00 .         .            .            *.
5.263E+02  3.185E+00 .         .            .             *
5.737E+02  2.553E+00 .         .            .           * .
6.211E+02  1.802E+00 .         .            .       *     .
6.684E+02  1.298E+00 .         .            .   *         .
7.158E+02  9.778E-01 .         .            *             .
7.632E+02  7.650E-01 .         .         *  .             .
8.105E+02  6.165E-01 .         .      *     .             .
8.579E+02  5.084E-01 .         .     *      .             .
9.053E+02  4.268E-01 .         .   *        .             .
9.526E+02  3.635E-01 .         . *          .             .
1.000E+03  3.133E-01 .         *            .             .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

What was supposed to be a low-pass filter turns out to be a band-pass filter with a peak somewhere around 526 Hz! The capacitance and inductance in this filter circuit are attaining resonance at that point, creating a large voltage drop around C_1, which is seen at the load, regardless of L_2's attenuating influence. The output voltage to the load at this point actually exceeds the input (source) voltage! A little more reflection reveals that if L_1 and C_2 are at resonance, they will impose a very heavy (very low impedance) load on the AC source, which might not be good either. We'll run the same analysis again, only this time plotting C_1's voltage and the source current along with load voltage :

```
legend :
* : v(3)
+ : v(2)
```

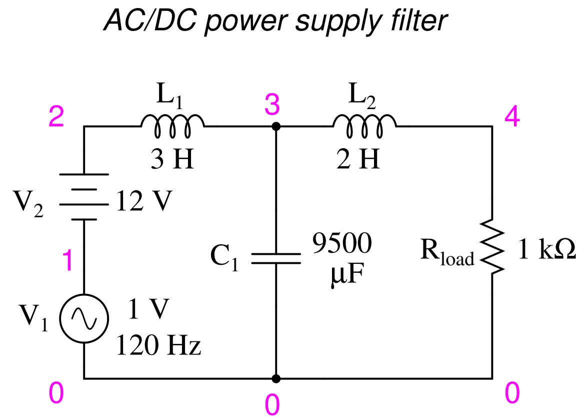
```
= : i(v1)
freq       v(3)
(*)----------  1.000E-01     3.162E-01     1.000E+00     3.162E+00
(+)----------  3.162E-01     1.000E+00     3.162E+00     1.000E+01
(=)----------  1.000E-03     3.162E-03     1.000E-02     3.162E-02
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
1.000E+02  1.033E+00 . =           +             *             .
1.474E+02  1.074E+00 .   =         .+            .*            .
1.947E+02  1.136E+00 .     =       . +           . *           .
2.421E+02  1.228E+00 .        =    .  +          . *           .
2.895E+02  1.361E+00 .           = .   +         .   *         .
3.368E+02  1.557E+00 .             .=    +       .    *        .
3.842E+02  1.853E+00 .             .   =   +     .       *     .
4.316E+02  2.308E+00 .             .       =  +  .         *   .
4.789E+02  2.919E+00 .             .           = +            *.
5.263E+02  3.185E+00 .             .             .x            *
5.737E+02  2.553E+00 .             .           +=.          *  .
6.211E+02  1.802E+00 .             .       + =   .      *      .
6.684E+02  1.298E+00 .             .   +  =      .  *          .
7.158E+02  9.778E-01 .             .+  =         *             .
7.632E+02  7.650E-01 .           + . =        *  .             .
8.105E+02  6.165E-01 .         +   =       *     .             .
8.579E+02  5.084E-01 .      +     =.     *       .             .
9.053E+02  4.268E-01 .    +      = .   *         .             .
9.526E+02  3.635E-01 .   +     =   . *           .             .
1.000E+03  3.133E-01 . +      =    *             .             .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

Sure enough, we see the voltage across C_1 and the source current spiking to a high point at the same frequency where the load voltage is maximum. If we were expecting this filter to provide a simple low-pass function, we might be disappointed by the results.

Despite this unintended resonance, low-pass filters made up of capacitors and inductors are frequently used as final stages in AC/DC power supplies to filter the unwanted AC "ripple" voltage out of the DC converted from AC. Why is this, if this particular filter design possesses a potentially troublesome resonant point?

The answer lies in the selection of filter component sizes and the frequencies encountered from an AC/DC converter (rectifier). What we're trying to do in an AC/DC power supply filter is separate DC voltage from a small amount of relatively high-frequency AC voltage. The filter inductors and capacitors are generally quite large (several Henrys for the inductors and thousands of μF for the capacitors is typical), making the filter's resonant frequency very, very low. DC of course, has a "frequency" of zero, so there's no way it can make an LC circuit resonate. The ripple voltage, on the other hand, is a non-sinusoidal AC voltage consisting of a fundamental frequency at least twice the frequency of the converted AC voltage, with harmonics many times that in addition. For plug-in-the-wall power supplies running on 60 Hz AC power (60 Hz United States; 50 Hz in Europe), the lowest frequency the filter will ever see is 120 Hz (100 Hz in Europe), which is well above its resonant point. Therefore, the potentially troublesome resonant point in a such a filter is completely avoided.

The following SPICE analysis calculates the voltage output (AC and DC) for such a filter, with series DC and AC (120 Hz) voltage sources providing a rough approximation of the mixed-frequency output of an AC/DC converter.




```
ac/dc power supply filter
v1 1 0 ac 1 sin
v2 2 1 dc
l1 2 3 3
c1 3 0 9500u
l2 3 4 2
rload 4 0 1k
.dc v2 12 12 1
.ac lin 1 120 120
.print dc v(4)
.print ac v(4)
.end
```

```
v2            v(4)
1.200E+01     1.200E+01    DC voltage at load = 12 volts

freq          v(4)
1.200E+02     3.412E-05    AC voltage at load = 34.12 microvolts
```

With a full 12 volts DC at the load and only 34.12 μV of AC left from the 1 volt AC source imposed across the load, this circuit design proves itself to be a very effective power supply filter.

The lesson learned here about resonant effects also applies to the design of high-pass filters using both capacitors and inductors. So long as the desired and undesired frequencies are well to either side of the resonant point, the filter will work okay. But if any signal of significant magnitude close to the resonant frequency is applied to the input of the filter, strange things will happen!

- **REVIEW :**

- Resonant combinations of capacitance and inductance can be employed to create very effective band-pass and band-stop filters without the need for added resistance in a circuit that would diminish the passage of desired frequencies.

- $$f_{resonant} = \frac{1}{2\pi\sqrt{LC}}$$

8.7 Summary

As lengthy as this chapter has been up to this point, it only begins to scratch the surface of filter design. A quick perusal of any advanced filter design textbook is sufficient to prove my point. The mathematics involved with component selection and frequency response prediction is daunting to say the least – well beyond the scope of the beginning electronics student. It has been my intent here to present the basic principles of filter design with as little math as possible, leaning on the power of the SPICE circuit analysis program to explore filter performance. The benefit of such computer simulation software cannot be understated, for the beginning student or for the working engineer.

Circuit simulation software empowers the student to explore circuit designs far beyond the reach of their math skills. With the ability to generate Bode plots and precise figures, an intuitive understanding of circuit concepts can be attained, which is something often lost when a student is burdened with the task of solving lengthy equations by hand. If you are not familiar with the use of SPICE or other circuit simulation programs, take the time to become so! It will be of great benefit to your study. To see SPICE analyses presented in this book is an aid to understanding circuits, but to actually set up and analyze your own circuit simulations is a much more engaging and worthwhile endeavor as a student.

8.8 Contributors

Les contributeurs de ce chapitre sont listés dans l'ordre chronologique de leurs contributions, depuis le plus récent jusqu'au premier. Voyez l'Annexe 2 (Liste des contributeur) pour les dates et les informations de contact.

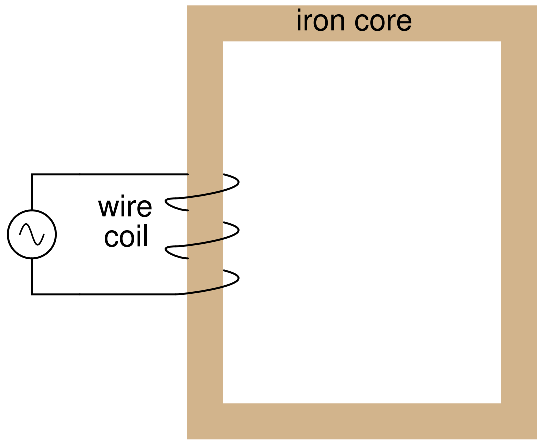
Jason Starck (June 2000) : HTML document formatting, which led to a much better-looking second edition.

Chapter 9

TRANSFORMERS

9.1 Mutual inductance and basic operation

Supposons que nous bobinions une self de fils isolés autour d'une boucle d'un matériau ferro-magnétique et alimentons cette bobine avec une source de tension AC :



As an inductor, we would expect this iron-core coil to oppose the applied voltage with its inductive reactance, limiting current through the coil as predicted by the equations $X_L = 2\pi fL$ and I=E/X (or I=E/Z). For the purposes of this example, though, we need to take a more detailed look at the interactions of voltage, current, and magnetic flux in the device.

Kirchhoff's voltage law describes how the algebraic sum of all voltages in a loop must equal zero. In this example, we could apply this fundamental law of electricity to describe the respective voltages of the source and of the inductor coil. Here, as in any one-source, one-load circuit, the voltage dropped across the load must equal the voltage supplied by the source, assuming zero voltage dropped along the resistance of any connecting wires. In other words, the load (inductor coil) must produce an opposing voltage equal in magnitude to the source, in order that it may balance against the source voltage and produce an algebraic loop voltage sum of zero. From where does this opposing voltage arise? If the load were a resistor, the opposing voltage would originate from the "friction" of electrons flowing through the resistance of the resistor. With a perfect inductor (no resistance in the coil wire), the opposing voltage comes from another mechanism : the *reaction* to a changing magnetic flux in the iron core.

Michael Faraday discovered the mathematical relationship between magnetic flux (Φ) and induced voltage with this equation :

$$e = N \frac{d\Phi}{dt}$$

Where,

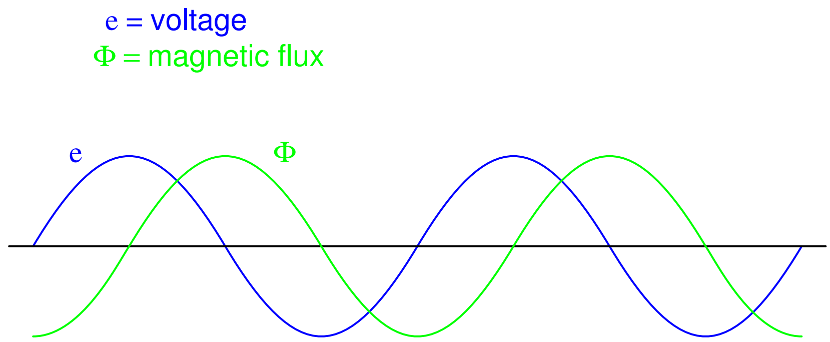
$e =$ (Instantaneous) induced voltage in volts

$N =$ Number of turns in wire coil (straight wire = 1)

$\Phi =$ Magnetic flux in Webers

$t =$ Time in seconds

The instantaneous voltage (voltage dropped at any instant in time) across a wire coil is equal to the number of turns of that coil around the core (N) multiplied by the instantaneous rate-of-change in magnetic flux ($d\Phi/dt$) linking with the coil. Graphed, this shows itself as a set of sine waves (assuming a sinusoidal voltage source), the flux wave 90^o lagging behind the voltage wave :

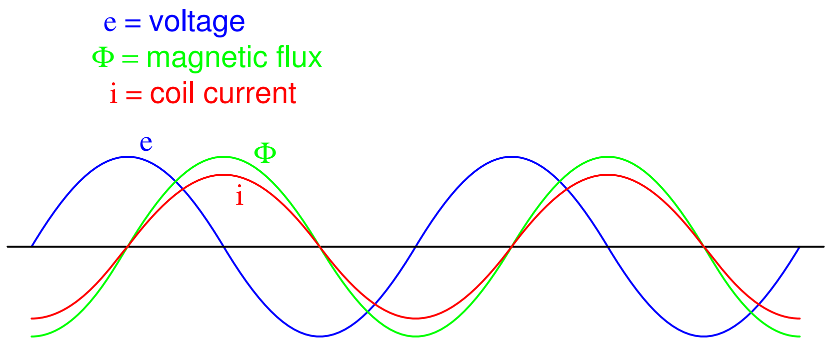


Magnetic flux through a ferromagnetic material is analogous to current through a conductor : it must be motivated by some force in order to occur. In electric circuits, this motivating force is voltage (a.k.a. electromotive force, or EMF). In magnetic "circuits," this motivating force is *magnetomotive force*, or *mmf*. Magnetomotive force (mmf) and magnetic flux (Φ) are related to each other by a property of magnetic materials known as *reluctance* (the latter quantity symbolized by a strange-looking letter "R") :

A comparison of "Ohm's Law" for electric and magnetic circuits:

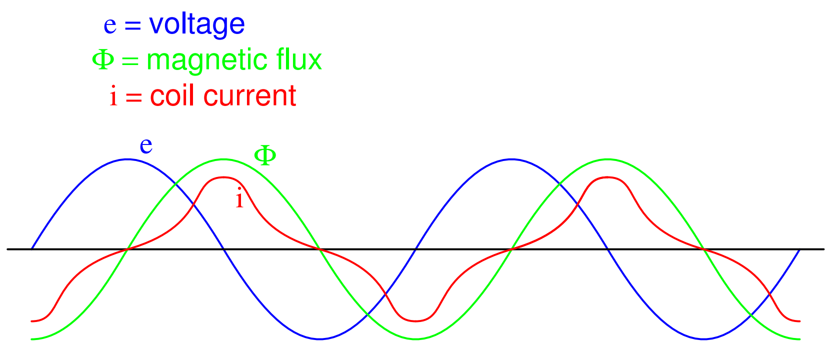
Electrical	Magnetic
$E = IR$	$mmf = \Phi\Re$

In our example, the mmf required to produce this changing magnetic flux (Φ) must be supplied by a changing current through the coil. Magnetomotive force generated by an electromagnet coil is equal to the amount of current through that coil (in amps) multiplied by the number of turns of that coil around the core (the SI unit for mmf is the *amp-turn*). Because the mathematical relationship between magnetic flux and mmf is directly proportional, and because the mathematical relationship between mmf and current is also directly proportional (no rates-of-change present in either equation), the current through the coil will be in-phase with the flux wave :



This is why alternating current through an inductor lags the applied voltage waveform by 90^o : because that is what is required to produce a changing magnetic flux whose rate-of-change produces an opposing voltage in-phase with the applied voltage. Due to its function in providing magnetizing force (mmf) for the core, this current is sometimes referred to as the *magnetizing current*.

It should be mentioned that the current through an iron-core inductor is not perfectly sinusoidal (sine-wave shaped), due to the nonlinear B/H magnetization curve of iron. In fact, if the inductor is cheaply built, using as little iron as possible, the magnetic flux density might reach high levels (approaching saturation), resulting in a magnetizing current waveform that looks something like this :

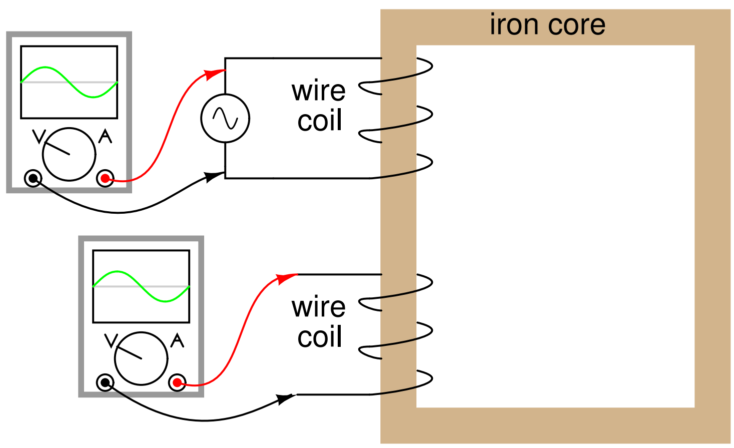


When a ferromagnetic material approaches magnetic flux saturation, disproportionately greater levels of magnetic field force (mmf) are required to deliver equal increases in magnetic field flux (Φ). Because mmf is proportional to current through the magnetizing coil (mmf = NI, where "N" is the number of turns of wire in the coil and "I" is the current through it), the large increases of mmf required to supply the needed increases in flux results in large increases in coil current. Thus, coil current increases dramatically at the peaks in order to maintain a flux waveform that isn't distorted, accounting for the bell-shaped half-cycles of the current waveform in the above plot.

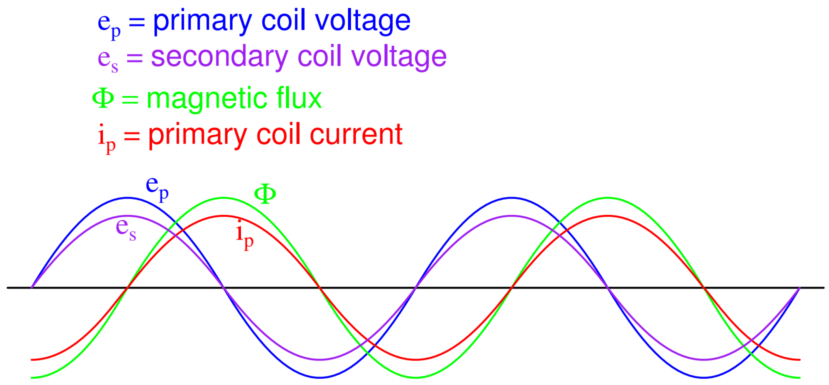
The situation is further complicated by energy losses within the iron core. The effects of hysteresis and eddy currents conspire to further distort and complicate the current waveform, making it even less sinusoidal and altering its phase to be lagging slightly less than 90^o behind the applied voltage waveform. This coil current resulting from the sum total of all magnetic effects in the core ($d\Phi/dt$ magnetization plus hysteresis losses, eddy current losses, etc.) is called the *exciting current*. The distortion of an iron-core inductor's exciting current may be minimized if it is designed for and operated at very low flux densities. Generally speaking, this requires a core with large cross-sectional area, which tends to make the inductor bulky and expensive. For the sake of simplicity, though, we'll assume that our example core is far from saturation and free from all losses, resulting in a perfectly sinusoidal exciting current.

As we've seen already in the inductors chapter, having a current waveform 90^o out of phase with the voltage waveform creates a condition where power is alternately absorbed and returned to the circuit by the inductor. If the inductor is perfect (no wire resistance, no magnetic core losses, etc.), it will dissipate zero power.

Let us now consider the same inductor device, except this time with a second coil wrapped around the same iron core. The first coil will be labeled the *primary* coil, while the second will be labeled the *secondary* :



If this secondary coil experiences the same magnetic flux change as the primary (which it should, assuming perfect containment of the magnetic flux through the common core), and has the same number of turns around the core, a voltage of equal magnitude and phase to the applied voltage will be induced along its length. In the following graph, the induced voltage waveform is drawn slightly smaller than the source voltage waveform simply to distinguish one from the other :



This effect is called *mutual inductance* : the induction of a voltage in one coil in response to a change in current in the other coil. Like normal (self-) inductance, it is measured in the unit of Henrys, but unlike normal inductance it is symbolized by the capital letter "M" rather than the letter "L" :

Inductance

$$e = L \frac{di}{dt}$$

Mutual inductance

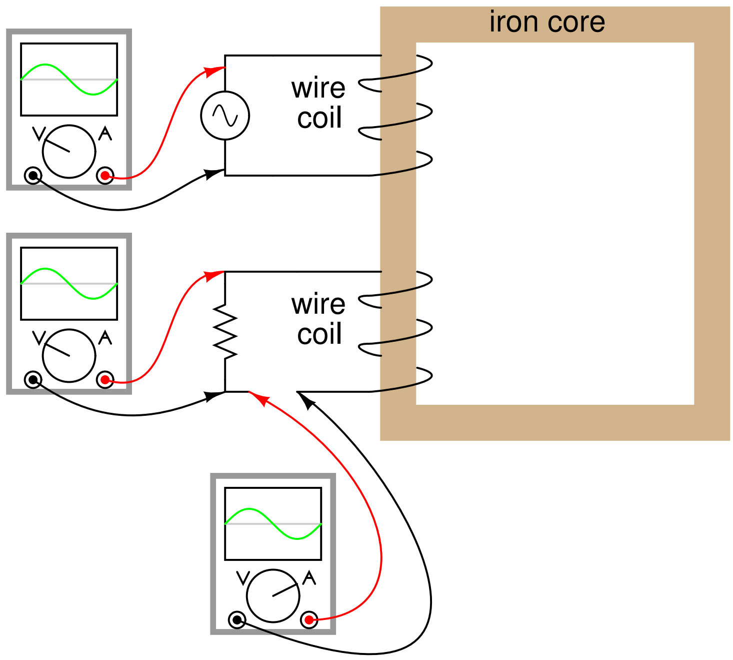
$$e_2 = M \frac{di_1}{dt}$$

Where,

e_2 = voltage induced in secondary coil

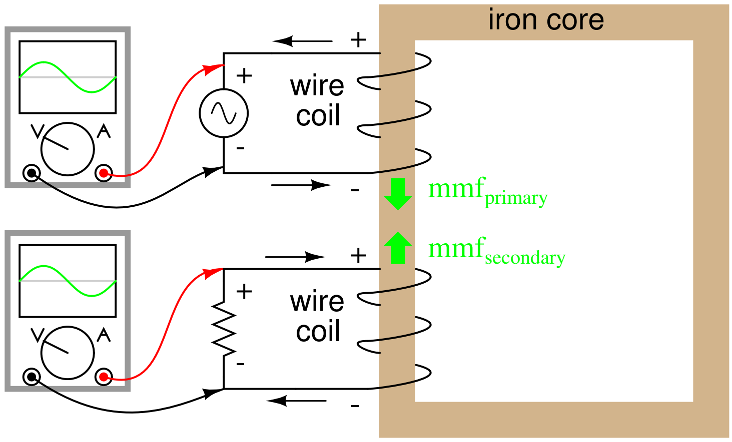
i_1 = current in primary coil

No current will exist in the secondary coil, since it is open-circuited. However, if we connect a load resistor to it, an alternating current will go through the coil, in phase with the induced voltage (because the voltage across a resistor and the current through it are *always* in phase with each other).



At first, one might expect this secondary coil current to cause additional magnetic flux in the core. In fact, it does not. If more flux were induced in the core, it would cause more voltage to be induced voltage in the primary coil (remember that e = $d\Phi/dt$). This cannot happen, because the primary coil's induced voltage must remain at the same magnitude and phase in order to balance with the applied voltage, in accordance with Kirchhoff's voltage law. Consequently, the magnetic flux in the core cannot be affected by secondary coil current. However, what *does* change is the amount of mmf in the magnetic circuit.

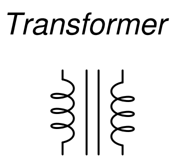
Magnetomotive force is produced any time electrons move through a wire. Usually, this mmf is accompanied by magnetic flux, in accordance with the mmf=ΦR "magnetic Ohm's Law" equation. In this case, though, additional flux is not permitted, so the only way the secondary coil's mmf may exist is if a counteracting mmf is generated by the primary coil, of equal magnitude and opposite phase. Indeed, this is what happens, an alternating current forming in the primary coil – 180^o out of phase with the secondary coil's current – to generate this counteracting mmf and prevent additional core flux. Polarity marks and current direction arrows have been added to the illustration to clarify phase relations :



If you find this process a bit confusing, do not worry. Transformer dynamics is a complex subject. What is important to understand is this : when an AC voltage is applied to the primary coil, it creates a magnetic flux in the core, which induces AC voltage in the secondary coil in-phase with the source voltage. Any current drawn through the secondary coil to power a load induces a corresponding current in the primary coil, drawing current from the source.

Notice how the primary coil is behaving as a load with respect to the AC voltage source, and how the secondary coil is behaving as a source with respect to the resistor. Rather than energy merely being alternately absorbed and returned the primary coil circuit, energy is now being *coupled* to the secondary coil where it is delivered to a dissipative (energy-consuming) load. As far as the source "knows," it's directly powering the resistor. Of course, there is also an additional primary coil current lagging the applied voltage by 90^o, just enough to magnetize the core to create the necessary voltage for balancing against the source (the *exciting current*).

We call this type of device a *transformer*, because it transforms electrical energy into magnetic energy, then back into electrical energy again. Because its operation depends on electromagnetic induction between two stationary coils and a magnetic flux of changing magnitude and "polarity," transformers are necessarily AC devices. Its schematic symbol looks like two inductors (coils) sharing the same magnetic core :

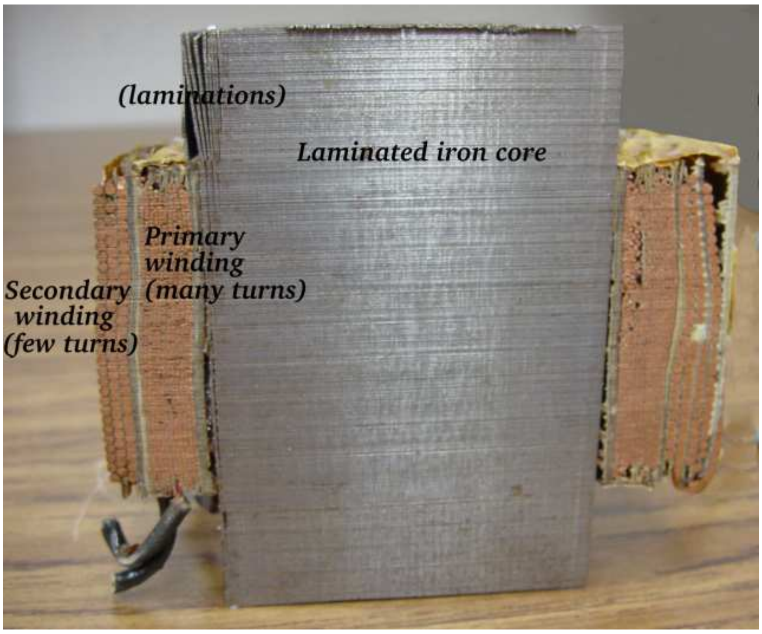


The two inductor coils are easily distinguished in the above symbol. The pair of vertical lines represent an iron core common to both inductors. While many transformers have ferromagnetic core materials, there are some that do not, their constituent inductors being magnetically linked together through the air.

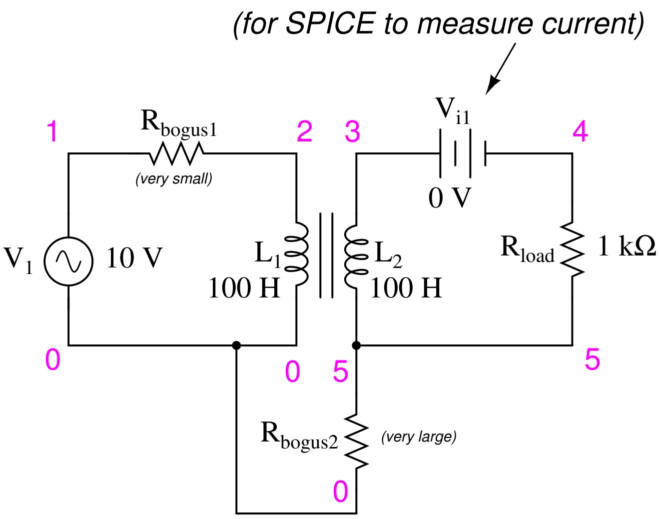
The following photograph shows a power transformer of the type used in gas-discharge lighting. Here, the two inductor coils can be clearly seen, wound around an iron core. While most transformer designs enclose the coils and core in a metal frame for protection, this particular transformer is open for viewing and so serves its illustrative purpose well :

Both coils of wire can be seen here with copper-colored varnish insulation. The top coil is larger than the bottom coil, having a greater number of "turns" around the core. In transformers, the inductor coils are often referred to as *windings*, in reference to the manufacturing process where wire is *wound* around the core material. As modeled in our initial example, the powered inductor of a transformer is called the *primary* winding, while the unpowered coil is called the *secondary* winding.

In the next photograph, a transformer is shown cut in half, exposing the cross-section of the iron core as well as both windings. Like the transformer shown previously, this unit also utilizes primary and secondary windings of differing turn counts. The wire gauge can also be seen to differ between primary and secondary windings. The reason for this disparity in wire gauge will be made clear in the next section of this chapter. Additionally, the iron core can be seen in this photograph to be made of many thin sheets (laminations) rather than a solid piece. The reason for this will also be explained in a later section of this chapter.



It is easy to demonstrate simple transformer action using SPICE, setting up the primary and secondary windings of the simulated transformer as a pair of "mutual" inductors. The coefficient of magnetic field coupling is given at the end of the "k" line in the SPICE circuit description, this example being set very nearly at perfection (1.000). This coefficient describes how closely "linked" the two inductors are, magnetically. The better these two inductors are magnetically coupled, the more efficient the energy transfer between them should be.




```
transformer
v1 1 0 ac 10 sin
```

```
rbogus1 1 2 1e-12
rbogus2 5 0 9e12
l1 2 0 100
l2 3 5 100
** This line tells SPICE that the two inductors
** l1 and l2 are magnetically "linked" together
k l1 l2 0.999
vi1 3 4 ac 0
rload 4 5 1k
.ac lin 1 60 60
.print ac v(2,0) i(v1)
.print ac v(3,5) i(vi1)
.end
```

Note : the R_{bogus} resistors are required to satisfy certain quirks of SPICE. The first breaks the otherwise continuous loop between the voltage source and L_1 which would not be permitted by SPICE. The second provides a path to ground (node 0) from the secondary circuit, necessary because SPICE cannot function with any ungrounded circuits.

```
freq          v(2)        i(v1)
6.000E+01     1.000E+01   9.975E-03   Primary winding

freq          v(3,5)      i(vi1)
6.000E+01     9.962E+00   9.962E-03   Secondary winding
```

Note that with equal inductances for both windings (100 Henrys each), the AC voltages and currents are nearly equal for the two. The difference between primary and secondary currents is the magnetizing current spoken of earlier : the 90^o lagging current necessary to magnetize the core. As is seen here, it is usually very small compared to primary current induced by the load, and so the primary and secondary currents are almost equal. What you are seeing here is quite typical of transformer efficiency. Anything less than 95% efficiency is considered poor for modern power transformer designs, and this transfer of power occurs with no moving parts or other components subject to wear.

If we decrease the load resistance so as to draw more current with the same amount of voltage, we see that the current through the primary winding increases in response. Even though the AC power source is not directly connected to the load resistance (rather, it is electromagnetically ”coupled”), the amount of current drawn from the source will be almost the same as the amount of current that would be drawn if the load were directly connected to the source. Take a close look at the next two SPICE simulations, showing what happens with different values of load resistors :

```
transformer
v1 1 0 ac 10 sin
rbogus1 1 2 1e-12
rbogus2 5 0 9e12
l1 2 0 100
l2 3 5 100
```

```
k l1 l2 0.999
vi1 3 4 ac 0
** Note load resistance value of 200 ohms
rload 4 5 200
.ac lin 1 60 60
.print ac v(2,0) i(v1)
.print ac v(3,5) i(vi1)
.end
```

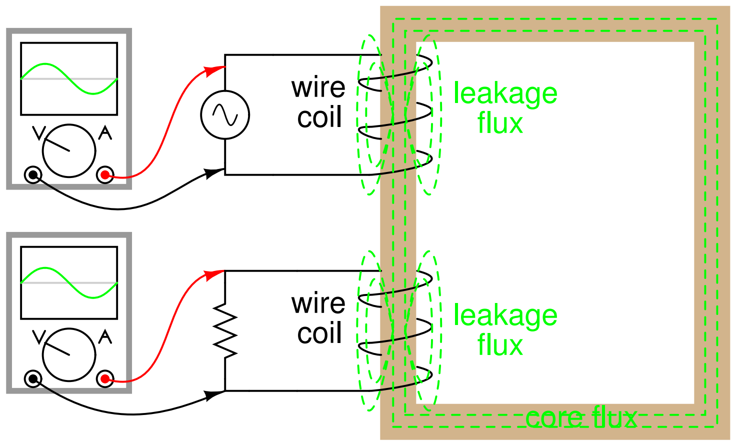
```
freq          v(2)        i(v1)
6.000E+01     1.000E+01   4.679E-02

freq          v(3,5)      i(vi1)
6.000E+01     9.348E+00   4.674E-02
```

Notice how the primary current closely follows the secondary current. In our first simulation, both currents were approximately 10 mA, but now they are both around 47 mA. In this second simulation, the two currents are closer to equality, because the magnetizing current remains the same as before while the load current has increased. Note also how the secondary voltage has decreased some with the heavier (greater current) load. Let's try another simulation with an even lower value of load resistance (15 Ω) :

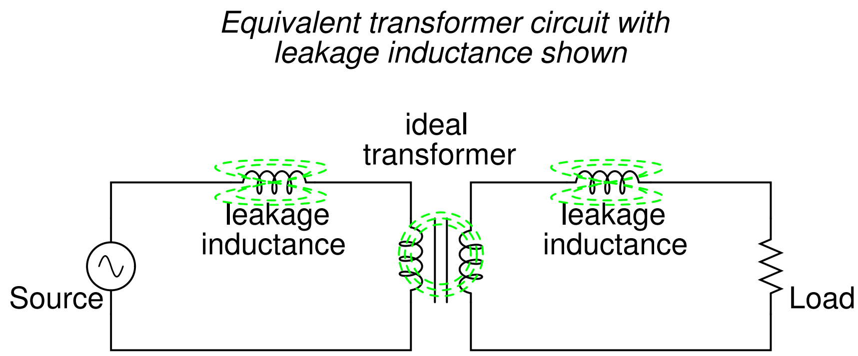
```
transformer
v1 1 0 ac 10 sin
rbogus1 1 2 1e-12
rbogus2 5 0 9e12
l1 2 0 100
l2 3 5 100
k l1 l2 0.999
vi1 3 4 ac 0
rload 4 5 15
.ac lin 1 60 60
.print ac v(2,0) i(v1)
.print ac v(3,5) i(vi1)
.end
```

```
freq          v(2)        i(v1)
6.000E+01     1.000E+01   1.301E-01

freq          v(3,5)      i(vi1)
6.000E+01     1.950E+00   1.300E-01
```

Our load current is now 0.13 amps, or 130 mA, which is substantially higher than the last time. The primary current is very close to being the same, but notice how the secondary voltage has fallen well below the primary voltage (1.95 volts versus 10 volts at the primary). The reason for this is an imperfection in our transformer design : because the primary and secondary inductances aren't *perfectly* linked (a k factor of 0.999 instead of 1.000) there is "stray" or "*leakage*" inductance. In other words, some of the magnetic field isn't linking with the secondary coil, and thus cannot couple energy to it :



Consequently, this "leakage" flux merely stores and returns energy to the source circuit via self-inductance, effectively acting as a series impedance in both primary and secondary circuits. Voltage gets dropped across this series impedance, resulting in a reduced load voltage : voltage across the load "sags" as load current increases.



If we change the transformer design to have better magnetic coupling between the primary and secondary coils, the figures for voltage between primary and secondary windings will be much closer to equality again :

```
transformer
v1 1 0 ac 10 sin
rbogus1 1 2 1e-12
rbogus2 5 0 9e12
l1 2 0 100
l2 3 5 100
** Coupling factor = 0.99999 instead of 0.999
k l1 l2 0.99999
```

```
vi1 3 4 ac 0
rload 4 5 15
.ac lin 1 60 60
.print ac v(2,0) i(v1)
.print ac v(3,5) i(vi1)
.end

freq          v(2)         i(v1)
6.000E+01     1.000E+01    6.658E-01

freq          v(3,5)       i(vi1)
6.000E+01     9.987E+00    6.658E-01
```

Here we see that our secondary voltage is back to being equal with the primary, and the secondary current is equal to the primary current as well. Unfortunately, building a real transformer with coupling this complete is very difficult. A compromise solution is to design both primary and secondary coils with less inductance, the strategy being that less inductance overall leads to less "leakage" inductance to cause trouble, for any given degree of magnetic coupling inefficiency. This results in a load voltage that is closer to ideal with the same (heavy) load and the same coupling factor :

```
transformer
v1 1 0 ac 10 sin
rbogus1 1 2 1e-12
rbogus2 5 0 9e12
** inductance = 1 henry instead of 100 henrys
l1 2 0 1
l2 3 5 1
k l1 l2 0.999
vi1 3 4 ac 0
rload 4 5 15
.ac lin 1 60 60
.print ac v(2,0) i(v1)
.print ac v(3,5) i(vi1)
.end

freq          v(2)         i(v1)
6.000E+01     1.000E+01    6.664E-01

freq          v(3,5)       i(vi1)
6.000E+01     9.977E+00    6.652E-01
```

Simply by using primary and secondary coils of less inductance, the load voltage for this heavy load has been brought back up to nearly ideal levels (9.977 volts). At this point, one might ask, "If less inductance is all that's needed to achieve near-ideal performance under heavy load, then why worry about coupling efficiency at all? If it's impossible to build a transformer with perfect coupling, but easy to design coils with low inductance, then why not just build all transformers with low-inductance coils and have excellent efficiency even with poor magnetic coupling?"

The answer to this question is found in another simulation : the same low-inductance transformer, but this time with a lighter load (1 kΩ instead of 15 Ω) :

```
transformer
v1 1 0 ac 10 sin
rbogus1 1 2 1e-12
rbogus2 5 0 9e12
l1 2 0 1
l2 3 5 1
k l1 l2 0.999
vi1 3 4 ac 0
rload 4 5 1k
.ac lin 1 60 60
.print ac v(2,0) i(v1)
.print ac v(3,5) i(vi1)
.end

freq          v(2)         i(v1)
6.000E+01     1.000E+01    2.835E-02

freq          v(3,5)       i(vi1)
6.000E+01     9.990E+00    9.990E-03
```

With lower winding inductances, the primary and secondary voltages are closer to being equal, but the primary and secondary currents are not. In this particular case, the primary current is 28.35 mA while the secondary current is only 9.990 mA : almost three times as much current in the primary as the secondary. Why is this? With less inductance in the primary winding, there is less inductive reactance, and consequently a much larger magnetizing current. A substantial amount of the current through the primary winding merely works to magnetize the core rather than *transfer* useful energy to the secondary winding and load.

An ideal transformer with identical primary and secondary windings would manifest equal voltage and current in both sets of windings for any load condition. In a perfect world, transformers would transfer electrical power from primary to secondary as smoothly as though the load were directly connected to the primary power source, with no transformer there at all. However, you can see this ideal goal can only be met if there is *perfect* coupling of magnetic flux between primary and secondary windings. Being that this is impossible to achieve, transformers must be designed to operate within certain expected ranges of voltages and loads in order to perform as close to ideal as possible. For now, the most important thing to keep in mind is a transformer's basic operating principle : the transfer of power from the primary to the secondary circuit via electromagnetic coupling.

- **REVIEW :**
- *Mutual inductance* is where the magnetic flux of two or more inductors are "linked" so that voltage is induced in one coil proportional to the rate-of-change of current in another.

- A *transformer* is a device made of two or more inductors, one of which is powered by AC, inducing an AC voltage across the second inductor. If the second inductor is connected to a load, power will be electromagnetically coupled from the first inductor's power source to that load.
- The powered inductor in a transformer is called the *primary winding*. The unpowered inductor in a transformer is called the *secondary winding*.
- Magnetic flux in the core (Φ) lags 90^o behind the source voltage waveform. The current drawn by the primary coil from the source to produce this flux is called the *magnetizing current*, and it also lags the supply voltage by 90^o.
- Total primary current in an unloaded transformer is called the *exciting current*, and is comprised of magnetizing current plus any additional current necessary to overcome core losses. It is never perfectly sinusoidal in a real transformer, but may be made more so if the transformer is designed and operated so that magnetic flux density is kept to a minimum.
- Core flux induces a voltage in any coil wrapped around the core. The induces voltage(s) are ideally in phase with the primary winding source voltage and share the same waveshape.
- Any current drawn through the secondary winding by a load will be "reflected" to the primary winding and drawn from the voltage source, as if the source were directly powering a similar load.

9.2 Step-up and step-down transformers

So far, we've observed simulations of transformers where the primary and secondary windings were of identical inductance, giving approximately equal voltage and current levels in both circuits. Equality of voltage and current between the primary and secondary sides of a transformer, however, is not the norm for all transformers. If the inductances of the two windings are not equal, something interesting happens :

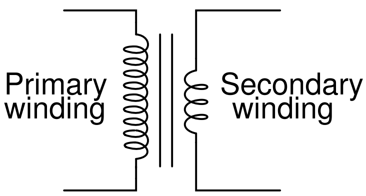
```
transformer
v1 1 0 ac 10 sin
rbogus1 1 2 1e-12
rbogus2 5 0 9e12
l1 2 0 10000
l2 3 5 100
k l1 l2 0.999
vi1 3 4 ac 0
rload 4 5 1k
.ac lin 1 60 60
.print ac v(2,0) i(v1)
.print ac v(3,5) i(vi1)
.end
```

```
freq          v(2)        i(v1)
```

```
6.000E+01     1.000E+01   9.975E-05    Primary winding
```

```
freq          v(3,5)      i(vi1)
6.000E+01     9.962E-01   9.962E-04    Secondary winding
```

Notice how the secondary voltage is approximately ten times less than the primary voltage (0.9962 volts compared to 10 volts), while the secondary current is approximately ten times greater (0.9962 mA compared to 0.09975 mA). What we have here is a device that steps voltage *down* by a factor of ten and current *up* by a factor of ten :

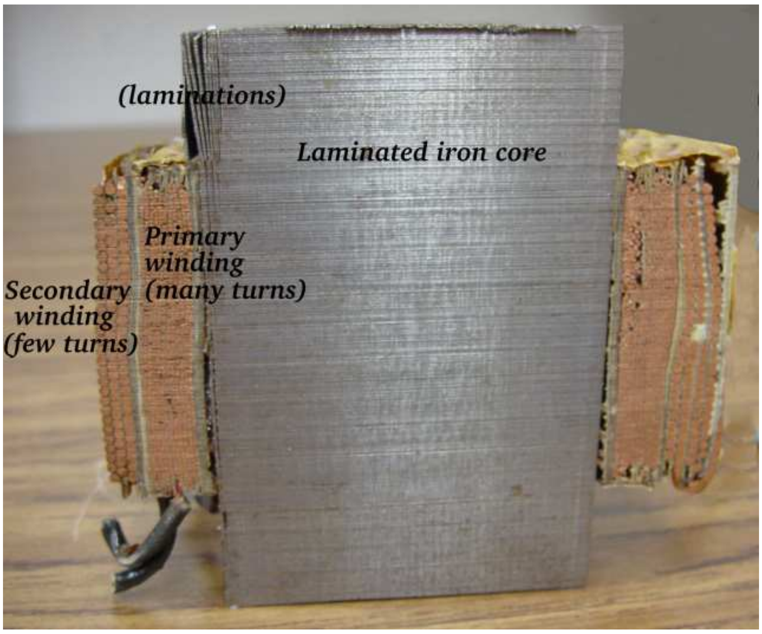


10:1 primary:secondary voltage ratio

1:10 primary:secondary current ratio

This is a very useful device, indeed. With it, we can easily multiply or divide voltage and current in AC circuits. Indeed, the transformer has made long-distance transmission of electric power a practical reality, as AC voltage can be "stepped up" and current "stepped down" for reduced wire resistance power losses along power lines connecting generating stations with loads. At either end (both the generator and at the loads), voltage levels are reduced by transformers for safer operation and less expensive equipment. A transformer that increases voltage from primary to secondary (more secondary winding turns than primary winding turns) is called a *step-up* transformer. Conversely, a transformer designed to do just the opposite is called a *step-down* transformer.

Let's re-examine a photograph shown in the previous section :



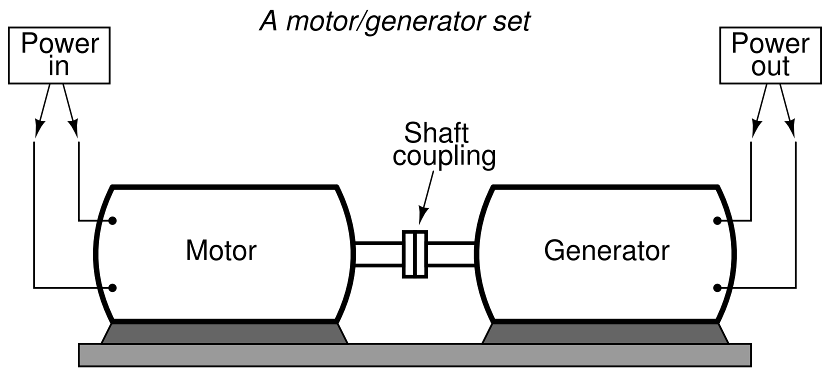
This is a step-down transformer, as evidenced by the high turn count of the primary winding and the low turn count of the secondary. As a step-down unit, this transformer converts high-voltage, low-current power into low-voltage, high-current power. The larger-gauge wire used in the secondary winding is necessary due to the increase in current. The primary winding, which doesn't have to conduct as much current, may be made of smaller-gauge wire.

In case you were wondering, it *is* possible to operate either of these transformer types backwards (powering the secondary winding with an AC source and letting the primary winding power a load) to perform the opposite function : a step-up can function as a step-down and visa-versa. However, as we saw in the first section of this chapter, efficient operation of a transformer requires that the individual winding inductances be engineered for specific operating ranges of voltage and current, so if a transformer is to be used "backwards" like this it must be employed within the original design parameters of voltage and current for each winding, lest it prove to be inefficient (or lest it be *damaged* by excessive voltage or current!).

Transformers are often constructed in such a way that it is not obvious which wires lead to the primary winding and which lead to the secondary. One convention used in the electric power industry to help alleviate confusion is the use of "H" designations for the higher-voltage winding (the primary winding in a step-down unit; the secondary winding in a step-up) and "X" designations for the lower-voltage winding. Therefore, a simple power transformer will have wires labeled "H_1", "H_2", "X_1", and "X_2". There is usually significance to the numbering of the wires (H_1 versus H_2, etc.), which we'll explore a little later in this chapter.

The fact that voltage and current get "stepped" in opposite directions (one up, the other down) makes perfect sense when you recall that power is equal to voltage times current, and realize that transformers cannot *produce* power, only convert it. Any device that could output more power than it took in would violate the *Law of Energy Conservation* in physics, namely that energy cannot be created or destroyed, only converted. As with the first transformer example we looked at, power transfer efficiency is very good from the primary to the secondary sides of the device.

The practical significance of this is made more apparent when an alternative is considered : before the advent of efficient transformers, voltage/current level conversion could only be achieved through the use of motor/generator sets. A drawing of a motor/generator set reveals the basic principle involved :



In such a machine, a motor is mechanically coupled to a generator, the generator designed to produce the desired levels of voltage and current at the rotating speed of the motor. While both motors and generators are fairly efficient devices, the use of both in this fashion compounds their inefficiencies so that the overall efficiency is in the range of 90% or less. Furthermore, because motor/generator sets obviously require moving parts, mechanical wear and balance are factors influencing both service life and performance. Transformers, on the other hand, are able to convert levels of AC voltage and current at very high efficiencies with no moving parts, making possible the widespread distribution and use of electric power we take for granted.

In all fairness it should be noted that motor/generator sets have not necessarily been obsoleted by transformers for *all* applications. While transformers are clearly superior over motor/generator sets for AC voltage and current level conversion, they cannot convert one frequency of AC power to another, or (by themselves) convert DC to AC or visa-versa. Motor/generator sets can do all these things with relative simplicity, albeit with the limitations of efficiency and mechanical factors already described. Motor/generator sets also have the unique property of kinetic energy storage : that is, if the motor's power supply is momentarily interrupted for any reason, its angular momentum (the inertia of that rotating mass) will maintain rotation of the generator for a short duration, thus isolating any loads powered by the generator from "glitches" in the main power system.

Looking closely at the numbers in the SPICE analysis, we should see a correspondence between the transformer's ratio and the two inductances. Notice how the primary inductor (l1) has 100 times more inductance than the secondary inductor (10000 H versus 100 H), and that the measured voltage step-down ratio was 10 to 1. The winding with more inductance will have higher voltage and less current than the other. Since the two inductors are wound around the same core material in the transformer (for the most efficient magnetic coupling between the two), the parameters affecting inductance for the two coils are equal except for the number of turns in each coil. If we take another look at our inductance formula, we see that inductance is proportional to the *square* of the number of coil turns :

$$L = \frac{N^2 \mu A}{l}$$

Where,

L = Inductance of coil in Henrys

N = Number of turns in wire coil (straight wire = 1)

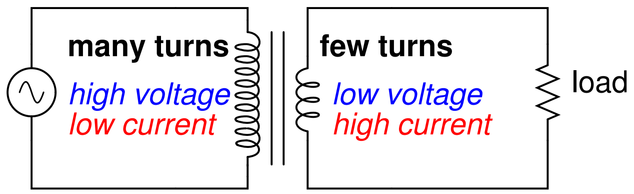
μ = Permeability of core material (absolute, not relative)

A = Area of coil in square meters

l = Average length of coil in meters

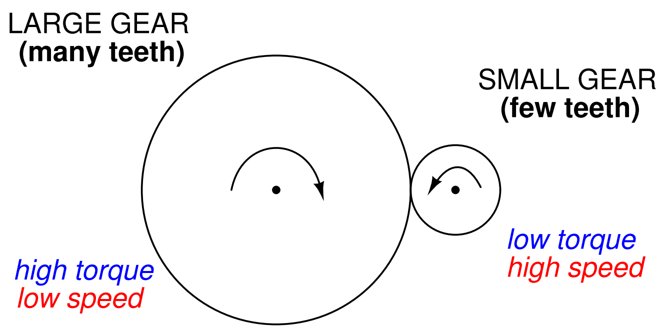
So, it should be apparent that our two inductors in the last SPICE transformer example circuit – with inductance ratios of 100 :1 – should have coil turn ratios of 10 :1, because 10 squared equals 100. This works out to be the same ratio we found between primary and secondary voltages and currents (10 :1), so we can say as a rule that the voltage and current transformation ratio is equal to the ratio of winding turns between primary and secondary.

Step-down transformer



The step-up/step-down effect of coil turn ratios in a transformer is analogous to gear tooth ratios in mechanical gear systems, transforming values of speed and torque in much the same way :

Torque reduction geartrain



Step-up and step-down transformers for power distribution purposes can be gigantic in proportion to the power transformers previously shown, some units standing as tall as a home. The following photograph shows a substation transformer standing about twelve feet tall :

- **REVIEW :**
- Transformers "step up" or "step down" voltage according to the ratios of primary to secondary wire turns.

$$\text{Voltage transformation ratio} = \frac{N_{secondary}}{N_{primary}}$$

$$\text{Current transformation ratio} = \frac{N_{primary}}{N_{secondary}}$$

Where,

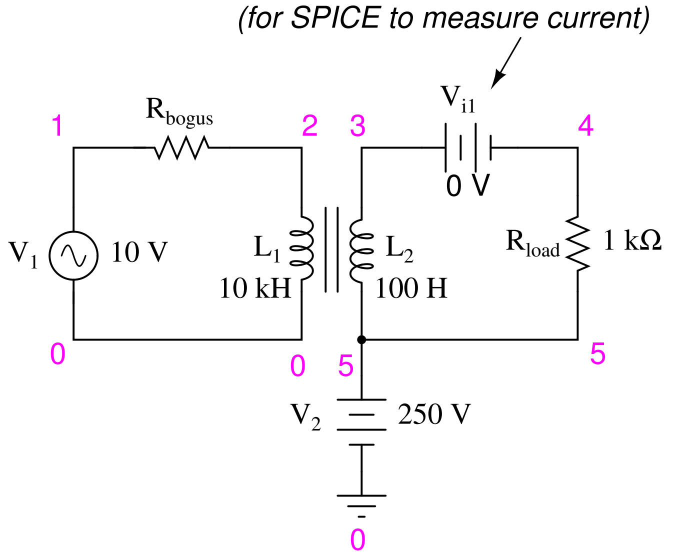
- N = number of turns in winding

- A transformer designed to increase voltage from primary to secondary is called a *step-up* transformer. A transformer designed to reduce voltage from primary to secondary is called a *step-down* transformer.
- The transformation ratio of a transformer will be equal to the square root of its primary to secondary inductance (L) ratio.

- $$\text{Voltage transformation ratio} = \sqrt{\frac{L_{secondary}}{L_{primary}}}$$

9.3 Electrical isolation

Aside from the ability to easily convert between different levels of voltage and current in AC and DC circuits, transformers also provide an extremely useful feature called *isolation*, which is the ability to couple one circuit to another without the use of direct wire connections. We can demonstrate an application of this effect with another SPICE simulation : this time showing "ground" connections for the two circuits, imposing a high DC voltage between one circuit and ground through the use of an additional voltage source :

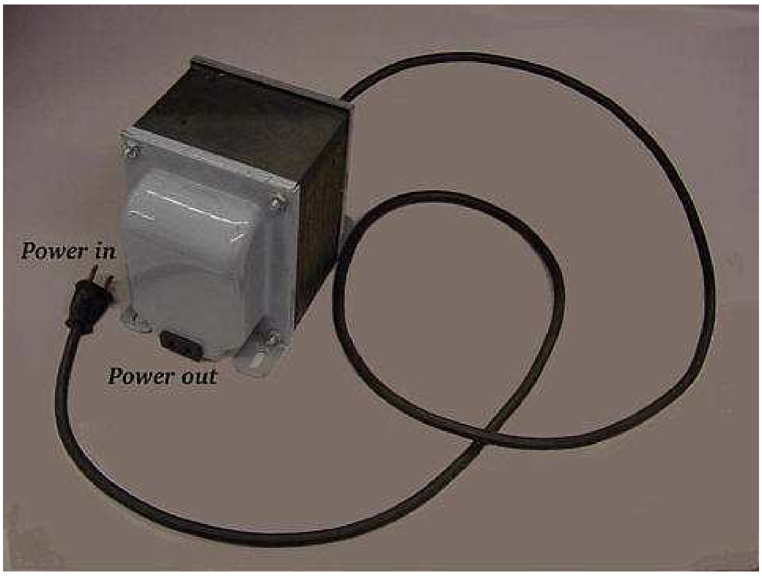



```
v1 1 0 ac 10 sin
rbogus1 1 2 1e-12
v2 5 0 dc 250
l1 2 0 10000
l2 3 5 100
k l1 l2 0.999
vi1 3 4 ac 0
rload 4 5 1k
.ac lin 1 60 60
.print ac v(2,0) i(v1)
.print ac v(3,5) i(vi1)
.end
```

```
DC voltages referenced to ground (node 0) :
(1)    0.0000    (2)    0.0000    (3)  250.0000
(4)  250.0000    (5)  250.0000

AC voltages :
freq          v(2)         i(v1)
6.000E+01     1.000E+01    9.975E-05    Primary winding

freq          v(3,5)       i(vi1)
6.000E+01     9.962E-01    9.962E-04    Secondary winding
```

SPICE shows the 250 volts DC being impressed upon the secondary circuit elements with respect to ground, but as you can see there is no effect on the primary circuit (zero DC voltage) at nodes 1 and 2, and the transformation of AC power from primary to secondary circuits remains the same as before. The impressed voltage in this example is often called a *common-mode* voltage because it is seen at more than one point in the circuit with reference to the common point of ground. The transformer isolates the common-mode voltage so that it is not impressed upon the primary circuit at all, but rather isolated to the secondary side. For the record, it does not matter that the common-mode voltage is DC, either. It could be AC, even at a different frequency, and the transformer would isolate it from the primary circuit all the same.

There are applications where electrical isolation is needed between two AC circuit without any transformation of voltage or current levels. In these instances, transformers called *isolation transformers* having 1 :1 transformation ratios are used. A benchtop isolation transformer is shown in the following photograph :



- **REVIEW :**
- By being able to transfer power from one circuit to another without the use of interconnecting conductors between the two circuits, transformers provide the useful feature of *electrical isolation.*
- Transformers designed to provide electrical isolation without stepping voltage and current either up or down are called *isolation transformers.*

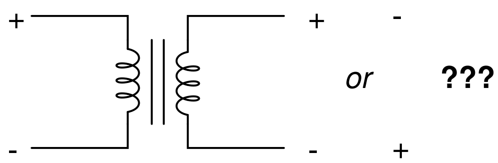
9.4 Phasing

Since transformers are essentially AC devices, we need to be aware of the phase relationships between the primary and secondary circuits. Using our SPICE example from before, we can plot the waveshapes for the primary and secondary circuits and see the phase relations for ourselves :

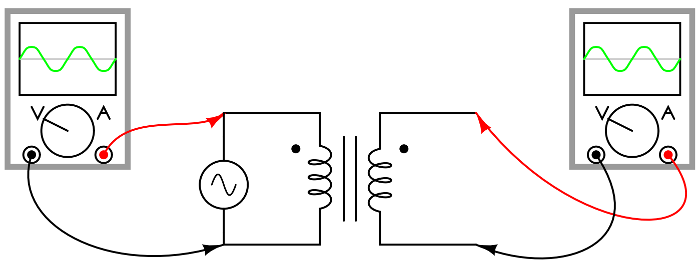
```
legend :
* : v(2)      Primary voltage
+ : v(3,5)    Secondary voltage
time        v(2)
(*)-----------      -10          -5           0           5          10
(+)-----------      -10          -5           0           5          10
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
0.000E+00  0.000E+00 .           .           x           .           .
1.000E-03  3.675E+00 .           .           .     + *   .           .
2.000E-03  6.803E+00 .           .           .           . + *       .
3.000E-03  9.008E+00 .           .           .           .      +*   .
4.000E-03  9.955E+00 .           .           .           .           x
5.000E-03  9.450E+00 .           .           .           .        *+.
6.000E-03  7.672E+00 .           .           .           .    * +    .
7.000E-03  4.804E+00 .           .           .          *.+          .
8.000E-03  1.245E+00 .           .           .  *  +     .           .
9.000E-03 -2.474E+00 .           .      * +  .           .           .
1.000E-02 -5.864E+00 .          *+           .           .           .
1.100E-02 -8.390E+00 .     *+    .           .           .           .
1.200E-02 -9.779E+00 .x          .           .           .           .
1.300E-02 -9.798E+00 +*          .           .           .           .
1.400E-02 -8.390E+00 .    +*     .           .           .           .
1.500E-02 -5.854E+00 .       + *.            .           .           .
1.600E-02 -2.479E+00 .           .     + *   .           .           .
1.700E-02  1.246E+00 .           .           .+ *        .           .
1.800E-02  4.795E+00 .           .           .       + *.            .
1.900E-02  7.686E+00 .           .           .           .   + *     .
2.000E-02  9.451E+00 .           .           .           .        x  .
2.100E-02  9.937E+00 .           .           .           .           x
2.200E-02  9.025E+00 .           .           .           .       *+  .
2.300E-02  6.802E+00 .           .           .           .  *+       .
2.400E-02  3.667E+00 .           .           .      * +  .           .
2.500E-02 -1.487E-03 .           .           * +         .           .
2.600E-02 -3.658E+00 .           .    * +    .           .           .
2.700E-02 -6.814E+00 .       * + .           .           .           .
2.800E-02 -9.026E+00 .   *+      .           .           .           .
2.900E-02 -9.917E+00 *+          .           .           .           .
3.000E-02 -9.511E+00 .x          .           .           .           .
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

legend :
* : i(v1)    Primary current
+ : i(vi1)   Secondary current
time       i(v1)
(*)---------- -2.000E-04   -1.000E-04      0   1.000E-04  2.000E-04
(+)---------- -1.000E-03   -5.000E-04      0   5.000E-04  1.000E-03
```

```
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
0.000E+00  0.000E+00 .          .          x          .          .
1.000E-03 -2.973E-05 .          .     +   *.          .          .
2.000E-03 -6.279E-05 .        + .    *     .          .          .
3.000E-03 -8.772E-05 .   +      . *        .          .          .
4.000E-03 -1.008E-04 +          *          .          .          .
5.000E-03 -9.954E-05 .+         *          .          .          .
6.000E-03 -8.522E-05 .    +     . *        .          .          .
7.000E-03 -5.919E-05 .         +.     *    .          .          .
8.000E-03 -2.500E-05 .          .       + *.          .          .
9.000E-03  1.212E-05 .          .          . *  +     .          .
1.000E-02  4.736E-05 .          .          .     *    .+         .
1.100E-02  7.521E-05 .          .          .       *  .     +    .
1.200E-02  9.250E-05 .          .          .         *.         +.
1.300E-02  9.648E-05 .          .          .         *.          +
1.400E-02  8.602E-05 .          .          .        * .      +   .
1.500E-02  6.362E-05 .          .          .     *    . +        .
1.600E-02  3.177E-05 .          .          .  *  +    .          .
1.700E-02 -4.998E-06 .          .          x          .          .
1.800E-02 -4.136E-05 .          .  +    *  .          .          .
1.900E-02 -7.246E-05 .     +    .   *      .          .          .
2.000E-02 -9.331E-05 . +        .*         .          .          .
2.100E-02 -1.019E-04 +          *          .          .          .
2.200E-02 -9.651E-05 . +        *          .          .          .
2.300E-02 -7.749E-05 .     +    .  *       .          .          .
2.400E-02 -4.842E-05 .          . +    *   .          .          .
2.500E-02 -1.275E-05 .          .         x.          .          .
2.600E-02  2.428E-05 .          .          .  *  +    .          .
2.700E-02  5.761E-05 .          .          .     *    .+         .
2.800E-02  8.261E-05 .          .          .        * .       +  .
2.900E-02  9.514E-05 .          .          .         *.         +.
3.000E-02  9.487E-05 .          .          .         *.         +.
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

It would appear that both voltage and current for the two transformer windings are in phase with each other, at least for our resistive load. This is simple enough, but it would be nice to know *which way* we should connect a transformer in order to ensure the proper phase relationships be kept. After all, a transformer is nothing more than a set of magnetically-linked inductors, and inductors don't usually come with polarity markings of any kind. If we were to look at an unmarked transformer, we would have no way of knowing which way to hook it up to a circuit to get in-phase (or 180^o out-of-phase) voltage and current :



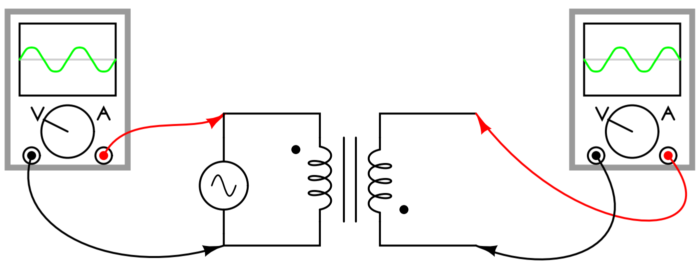
Since this is a practical concern, transformer manufacturers have come up with a sort of polarity marking standard to denote phase relationships. It is called the *dot convention*, and is nothing more than a dot placed next to each corresponding leg of a transformer winding :



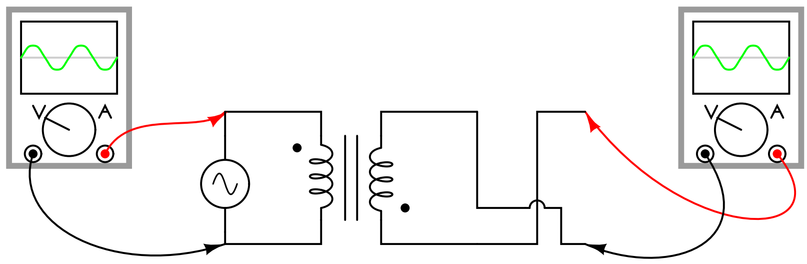
Typically, the transformer will come with some kind of schematic diagram labeling the wire leads for primary and secondary windings. On the diagram will be a pair of dots similar to what is seen above. Sometimes dots will be omitted, but when "H" and "X" labels are used to label transformer winding wires, the subscript numbers are supposed to represent winding polarity. The "1" wires (H_1 and X_1) represent where the polarity-marking dots would normally be placed.

The similar placement of these dots next to the top ends of the primary and secondary windings tells us that whatever instantaneous voltage polarity seen across the primary winding will be the same as that across the secondary winding. In other words, the phase shift from primary to secondary will be zero degrees.

On the other hand, if the dots on each winding of the transformer do *not* match up, the phase shift will be 180^o between primary and secondary, like this :



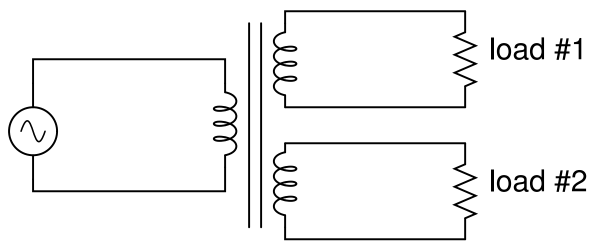
Of course, the dot convention only tells you which end of each winding is which, relative to the other winding(s). If you want to reverse the phase relationship yourself, all you have to do is swap the winding connections like this :



- **REVIEW :**
- The phase relationships for voltage and current between primary and secondary circuits of a transformer are direct : ideally, zero phase shift.
- The *dot convention* is a type of polarity marking for transformer windings showing which end of the winding is which, relative to the other windings.

9.5 Winding configurations

Transformers are very versatile devices. The basic concept of energy transfer between mutual inductors is useful enough between a single primary and single secondary coil, but transformers don't have to be made with just two sets of windings. Consider this transformer circuit :

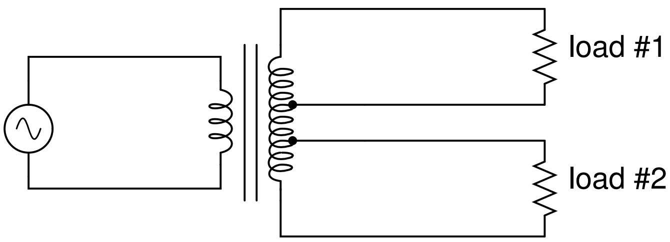


Here, three inductor coils share a common magnetic core, magnetically "coupling" or "linking" them together. The relationship of winding turn ratios and voltage ratios seen with a single pair of mutual inductors still holds true here for multiple pairs of coils. It is entirely possible to assemble a transformer such as the one above (one primary winding, two secondary windings) in which one secondary winding is a step-down and the other is a step-up. In fact, this design of transformer was quite common in vacuum tube power supply circuits, which were required to supply low voltage for the tubes' filaments (typically 6 or 12 volts) and high voltage for the tubes' plates (several hundred volts) from a nominal primary voltage of 110 volts AC. Not only are voltages and currents of completely different magnitudes possible with such a transformer, but all circuits are electrically isolated from one another.

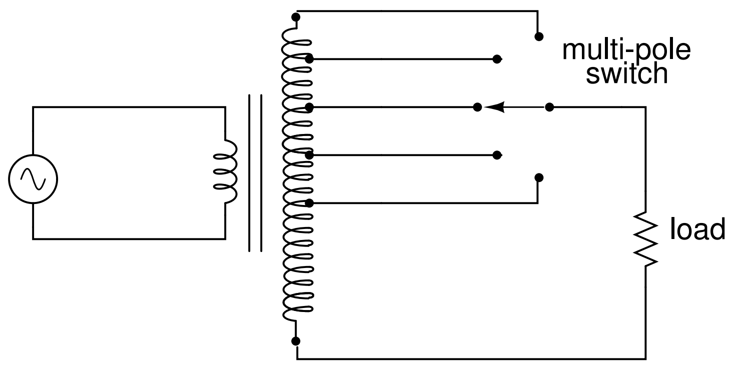
A photograph of a multiple-winding transformer is shown here :

This particular transformer is intended to provide both high and low voltages necessary in an electronic system using vacuum tubes. Low voltage is required to power the filaments of vacuum tubes, while high voltage is required to create the potential difference between the plate and cathode elements of each tube. One transformer with multiple windings suffices elegantly to provide all the necessary voltage levels from a single 115 V source. The wires for this transformer (15 of them!) are not shown in the photograph, being hidden from view.

If electrical isolation between secondary circuits is not of great importance, a similar effect can be obtained by "tapping" a single secondary winding at multiple points along its length, like this :

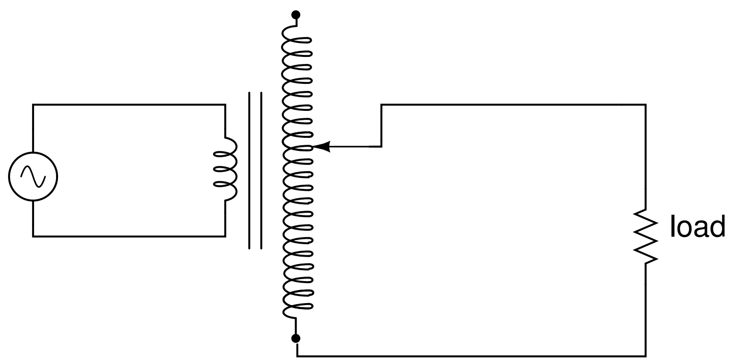


A tap is nothing more than a wire connection made at some point on a winding between the very ends. Not surprisingly, the winding turn/voltage magnitude relationship of a normal transformer holds true for all tapped segments of windings. This fact can be exploited to produce a transformer capable of multiple ratios :



Carrying the concept of winding taps further, we end up with a "variable transformer," where a sliding contact is moved along the length of an exposed secondary winding, able to connect with it at any point along its length. The effect is equivalent to having a winding tap at every turn of the winding, and a switch with poles at every tap position :

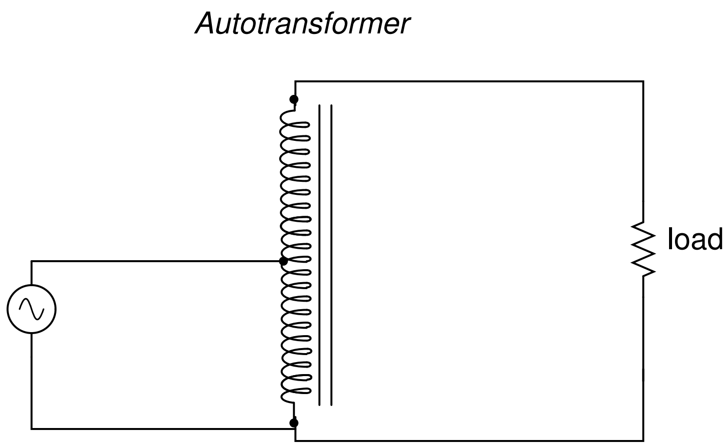
Variable transformer



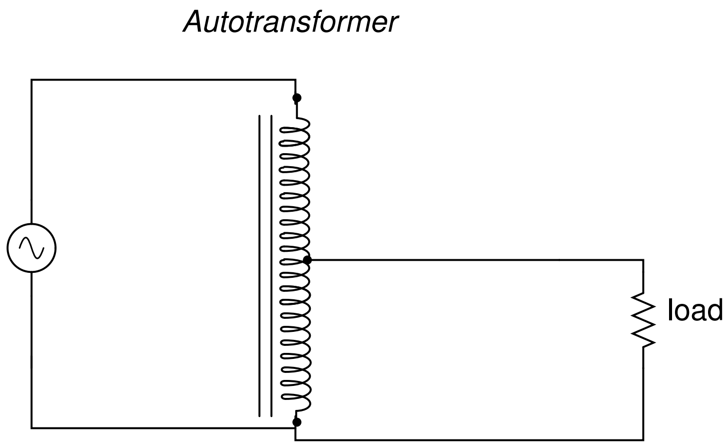
One consumer application of the variable transformer is in speed controls for model train sets, especially the train sets of the 1950's and 1960's. These transformers were essentially step-down units, the highest voltage obtainable from the secondary winding being substantially less than the primary voltage of 110 to 120 volts AC. The variable-sweep contact provided a simple means of voltage control with little wasted power, much more efficient than control using a variable resistor!

Moving-slide contacts are too impractical to be used in large industrial power transformer designs, but multi-pole switches and winding taps are common for voltage adjustment. Adjustments need to be made periodically in power systems to accommodate changes in loads over months or years in time, and these switching circuits provide a convenient means. Typically, such "tap switches" are not engineered to handle full-load current, but must be actuated only when the transformer has been de-energized (no power).

Seeing as how we can tap any transformer winding to obtain the equivalent of several windings (albeit with loss of electrical isolation between them), it makes sense that it should be possible to forego electrical isolation altogether and build a transformer from a single winding. Indeed this is possible, and the resulting device is called an *autotransformer* :

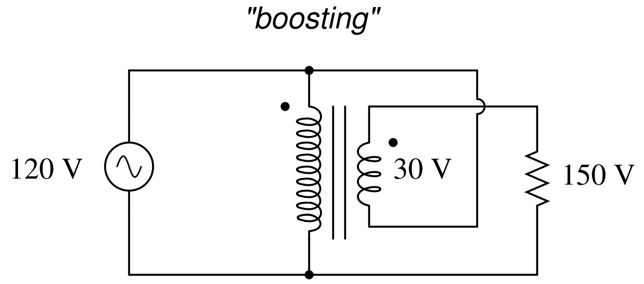


The autotransformer depicted above performs a voltage step-up function. A step-down autotransformer would look something like this :

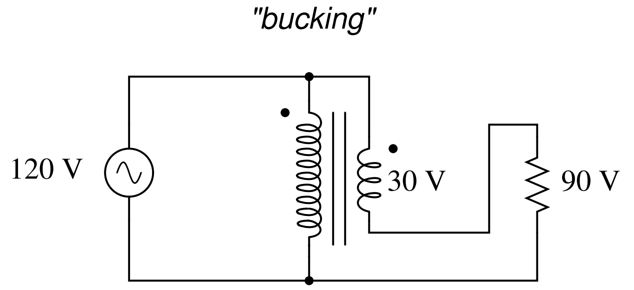


Autotransformers find popular use in applications requiring a slight boost or reduction in voltage to a load. The alternative with a normal (isolated) transformer would be to either have just the right primary/secondary winding ratio made for the job or use a step-down configuration with the secondary winding connected in series-aiding ("boosting") or series-opposing ("bucking") fashion. Primary, secondary, and load voltages are given to illustrate how this would work.

First, the "boosting" configuration. Here, the secondary coil's polarity is oriented so that its voltage directly adds to the primary voltage :

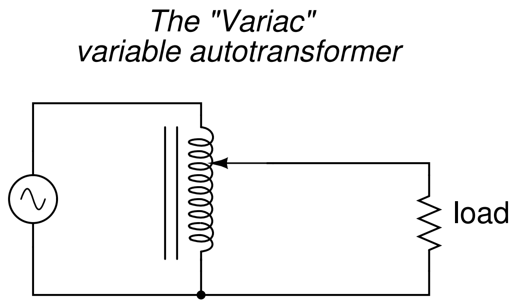


Next, the "bucking" configuration. Here, the secondary coil's polarity is oriented so that its voltage directly subtracts from the primary voltage :



The prime advantage of an autotransformer is that the same boosting or bucking function is obtained with only a single winding, making it cheaper and lighter to manufacture than a regular (isolating) transformer having both primary and secondary windings.

Like regular transformers, autotransformer windings can be tapped to provide variations in ratio. Additionally, they can be made continuously variable with a sliding contact to tap the winding at any point along its length. The latter configuration is popular enough to have earned itself its own name : the *Variac.*



Small variacs for benchtop use are popular pieces of equipment for the electronics experimenter, being able to step household AC voltage down (or sometimes up as well) with a wide, fine range of control by a simple twist of a knob.

- **REVIEW :**
- Transformers can be equipped with more than just a single primary and single secondary winding pair. This allows for multiple step-up and/or step-down ratios in the same device.

- Transformer windings can also be "tapped :" that is, intersected at many points to segment a single winding into sections.
- Variable transformers can be made by providing a movable arm that sweeps across the length of a winding, making contact with the winding at any point along its length. The winding, of course, has to be bare (no insulation) in the area where the arm sweeps.
- An autotransformer is a single, tapped inductor coil used to step up or step down voltage like a transformer, except without providing electrical isolation.
- A *Variac* is a variable autotransformer.

9.6 Voltage regulation

As we saw in a few SPICE analyses earlier in this chapter, the output voltage of a transformer varies some with varying load resistances, even with a constant voltage input. The degree of variance is affected by the primary and secondary winding inductances, among other factors, not the least of which includes winding resistance and the degree of mutual inductance (magnetic coupling) between the primary and secondary windings. For power transformer applications, where the transformer is seen by the load (ideally) as a constant source of voltage, it is good to have the secondary voltage vary as little as possible for wide variances in load current.

The measure of how well a power transformer maintains constant secondary voltage over a range of load currents is called the transformer's *voltage regulation.* It can be calculated from the following formula :

$$\text{Regulation percentage} = \frac{E_{\text{no-load}} - E_{\text{full-load}}}{E_{\text{full-load}}} \ (100\%)$$

"Full-load" means the point at which the transformer is operating at maximum permissible secondary current. This operating point will be determined primarily by the winding wire size (ampacity) and the method of transformer cooling. Taking our first SPICE transformer simulation as an example, let's compare the output voltage with a 1 kΩ load versus a 200 Ω load (assuming that the 200 Ω load will be our "full load" condition). Recall if you will that our constant primary voltage was 10.00 volts AC :

```
freq          v(3,5)       i(vi1)
6.000E+01     9.962E+00    9.962E-03     Output with 1k ohm load

freq          v(3,5)       i(vi1)
6.000E+01     9.348E+00    4.674E-02     Output with 200 ohm load
```

Notice how the output voltage decreases as the load gets heavier (more current). Now let's take that same transformer circuit and place a load resistance of extremely high magnitude across the secondary winding to simulate a "no-load" condition :

```
transformer
v1 1 0 ac 10 sin
```

```
rbogus1 1 2 1e-12
rbogus2 5 0 9e12
l1 2 0 100
l2 3 5 100
k l1 l2 0.999
vi1 3 4 ac 0
rload 4 5 9e12
.ac lin 1 60 60
.print ac v(2,0) i(v1)
.print ac v(3,5) i(vi1)
.end

freq          v(2)        i(v1)
6.000E+01     1.000E+01   2.653E-04

freq          v(3,5)      i(vi1)
6.000E+01     9.990E+00   1.110E-12   Output with (almost) no load
```

So, we see that our output (secondary) voltage spans a range of 9.990 volts at (virtually) no load and 9.348 volts at the point we decided to call "full load." Calculating voltage regulation with these figures, we get :

$$\text{Regulation percentage} = \frac{9.990\text{ V} - 9.348\text{ V}}{9.990\text{ V}} (100\%)$$

$$\text{Regulation percentage} = 6.4264\ \%$$

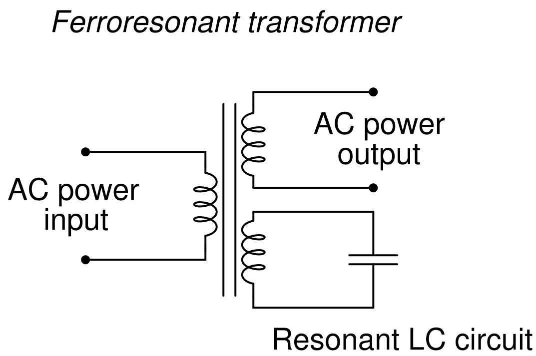
Incidentally, this would be considered rather poor (or "loose") regulation for a power transformer. Powering a simple resistive load like this, a good power transformer should exhibit a regulation percentage of less than 3%. Inductive loads tend to create a condition of worse voltage regulation, so this analysis with purely resistive loads was a "best-case" condition.

There are some applications, however, where poor regulation is actually desired. One such case is in discharge lighting, where a step-up transformer is required to initially generate a high voltage (necessary to "ignite" the lamps), then the voltage is expected to drop off once the lamp begins to draw current. This is because discharge lamps' voltage requirements tend to be much lower after a current has been established through the arc path. In this case, a step-up transformer with poor voltage regulation suffices nicely for the task of conditioning power to the lamp.

Another application is in current control for AC arc welders, which are nothing more than step-down transformers supplying low-voltage, high-current power for the welding process. A high voltage is desired to assist in "striking" the arc (getting it started), but like the discharge lamp, an arc doesn't require as much voltage to sustain itself once the air has been heated to the point of ionization. Thus, a decrease of secondary voltage under high load current would be a good thing. Some arc welder designs provide arc current adjustment by means of a movable iron core in the transformer, cranked in or out of the winding assembly by the operator. Moving the iron slug away from the windings reduces the strength of magnetic coupling between the windings, which diminishes no-load secondary voltage *and* makes for poorer voltage regulation.

No exposition on transformer regulation could be called complete without mention of an unusual device called a *ferroresonant transformer*. "Ferroresonance" is a phenomenon associated with the behavior of iron cores while operating near a point of magnetic saturation (where the core is so strongly magnetized that further increases in winding current results in little or no increase in magnetic flux).

While being somewhat difficult to describe without going deep into electromagnetic theory, the ferroresonant transformer is a power transformer engineered to operate in a condition of persistent core saturation. That is, its iron core is "stuffed full" of magnetic lines of flux for a large portion of the AC cycle so that variations in supply voltage (primary winding current) have little effect on the core's magnetic flux density, which means the secondary winding outputs a nearly constant voltage despite significant variations in supply (primary winding) voltage. Normally, core saturation in a transformer results in distortion of the sinewave shape, and the ferroresonant transformer is no exception. To combat this side effect, ferroresonant transformers have an auxiliary secondary winding paralleled with one or more capacitors, forming a resonant circuit tuned to the power supply frequency. This "tank circuit" serves as a filter to reject harmonics created by the core saturation, and provides the added benefit of storing energy in the form of AC oscillations, which is available for sustaining output winding voltage for brief periods of input voltage loss (milliseconds' worth of time, but certainly better than nothing).



In addition to blocking harmonics created by the saturated core, this resonant circuit also "filters out" harmonic frequencies generated by nonlinear (switching) loads in the secondary winding circuit and any harmonics present in the source voltage, providing "clean" power to the load.

Ferroresonant transformers offer several features useful in AC power conditioning : constant output voltage given substantial variations in input voltage, harmonic filtering between the power source and the load, and the ability to "ride through" brief losses in power by keeping a reserve of energy in its resonant tank circuit. These transformers are also highly tolerant of excessive loading and transient (momentary) voltage surges. They are so tolerant, in fact, that some may be briefly paralleled with unsynchronized AC power sources, allowing a load to be switched from one source of power to another in a "make-before-break" fashion with no interruption of power on the secondary side!

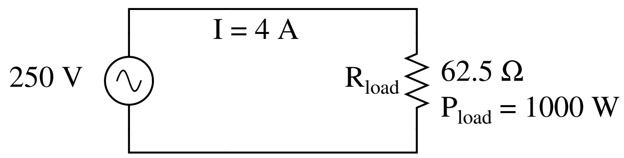
Unfortunately, these devices have equally noteworthy disadvantages : they waste a lot of energy (due to hysteresis losses in the saturated core), generating *significant* heat in the process, and are intolerant of frequency variations, which means they don't work very well when powered by small engine-driven generators having poor speed regulation. Voltages produced in the resonant winding/capacitor circuit tend to be very high, necessitating expensive capacitors and presenting the service technician with very dangerous working voltages. Some applications, though, may prioritize the ferroresonant transformer's advantages over its disadvantages. Semiconductor circuits exist to "condition" AC power as an alternative to ferroresonant devices, but none can compete with this transformer in terms of sheer simplicity.

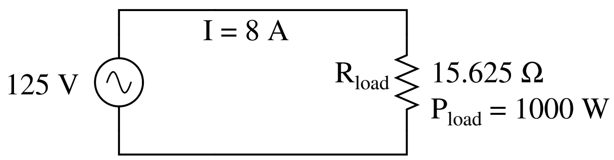
- **REVIEW :**
- *Voltage regulation* is the measure of how well a power transformer can maintain constant secondary voltage given a constant primary voltage and wide variance in load current. The lower the percentage (closer to zero), the more stable the secondary voltage and the better the regulation it will provide.
- A *ferroresonant* transformer is a special transformer designed to regulate voltage at a stable level despite wide variation in input voltage.

9.7 Special transformers and applications

Because transformers can step voltage and current to different levels, and because power is transferred equivalently between primary and secondary windings, they can be used to "convert" the impedance of a load to a different level. That last phrase deserves some explanation, so let's investigate what it means.

The purpose of a load (usually) is to do something productive with the power it dissipates. In the case of a resistive heating element, the practical purpose for the power dissipated is to heat something up. Loads are engineered to safely dissipate a certain maximum amount of power, but two loads of equal power rating are not necessarily identical. Consider these two 1000 watt resistive heating elements :

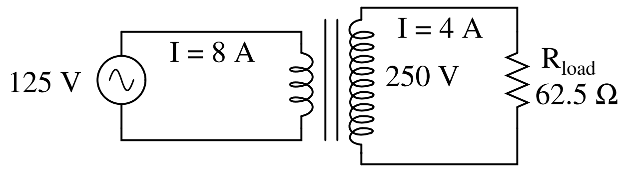




Both heaters dissipate exactly 1000 watts of power, but they do so at different voltage and current levels (either 250 volts and 4 amps, or 125 volts and 8 amps). Using Ohm's Law to determine the necessary resistance of these heating elements (R=E/I), we arrive at figures of 62.5 Ω and 15.625 Ω, respectively. If these are AC loads, we might refer to their opposition to current in terms of impedance rather than plain resistance, although in this case that's all they're composed of (no reactance). The 250 volt heater would be said to be a higher impedance load than the 125 volt heater.

If we desired to operate the 250 volt heater element directly on a 125 volt power system, we would end up being disappointed. With 62.5 Ω of impedance (resistance), the current would only be 2 amps (I=E/R; 125/62.5), and the power dissipation would only be 250 watts (P=IE; 125 x 2), or one-quarter of its rated power. The impedance of the heater and the voltage of our source would be mismatched, and we couldn't obtain the full rated power dissipation from the heater.

All hope is not lost, though. With a step-up transformer, we could operate the 250 volt heater element on the 125 volt power system like this :



1000 watts dissipation at the load resistor !

The ratio of the transformer's windings provides the voltage step-up *and* current step-down we need for the otherwise mismatched load to operate properly on this system. Take a close look at the primary circuit figures : 125 volts at 8 amps. As far as the power supply "knows," it's powering a 15.625 Ω (R=E/I) load at 125 volts, not a 62.5 Ω load! The voltage and current figures for the primary winding are indicative of 15.625 Ω load impedance, not the actual 62.5 Ω of the load itself. In other words, not only has our step-up transformer transformed voltage and current, but it has transformed *impedance* as well.

The transformation ratio of impedance is the square of the voltage/current transformation ratio, the same as the winding inductance ratio :

$$\text{Voltage transformation ratio} = \frac{N_{secondary}}{N_{primary}}$$

$$\text{Current transformation ratio} = \frac{N_{primary}}{N_{secondary}}$$

$$\text{Impedance transformation ratio} = \left(\frac{N_{secondary}}{N_{primary}}\right)^2$$

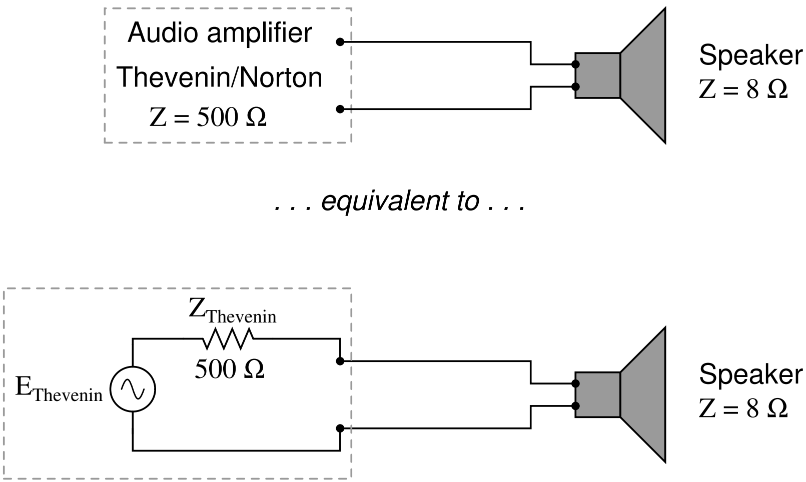
$$\text{Inductance ratio} = \left(\frac{N_{secondary}}{N_{primary}}\right)^2$$

Where,

N = number of turns in winding

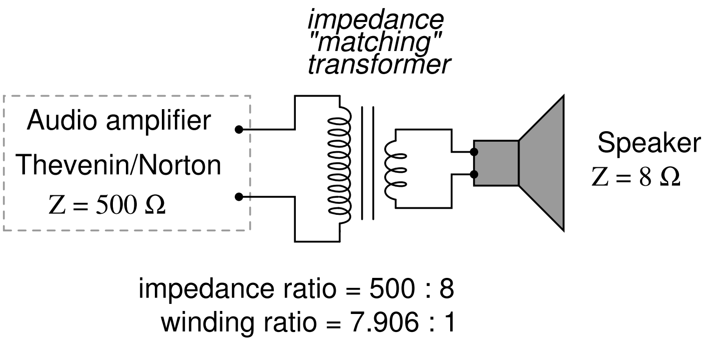
This concurs with our example of the 2 :1 step-up transformer and the impedance ratio of 62.5 Ω to 15.625 Ω (a 4 :1 ratio, which is 2 :1 squared). Impedance transformation is a highly useful ability of transformers, for it allows a load to dissipate its full rated power even if the power system is not at the proper voltage to directly do so.

Recall from our study of network analysis the *Maximum Power Transfer Theorem*, which states that the maximum amount of power will be dissipated by a load resistance when that load resistance is equal to the Thevenin/Norton resistance of the network supplying the power. Substitute the word "impedance" for "resistance" in that definition and you have the AC version of that Theorem. If we're trying to obtain theoretical maximum power dissipation from a load, we must be able to properly match the load impedance and source (Thevenin/Norton) impedance together. This is generally more of a concern in specialized electric circuits such as radio transmitter/antenna and audio amplifier/speaker systems. Let's take an audio amplifier system and see how it works :



With an internal impedance of 500 Ω, the amplifier can only deliver full power to a load (speaker) also having 500 Ω of impedance. Such a load would drop higher voltage and draw less current than an 8 Ω speaker dissipating the same amount of power. If an 8 Ω speaker were connected directly to the 500 Ω amplifier as shown, the *impedance mismatch* would result in very poor (low peak power) performance. Additionally, the amplifier would tend to dissipate more than its fair share of power in the form of heat trying to drive the low impedance speaker.

To make this system work better, we can use a transformer to match these mismatched impedances. Since we're going from a high impedance (high voltage, low current) supply to a low impedance (low voltage, high current) load, we'll need to use a step-down transformer :

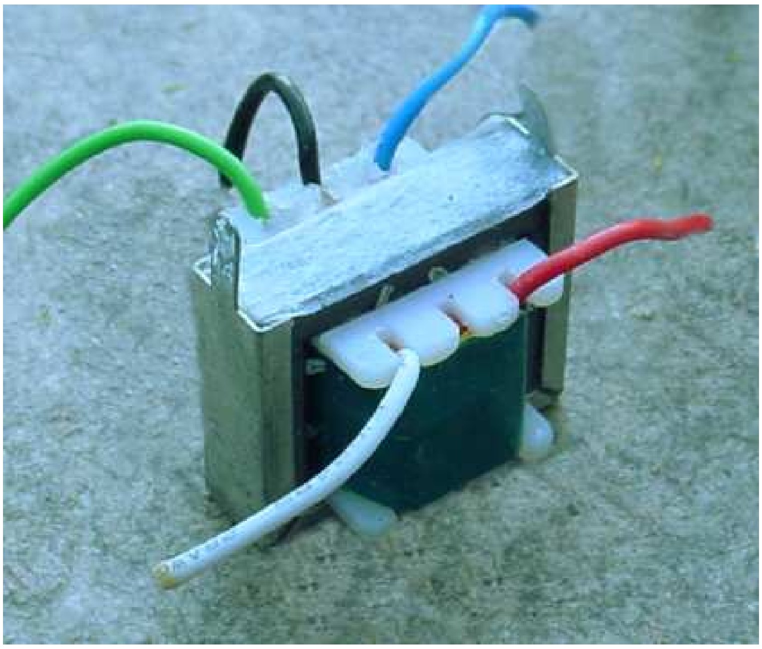


To obtain an impedance transformation ratio of 500 :8, we would need a winding ratio equal to the square root of 500 :8 (the square root of 62.5 :1, or 7.906 :1). With such a transformer in place, the speaker will load the amplifier to just the right degree, drawing power at the correct voltage and current levels to satisfy the Maximum Power Transfer Theorem and make for the most efficient power delivery to the load. The use of a transformer in this capacity is called *impedance matching*.

Anyone who has ridden a multi-speed bicycle can intuitively understand the principle of impedance matching. A human's legs will produce maximum power when spinning the bicycle crank at a particular speed (about 60 to 90 revolution per minute). Above or below that rotational speed, human leg muscles are less efficient at generating power. The purpose of the bicycle's "gears" is to impedance-match the rider's legs to the riding conditions so that they always spin the crank at the optimum speed.

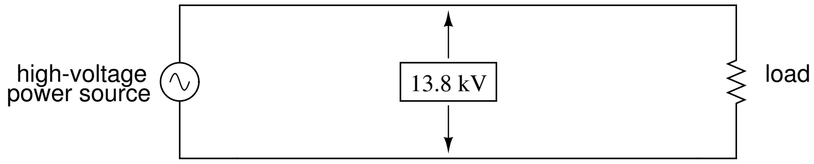
If the rider attempts to start moving while the bicycle is shifted into its "top" gear, he or she will find it very difficult to get moving. Is it because the rider is weak? No, it's because the high step-up ratio of the bicycle's chain and sprockets in that top gear presents a mismatch between the conditions (lots of inertia to overcome) and their legs (needing to spin at 60-90 RPM for maximum power output). On the other hand, selecting a gear that is too low will enable the rider to get moving immediately, but limit the top speed they will be able to attain. Again, is the lack of speed an indication of weakness in the bicyclist's legs? No, it's because the lower speed ratio of the selected gear creates another type of mismatch between the conditions (low load) and the rider's legs (losing power if spinning faster than 90 RPM). It is much the same with electric power sources and loads : there must be an impedance match for maximum system efficiency. In AC circuits, transformers perform the same matching function as the sprockets and chain ("gears") on a bicycle to match otherwise mismatched sources and loads.

Impedance matching transformers are not fundamentally different from any other type of transformer in construction or appearance. A small impedance-matching transformer (about two centimeters in width) for audio-frequency applications is shown in the following photograph :



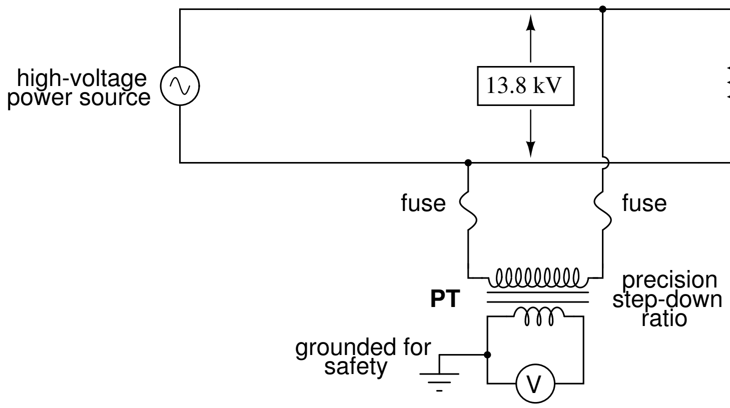
Another impedance-matching transformer can be seen on this printed circuit board, in the upper right corner, to the immediate left of resistors R_2 and R_1. It is labeled "T1" :

Transformers can also be used in electrical instrumentation systems. Due to transformers' ability to step up or step down voltage and current, and the electrical isolation they provide, they can serve as a way of connecting electrical instrumentation to high-voltage, high current power systems. Suppose we wanted to accurately measure the voltage of a 13.8 kV power system (a very common power distribution voltage in American industry) :



Designing, installing, and maintaining a voltmeter capable of directly measuring 13,800 volts AC would be no easy task. The safety hazard alone of bringing 13.8 kV conductors into an instrument panel would be severe, not to mention the design of the voltmeter itself. However, by using a precision step-down transformer, we can reduce the 13.8 kV down to a safe level of voltage at a constant ratio, and isolate it from the instrument connections, adding an additional level of safety to the metering system :

Instrument application: the "Potential Transformer"



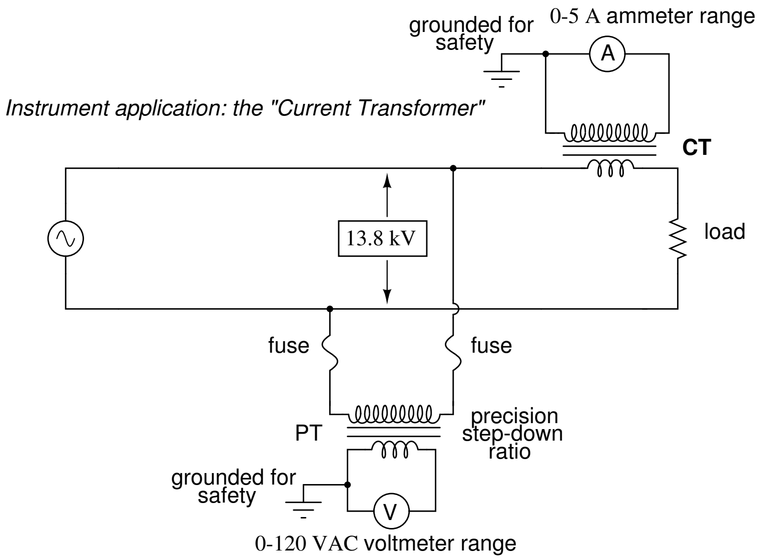
0-120 VAC voltmeter range

Now the voltmeter reads a precise fraction, or ratio, of the actual system voltage, its scale set to read as though it were measuring the voltage directly. The transformer keeps the instrument voltage at a safe level and electrically isolates it from the power system, so there is no direct connection between the power lines and the instrument or instrument wiring. When used in this capacity, the transformer is called a *Potential Transformer*, or simply *PT*.

Potential transformers are designed to provide as accurate a voltage step-down ratio as possible. To aid in precise voltage regulation, loading is kept to a minimum : the voltmeter is made to have high input impedance so as to draw as little current from the PT as possible. As you can see, a fuse has been connected in series with the PTs primary winding, for safety and ease of disconnecting the PT from the circuit.

A standard secondary voltage for a PT is 120 volts AC, for full-rated power line voltage. The standard voltmeter range to accompany a PT is 150 volts, full-scale. PTs with custom winding ratios can be manufactured to suit any application. This lends itself well to industry standardization of the actual voltmeter instruments themselves, since the PT will be sized to step the system voltage down to this standard instrument level.

Following the same line of thinking, we can use a transformer to step down current through a power line so that we are able to safely and easily measure high system currents with inexpensive ammeters. Of course, such a transformer would be connected in series with the power line, like this :



Note that while the PT is a step-down device, the *Current Transformer* (or *CT*) is a step-up device (with respect to voltage), which is what is needed to step *down* the power line current. Quite often, CTs are built as donut-shaped devices through which the power line conductor is run, the power line itself acting as a single-turn primary winding :

Some CTs are made to hinge open, allowing insertion around a power conductor without disturbing the conductor at all. The industry standard secondary current for a CT is a range of 0 to 5 amps AC. Like PTs, CTs can be made with custom winding ratios to fit almost any application. Because their "full load" secondary current is 5 amps, CT ratios are usually described in terms of full-load primary amps to 5 amps, like this :

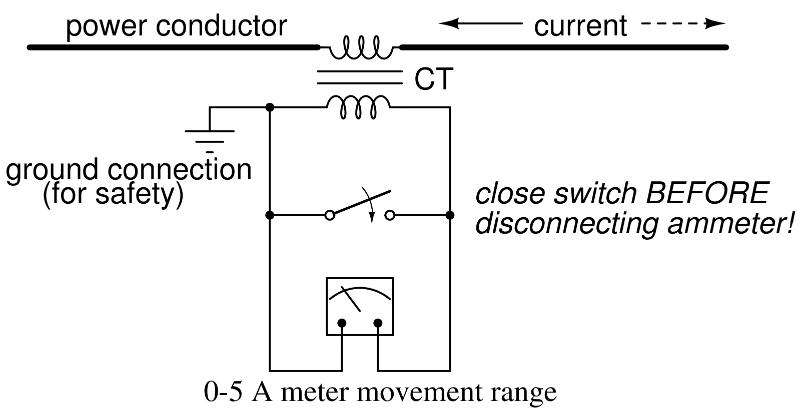
600 : 5 ratio *(for measuring up to 600 A line current)*

100 : 5 ratio *(for measuring up to 100 A line current)*

1k : 5 ratio *(for measuring up to 1000 A line current)*

The "donut" CT shown in the photograph has a ratio of 50 :5. That is, when the conductor through the center of the torus is carrying 50 amps of current (AC), there will be 5 amps of current induced in the CT's winding.

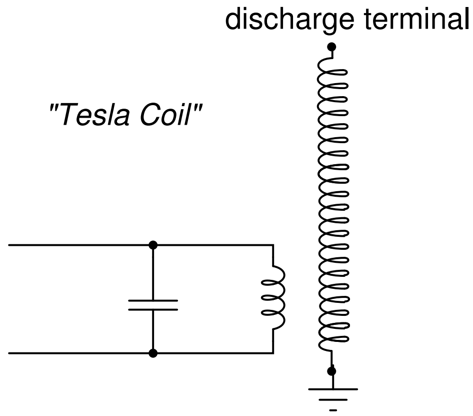
Because CTs are designed to be powering ammeters, which are low-impedance loads, and they are wound as voltage step-up transformers, they should never, *ever* be operated with an open-circuited secondary winding. Failure to heed this warning will result in the CT producing extremely high secondary voltages, dangerous to equipment and personnel alike. To facilitate maintenance of ammeter instrumentation, short-circuiting switches are often installed in parallel with the CT's secondary winding, to be closed whenever the ammeter is removed for service :



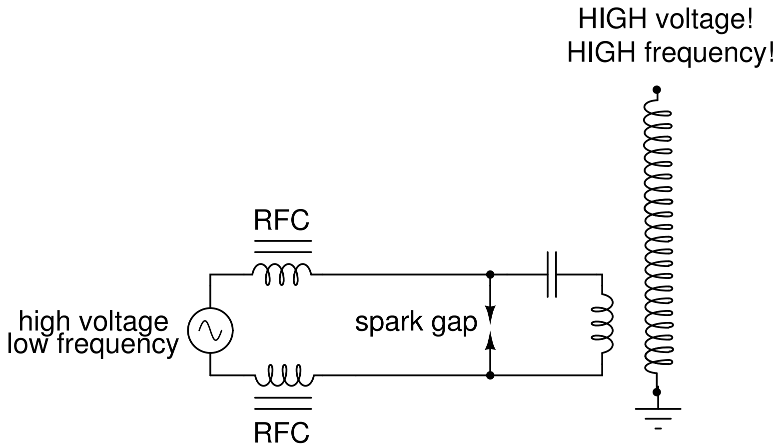
Though it may seem strange to *intentionally* short-circuit a power system component, it is perfectly proper and quite necessary when working with current transformers.

Another kind of special transformer, seen often in radio-frequency circuits, is the *air core* transformer. True to its name, an air core transformer has its windings wrapped around a nonmagnetic form, usually a hollow tube of some material. The degree of coupling (mutual inductance) between windings in such a transformer is many times less than that of an equivalent iron-core transformer, but the undesirable characteristics of a ferromagnetic core (eddy current losses, hysteresis, saturation, etc.) are completely eliminated. It is in high-frequency applications that these effects of iron cores are most problematic.

One notable example of air-core transformer is the *Tesla Coil*, named after the Serbian electrical genius Nikola Tesla, who was also the inventor of the rotating magnetic field AC motor, polyphase AC power systems, and many elements of radio technology. The Tesla Coil is a resonant, high-frequency step-up transformer used to produce extremely high voltages. One of Tesla's dreams was to employ his coil technology to distribute electric power without the need for wires, simply broadcasting it in the form of radio waves which could be received and conducted to loads by means of antennas. The basic schematic for a Tesla Coil looks like this :



The capacitor, in conjunction with the transformer's primary winding, forms a tank circuit. The secondary winding is wound in close proximity to the primary, usually around the same nonmagnetic form. Several options exist for "exciting" the primary circuit, the simplest being a high-voltage, low-frequency AC source and spark gap :



The purpose of the high-voltage, low-frequency AC power source is to "charge" the primary tank circuit. When the spark gap fires, its low impedance acts to complete the capacitor/primary coil tank circuit, allowing it to oscillate at its resonant frequency. The "RFC" inductors are "Radio Frequency Chokes," which act as high impedances to prevent the AC source from interfering with the oscillating tank circuit.

The secondary side of the Tesla coil transformer is also a tank circuit, relying on the parasitic (stray) capacitance existing between the discharge terminal and earth ground to complement the secondary winding's inductance. For optimum operation, this secondary tank circuit is tuned to the same resonant frequency as the primary circuit, with energy exchanged not only between capacitors and inductors during resonant oscillation, but also back-and-forth between primary and secondary windings. The visual results are spectacular :

Tesla Coils find application primarily as novelty devices, showing up in high school science fairs, basement workshops, and the occasional low budget science-fiction movie.

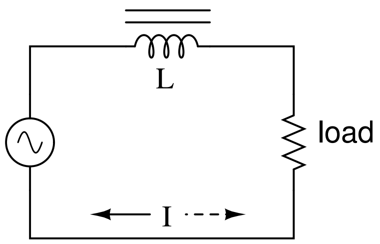
It should be noted that Tesla coils can be extremely dangerous devices. Burns caused by radio-frequency ("RF") current, like all electrical burns, can be very deep, unlike skin burns caused by contact with hot objects or flames. Although the high-frequency discharge of a Tesla coil has the curious property of being beyond the "shock perception" frequency of the human nervous system, this does not mean Tesla coils cannot hurt or even kill you! I strongly advise seeking the assistance of an experienced Tesla coil experimenter if you would embark on building one yourself.

So far, we've explored the transformer as a device for converting different levels of voltage, current, and even impedance from one circuit to another. Now we'll take a look at it as a completely different kind of device : one that allows a small electrical signal to exert *control* over a much larger quantity of electrical power. In this mode, a transformer acts as an *amplifier.*

The device I'm referring to is called a *saturable-core reactor*, or simply *saturable reactor.* Actually, it is not really a transformer at all, but rather a special kind of inductor whose inductance can be varied by the application of a DC current through a second winding wound around the same iron core. Like the ferroresonant transformer, the saturable reactor relies on the principle of magnetic saturation. When a material such as iron is completely saturated (that is, all its magnetic domains are lined up with the applied magnetizing force), additional increases in current through the magnetizing winding will not result in further increases of magnetic flux.

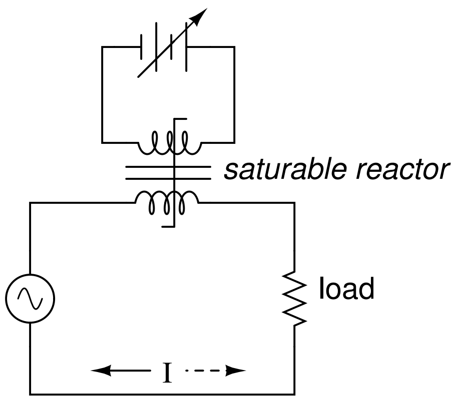
Now, inductance is the measure of how well an inductor opposes changes in current by developing a voltage in an opposing direction. The ability of an inductor to generate this opposing voltage is directly connected with the change in magnetic flux inside the inductor resulting from the change in current, and the number of winding turns in the inductor. If an inductor has a saturated core, no further magnetic flux will result from further increases in current, and so there will be no voltage induced in opposition to the change in current. In other words, an inductor loses its inductance (ability to oppose changes in current) when its core becomes magnetically saturated.

If an inductor's inductance changes, then its reactance (and impedance) to AC current changes as well. In a circuit with a constant voltage source, this will result in a change in current :



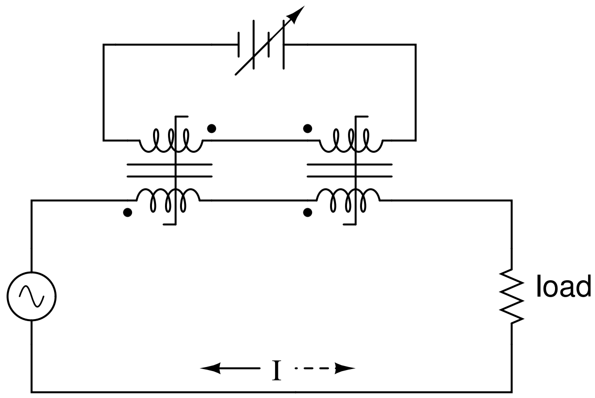
If L changes in inductance, Z_L will correspondingly change, thus changing the circuit current.

A saturable reactor capitalizes on this effect by forcing the core into a state of saturation with a strong magnetic field generated by current through another winding. The reactor's "power" winding is the one carrying the AC load current, and the "control" winding is one carrying a DC current strong enough to drive the core into saturation :



The strange-looking transformer symbol shown in the above schematic represents a saturable-core reactor, the upper winding being the DC control winding and the lower being the "power" winding through which the controlled AC current goes. Increased DC control current produces more magnetic flux in the reactor core, driving it closer to a condition of saturation, thus decreasing the power winding's inductance, decreasing its impedance, and increasing current to the load. Thus, the DC control current is able to exert *control* over the AC current delivered to the load.

The circuit shown would work, but it would not work very well. The first problem is the natural transformer action of the saturable reactor : AC current through the power winding will induce a voltage in the control winding, which may cause trouble for the DC power source. Also, saturable reactors tend to regulate AC power only in one direction : in one half of the AC cycle, the mmf's from both windings add; in the other half, they subtract. Thus, the core will have more flux in it during one half of the AC cycle than the other, and will saturate first in that cycle half, passing load current more easily in one direction than the other. Fortunately, both problems can be overcome with a little ingenuity :



Notice the placement of the phasing dots on the two reactors : the power windings are "in phase" while the control windings are "out of phase." If both reactors are identical, any voltage induced in the control windings by load current through the power windings will cancel out to zero at the battery terminals, thus eliminating the first problem mentioned. Furthermore, since the DC control current through both reactors produces magnetic fluxes in different directions through the reactor cores, one reactor will saturate more in one cycle of the AC power while the other reactor will saturate more in the other, thus equalizing the control action through each half-cycle so that the AC power is "throttled" symmetrically. This phasing of control windings can be accomplished with two separate reactors as shown, or in a single reactor design with intelligent layout of the windings and core.

Saturable reactor technology has even been miniaturized to the circuit-board level in compact packages more generally known as *magnetic amplifiers*. I personally find this to be fascinating : the effect of amplification (one electrical signal controlling another), normally requiring the use of physically fragile vacuum tubes or electrically "fragile" semiconductor devices, can be realized in a device both physically and electrically rugged. Magnetic amplifiers do have disadvantages over their more fragile counterparts, namely size, weight, nonlinearity, and bandwidth (frequency response), but their utter simplicity still commands a certain degree of appreciation, if not practical application.

Saturable-core reactors are less commonly known as "saturable-core inductors" or *transductors*.

- **REVIEW :**
- Transformers can be used to transform impedance as well as voltage and current. When this is done to improve power transfer to a load, it is called *impedance matching*.
- A *Potential Transformer* (PT) is a special instrument transformer designed to provide a precise voltage step-down ratio for voltmeters measuring high power system voltages.

- A *Current Transformer* (CT) is another special instrument transformer designed to step down the current through a power line to a safe level for an ammeter to measure.
- An *air-core* transformer is one lacking a ferromagnetic core.
- A *Tesla Coil* is a resonant, air-core, step-up transformer designed to produce very high AC voltages at high frequency.
- A *saturable reactor* is a special type of inductor, the inductance of which can be controlled by the DC current through a second winding around the same core. With enough DC current, the magnetic core can be saturated, decreasing the inductance of the power winding in a controlled fashion.

9.8 Practical considerations

9.8.1 Power capacity

As has already been observed, transformers must be well designed in order to achieve acceptable power coupling, tight voltage regulation, and low exciting current distortion. Also, transformers must be designed to carry the expected values of primary and secondary winding current without any trouble. This means the winding conductors must be made of the proper gauge wire to avoid any heating problems. An ideal transformer would have perfect coupling (no leakage inductance), perfect voltage regulation, perfectly sinusoidal exciting current, no hysteresis or eddy current losses, and wire thick enough to handle any amount of current. Unfortunately, the ideal transformer would have to be infinitely large and heavy to meet these design goals. Thus, in the business of *practical* transformer design, compromises must be made.

Additionally, winding conductor insulation is a concern where high voltages are encountered, as they often are in step-up and step-down power distribution transformers. Not only do the windings have to be well insulated from the iron core, but each winding has to be sufficiently insulated from the other in order to maintain electrical isolation between windings.

Respecting these limitations, transformers are rated for certain levels of primary and secondary winding voltage and current, though the current rating is usually derived from a volt-amp (VA) rating assigned to the transformer. For example, take a step-down transformer with a primary voltage rating of 120 volts, a secondary voltage rating of 48 volts, and a VA rating of 1 kVA (1000 VA). The maximum winding currents can be determined as such :

$$\frac{1000 \text{ VA}}{120 \text{ V}} = 8.333 \text{ A} \quad \textit{(maximum primary winding current)}$$

$$\frac{1000 \text{ VA}}{48 \text{ V}} = 20.833 \text{ A} \quad \textit{(maximum secondary winding current)}$$

Sometimes windings will bear current ratings in amps, but this is typically seen on small transformers. Large transformers are almost always rated in terms of winding voltage and VA or kVA.

9.8.2 Energy losses

When transformers transfer power, they do so with a minimum of loss. As it was stated earlier, modern power transformer designs typically exceed 95% efficiency. It is good to know where some of this lost power goes, however, and what causes it to be lost.

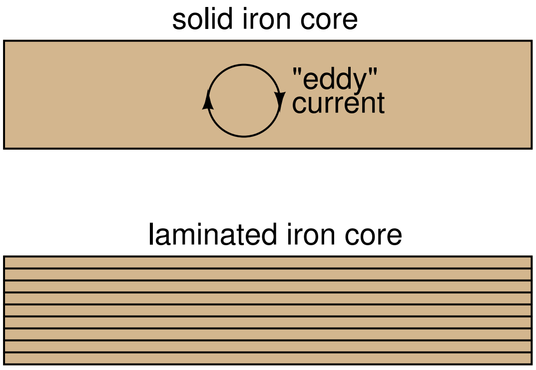
There is, of course, power lost due to resistance of the wire windings. Unless superconducting wires are used, there will always be power dissipated in the form of heat through the resistance of current-carrying conductors. Because transformers require such long lengths of wire, this loss can be a significant factor. Increasing the gauge of the winding wire is one way to minimize this loss, but only with substantial increases in cost, size, and weight.

Resistive losses aside, the bulk of transformer power loss is due to magnetic effects in the core. Perhaps the most significant of these "core losses" is *eddy-current loss*, which is resistive power dissipation due to the passage of induced currents through the iron of the core. Because iron is a conductor of electricity as well as being an excellent "conductor" of magnetic flux, there will be currents induced in the iron just as there are currents induced in the secondary windings from the alternating magnetic field. These induced currents – as described by the perpendicularity clause of Faraday's Law – tend to circulate through the cross-section of the core perpendicularly to the primary winding turns. Their circular motion gives them their unusual name : like eddies in a stream of water that circulate rather than move in straight lines.

Iron is a fair conductor of electricity, but not as good as the copper or aluminum from which wire windings are typically made. Consequently, these "eddy currents" must overcome significant electrical resistance as they circulate through the core. In overcoming the resistance offered by the iron, they dissipate power in the form of heat. Hence, we have a source of inefficiency in the transformer that is difficult to eliminate.

This phenomenon is so pronounced that it is often exploited as a means of heating ferrous (iron-containing) materials. The following photograph shows an "induction heating" unit raising the temperature of a large pipe section. Loops of wire covered by high-temperature insulation encircle the pipe's circumference, inducing eddy currents within the pipe wall by electromagnetic induction. In order to maximize the eddy current effect, high-frequency alternating current is used rather than power line frequency (60 Hz). The box units at the right of the picture produce the high-frequency AC and control the amount of current in the wires to stabilize the pipe temperature at a pre-determined "set-point."

The main strategy in mitigating these wasteful eddy currents in transformer cores is to form the iron core in sheets, each sheet covered with an insulating varnish so that the core is divided up into thin slices. The result is very little width in the core for eddy currents to circulate in :



Laminated cores like the one shown here are standard in almost all low-frequency transformers. Recall from the photograph of the transformer cut in half that the iron core was composed of many thin sheets rather than one solid piece. Eddy current losses increase with frequency, so transformers designed to run on higher-frequency power (such as 400 Hz, used in many military and aircraft applications) must use thinner laminations to keep the losses down to a respectable minimum. This has the undesirable effect of increasing the manufacturing cost of the transformer.

Another, similar technique for minimizing eddy current losses which works better for high-frequency applications is to make the core out of iron powder instead of thin iron sheets. Like the lamination sheets, these granules of iron are individually coated in an electrically insulating material, which makes the core nonconductive except for within the width of each granule. Powdered iron cores are often found in transformers handling radio-frequency currents.

Another "core loss" is that of magnetic *hysteresis*. All ferromagnetic materials tend to retain some degree of magnetization after exposure to an external magnetic field. This tendency to stay magnetized is called "hysteresis," and it takes a certain investment in energy to overcome this opposition to change every time the magnetic field produced by the primary winding changes polarity (twice per AC cycle). This type of loss can be mitigated through good core material selection (choosing a core alloy with low hysteresis, as evidenced by a "thin" B/H hysteresis curve), and designing the core for minimum flux density (large cross-sectional area).

Transformer energy losses tend to worsen with increasing frequency. The skin effect within winding conductors reduces the available cross-sectional area for electron flow, thereby increasing effective resistance as the frequency goes up and creating more power lost through resistive dissipation. Magnetic core losses are also exaggerated with higher frequencies, eddy currents and hysteresis effects becoming more severe. For this reason, transformers of significant size are designed to operate efficiently in a limited range of frequencies. In most power distribution systems where the line frequency is very stable, one would think excessive frequency would never pose a problem. Unfortunately it does, in the form of harmonics created by nonlinear loads.

As we've seen in earlier chapters, nonsinusoidal waveforms are equivalent to additive series of multiple sinusoidal waveforms at different amplitudes and frequencies. In power systems, these other frequencies are whole-number multiples of the fundamental (line) frequency, meaning that they will always be higher, not lower, than the design frequency of the transformer. In significant measure, they can cause severe transformer overheating. Power transformers can be engineered to handle certain levels of power system harmonics, and this capability is sometimes denoted with a "K factor" rating.

9.8.3 Stray capacitance and inductance

Aside from power ratings and power losses, transformers often harbor other undesirable limitations which circuit designers must be made aware of. Like their simpler counterparts – inductors – transformers exhibit capacitance due to the insulation dielectric between conductors : from winding to winding, turn to turn (in a single winding), and winding to core. Usually this capacitance is of no concern in a power application, but small signal applications (especially those of high frequency) may not tolerate this quirk well. Also, the effect of having capacitance along with the windings' designed inductance gives transformers the ability to *resonate* at a particular frequency, definitely a design concern in signal applications where the applied frequency may reach this point (usually the resonant frequency of a power transformer is well beyond the frequency of the AC power it was designed to operate on).

Flux containment (making sure a transformer's magnetic flux doesn't escape so as to interfere with another device, and making sure other devices' magnetic flux is shielded from the transformer core) is another concern shared both by inductors and transformers.

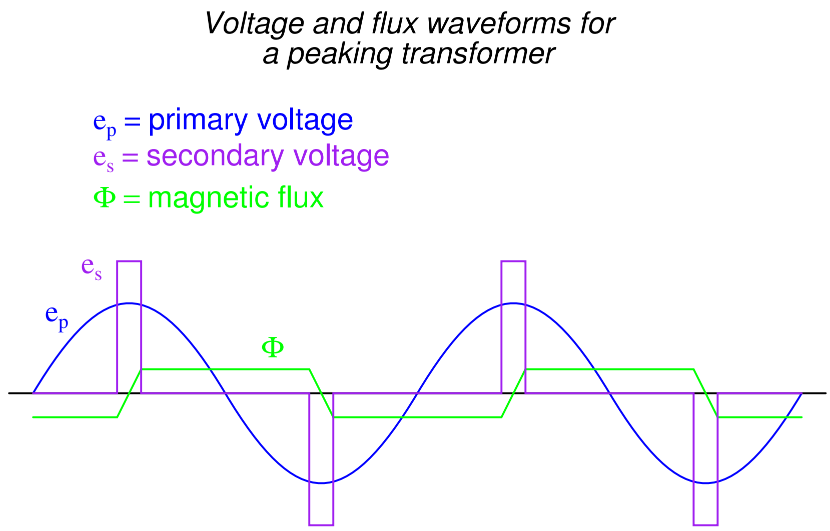
Closely related to the issue of flux containment is leakage inductance. We've already seen the detrimental effects of leakage inductance on voltage regulation with SPICE simulations early in this chapter. Because leakage inductance is equivalent to an inductance connected in series with the transformer's winding, it manifests itself as a series impedance with the load. Thus, the more current drawn by the load, the less voltage available at the secondary winding terminals. Usually, good voltage regulation is desired in transformer design, but there are exceptional applications. As was stated before, discharge lighting circuits require a step-up transformer with "loose" (poor) voltage regulation to ensure reduced voltage after the establishment of an arc through the lamp. One way to meet this design criterion is to engineer the transformer with flux leakage paths for magnetic flux to bypass the secondary winding(s). The resulting leakage flux will produce leakage inductance, which will in turn produce the poor regulation needed for discharge lighting.

9.8.4 Core saturation

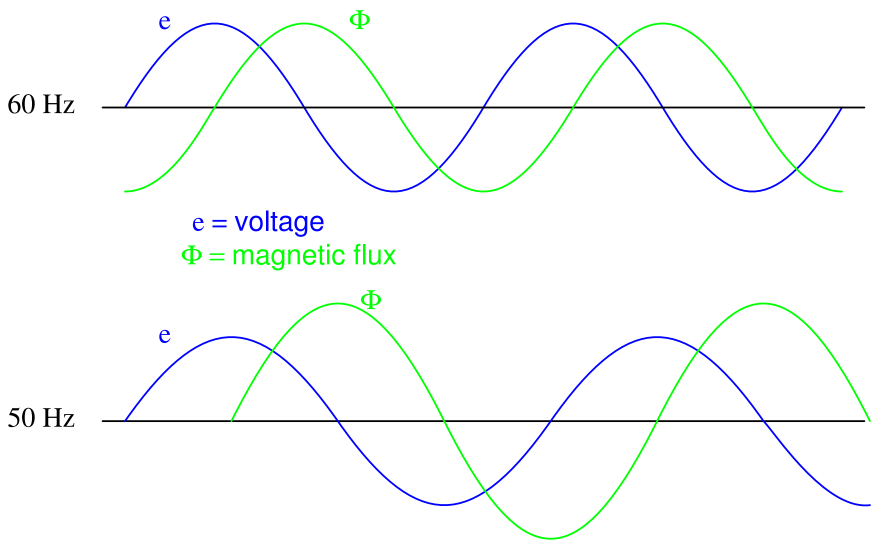
Transformers are also constrained in their performance by the magnetic flux limitations of the core. For ferromagnetic core transformers, we must be mindful of the saturation limits of the core. Remember that ferromagnetic materials cannot support infinite magnetic flux densities : they tend to "saturate" at a certain level (dictated by the material and core dimensions), meaning that further increases in magnetic field force (mmf) do not result in proportional increases in magnetic field flux (Φ).

When a transformer's primary winding is overloaded from excessive applied voltage, the core flux may reach saturation levels during peak moments of the AC sinewave cycle. If this happens, the voltage induced in the secondary winding will no longer match the wave-shape as the voltage powering the primary coil. In other words, the overloaded transformer will *distort* the waveshape from primary to secondary windings, creating harmonics in the secondary winding's output. As we discussed before, harmonic content in AC power systems typically causes problems.

Special transformers known as *peaking transformers* exploit this principle to produce brief voltage pulses near the peaks of the source voltage waveform. The core is designed to saturate quickly and sharply, at voltage levels well below peak. This results in a severely cropped sine-wave flux waveform, and secondary voltage pulses only when the flux is changing (below saturation levels) :



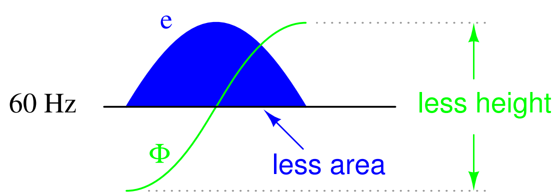
Another cause of abnormal transformer core saturation is operation at frequencies lower than normal. For example, if a power transformer designed to operate at 60 Hz is forced to operate at 50 Hz instead, the flux must reach greater peak levels than before in order to produce the same opposing voltage needed to balance against the source voltage. This is true even if the source voltage is the same as before.

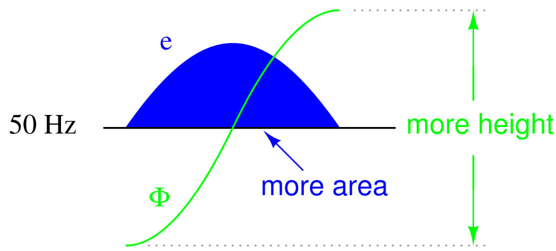


Since instantaneous winding voltage is proportional to the instantaneous magnetic flux's *rate of change* in a transformer, a voltage waveform reaching the same peak value, but taking a longer amount of time to complete each half-cycle, demands that the flux maintain the same rate of change as before, but for longer periods of time. Thus, if the flux has to climb at the same rate as before, but for longer periods of time, it will climb to a greater peak value.

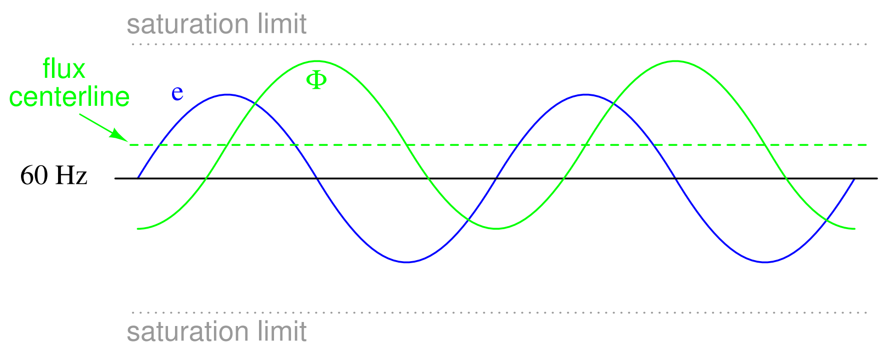
Mathematically, this is another example of calculus in action. Because the voltage is proportional to the flux's rate-of-change, we say that the voltage waveform is the *derivative* of the flux waveform, "derivative" being that calculus operation defining one mathematical function (waveform) in terms of the rate-of-change of another. If we take the opposite perspective, though, and relate the original waveform to its derivative, we may call the original waveform the *integral* of the derivative waveform. In this case, the voltage waveform is the derivative of the flux waveform, and the flux waveform is the integral of the voltage waveform.

The integral of any mathematical function is proportional to the area accumulated underneath the curve of that function. Since each half-cycle of the 50 Hz waveform accumulates more area between it and the zero line of the graph than the 60 Hz waveform will – and we know that the magnetic flux is the integral of the voltage – the flux will attain higher values :





Yet another cause of transformer saturation is the presence of DC current in the primary winding. Any amount of DC voltage dropped across the primary winding of a transformer will cause additional magnetic flux in the core. This additional flux "bias" or "offset" will push the alternating flux waveform closer to saturation in one half-cycle than the other :

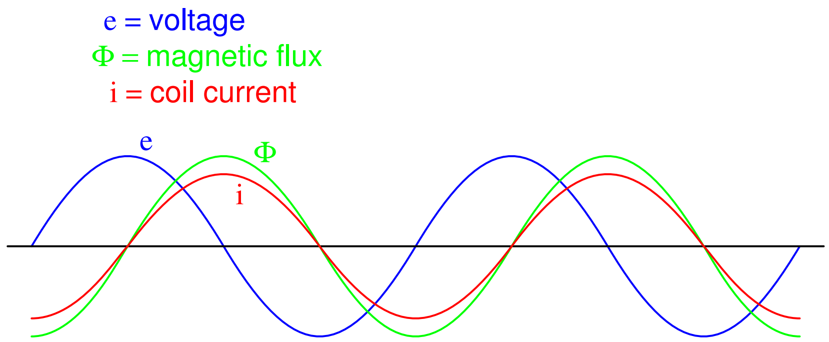


For most transformers, core saturation is a very undesirable effect, and it is avoided through good design : engineering the windings and core so that magnetic flux densities remain well below the saturation levels. This ensures that the relationship between mmf and Φ is more linear throughout the flux cycle, which is good because it makes for less distortion in the magnetization current waveform. Also, engineering the core for low flux densities provides a safe margin between the normal flux peaks and the core saturation limits to accommodate occasional, abnormal conditions such as frequency variation and DC offset.

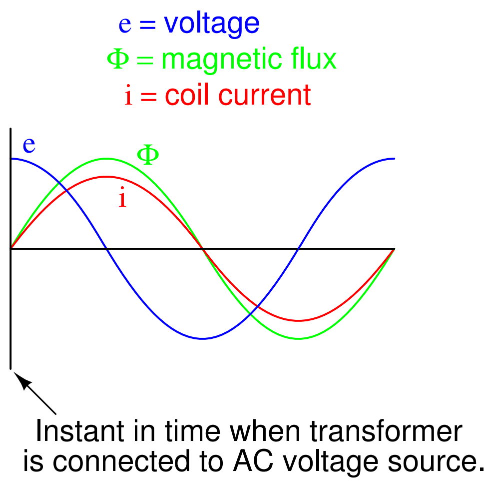
9.8.5 Inrush current

When a transformer is initially connected to a source of AC voltage, there may be a substantial surge of current through the primary winding called *inrush current*. This is analogous to the inrush current exhibited by an electric motor that is started up by sudden connection to a power source, although transformer inrush is caused by a different phenomenon.

We know that the rate of change of instantaneous flux in a transformer core is proportional to the instantaneous voltage drop across the primary winding. Or, as stated before, the voltage waveform is the derivative of the flux waveform, and the flux waveform is the integral of the voltage waveform. In a continuously-operating transformer, these two waveforms are phase-shifted by 90^o. Since flux (Φ) is proportional to the magnetomotive force (mmf) in the core, and the mmf is proportional to winding current, the current waveform will be in-phase with the flux waveform, and both will be lagging the voltage waveform by 90^o :

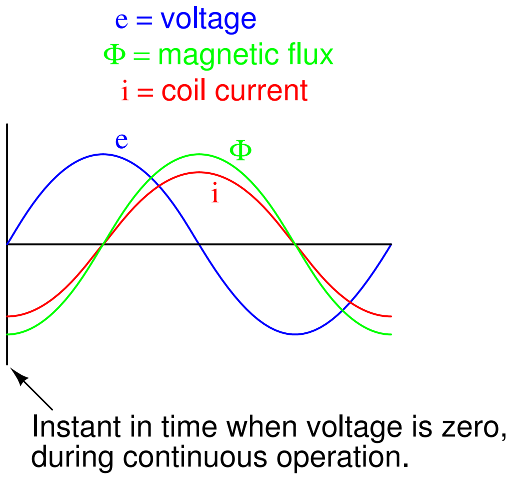


Let us suppose that the primary winding of a transformer is suddenly connected to an AC voltage source at the exact moment in time when the instantaneous voltage is at its positive peak value. In order for the transformer to create an opposing voltage drop to balance against this applied source voltage, a magnetic flux of rapidly increasing value must be generated. The result is that winding current increases rapidly, but actually no more rapidly than under normal conditions :

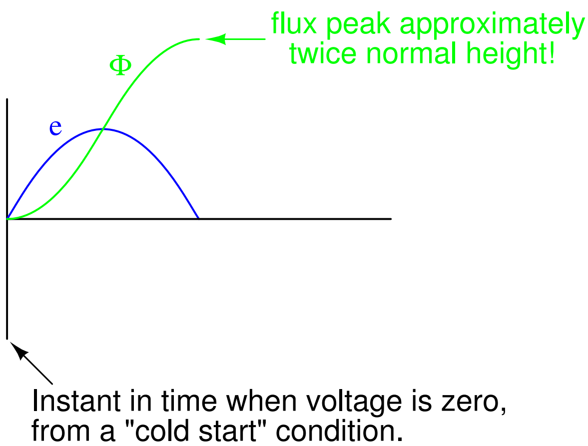


Both core flux and coil current start from zero and build up to the same peak values experienced during continuous operation. Thus, there is no "surge" or "inrush" or current in this scenario.

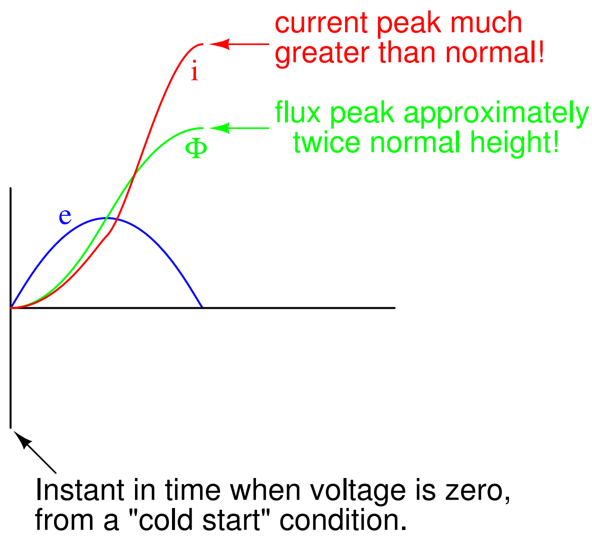
Alternatively, let us consider what happens if the transformer's connection to the AC voltage source occurs at the exact moment in time when the instantaneous voltage is at zero. During continuous operation (when the transformer has been powered for quite some time), this is the point in time where both flux and winding current are at their negative peaks, experiencing zero rate-of-change ($d\Phi/dt = 0$ and $di/dt = 0$). As the voltage builds to its positive peak, the flux and current waveforms build to their maximum positive rates-of-change, and on upward to their positive peaks as the voltage descends to a level of zero :



A significant difference exists, however, between continuous-mode operation and the sudden starting condition assumed in this scenario : during continuous operation, the flux and current levels were at their negative peaks when voltage was at its zero point; in a transformer that has been sitting idle, however, both magnetic flux and winding current should start at *zero*. When the magnetic flux increases in response to a rising voltage, it will increase from zero upwards, not from a previously negative (magnetized) condition as we would normally have in a transformer that's been powered for awhile. Thus, in a transformer that's just "starting," the flux will reach approximately twice its normal peak magnitude as it "integrates" the area under the voltage waveform's first half-cycle :



In an ideal transformer, the magnetizing current would rise to approximately twice its normal peak value as well, generating the necessary mmf to create this higher-than-normal flux. However, most transformers aren't designed with enough of a margin between normal flux peaks and the saturation limits to avoid saturating in a condition like this, and so the core will almost certainly saturate during this first half-cycle of voltage. During saturation, disproportionate amounts of mmf are needed to generate magnetic flux. This means that winding current, which creates the mmf to cause flux in the core, will disproportionately rise to a value *easily exceeding* twice its normal peak :

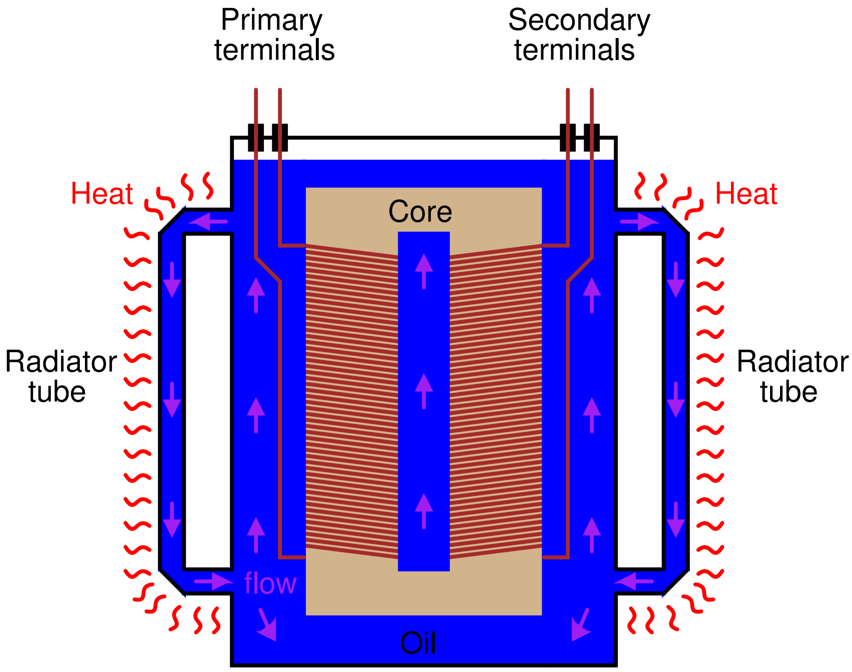


This is the mechanism causing inrush current in a transformer's primary winding when connected to an AC voltage source. As you can see, the magnitude of the inrush current strongly depends on the exact time that electrical connection to the source is made. If the transformer happens to have some residual magnetism in its core at the moment of connection to the source, the inrush could be even more severe. Because of this, transformer overcurrent protection devices are usually of the "slow-acting" variety, so as to tolerate current surges such as this without opening the circuit.

9.8.6 Heat and Noise

In addition to unwanted electrical effects, transformers may also exhibit undesirable physical effects, the most notable being the production of heat and noise. Noise is primarily a nuisance effect, but heat is a potentially serious problem because winding insulation will be damaged if allowed to overheat. Heating may be minimized by good design, ensuring that the core does not approach saturation levels, that eddy currents are minimized, and that the windings are not overloaded or operated too close to maximum ampacity.

Large power transformers have their core and windings submerged in an oil bath to transfer heat and muffle noise, and also to displace moisture which would otherwise compromise the integrity of the winding insulation. Heat-dissipating "radiator" tubes on the outside of the transformer case provide a convective oil flow path to transfer heat from the transformer's core to ambient air :



Oil-less, or "dry," transformers are often rated in terms of maximum operating temperature "rise" (temperature increase beyond ambient) according to a letter-class system : A, B, F, or H. These letter codes are arranged in order of lowest heat tolerance to highest :

- **Class A :** No more than 55^o Celsius winding temperature rise, at 40^o Celsius (maximum) ambient air temperature.
- **Class B :** No more than 80^o Celsius winding temperature rise, at 40^o Celsius (maximum)ambient air temperature.
- **Class F :** No more than 115^o Celsius winding temperature rise, at 40^o Celsius (maximum)ambient air temperature.
- **Class H :** No more than 150^o Celsius winding temperature rise, at 40^o Celsius (maximum)ambient air temperature.

Audible noise is an effect primarily originating from the phenomenon of *magnetostriction* : the slight change of length exhibited by a ferromagnetic object when magnetized. The familiar "hum" heard around large power transformers is the sound of the iron core expanding and contracting at 120 Hz (twice the system frequency, which is 60 Hz in the United States) – one cycle of core contraction and expansion for every peak of the magnetic flux waveform – plus noise created by mechanical forces between primary and secondary windings. Again, maintaining low magnetic flux levels in the core is the key to minimizing this effect, which explains why ferroresonant transformers – which must operate in saturation for a large portion of the current waveform – operate both hot and noisy.

Another noise-producing phenomenon in power transformers is the physical reaction force between primary and secondary windings when heavily loaded. If the secondary winding is open-circuited, there will be no current through it, and consequently no magneto-motive force (mmf) produced by it. However, when the secondary is "loaded" (current supplied to a load), the winding generates an mmf, which becomes counteracted by a "reflected" mmf in the primary winding to prevent core flux levels from changing. These opposing mmf's generated between primary and secondary windings as a result of secondary (load) current produce a repulsive, physical force between the windings which will tend to make them vibrate. Transformer designers have to consider these physical forces in the construction of the winding coils, to ensure there is adequate mechanical support to handle the stresses. Under heavy load conditions, though, these stresses may be great enough to cause audible noise to emanate from the transformer.

- **REVIEW :**
- Power transformers are limited in the amount of power they can transfer from primary to secondary winding(s). Large units are typically rated in VA (volt-amps) or kVA (kilo volt-amps).
- Resistance in transformer windings contributes to inefficiency, as current will dissipate heat, wasting energy.
- Magnetic effects in a transformer's iron core also contribute to inefficiency. Among the effects are *eddy currents* (circulating induction currents in the iron core) and *hysteresis* (power lost due to overcoming the tendency of iron to magnetize in a particular direction).
- Increased frequency results in increased power losses within a power transformer. The presence of harmonics in a power system is a source of frequencies significantly higher than normal, which may cause overheating in large transformers.
- Both transformers and inductors harbor certain unavoidable amounts of capacitance due to wire insulation (dielectric) separating winding turns from the iron core and from each other. This capacitance can be significant enough to give the transformer a natural *resonant frequency*, which can be problematic in signal applications.
- *Leakage inductance* is caused by magnetic flux not being 100% coupled between windings in a transformer. Any flux not involved with *transferring* energy from one winding to another will store and release energy, which is how (self-) inductance works. Leakage inductance tends to worsen a transformer's voltage regulation (secondary voltage "sags" more for a given amount of load current).
- Magnetic *saturation* of a transformer core may be caused by excessive primary voltage, operation at too low of a frequency, and/or by the presence of a DC current in any of the windings. Saturation may be minimized or avoided by conservative design, which provides an adequate margin of safety between peak magnetic flux density values and the saturation limits of the core.
- Transformers often experience significant *inrush currents* when initially connected to an AC voltage source. Inrush current is most severe when connection to the AC source is made at the moment instantaneous source voltage is zero.

- Noise is a common phenomenon exhibited by transformers – especially power transformers – and is primarily caused by *magnetostriction* of the core. Physical forces causing winding vibration may also generate noise under conditions of heavy (high current) secondary winding load.

9.9 Contributors

Les contributeurs de ce chapitre sont listés dans l'ordre chronologique de leurs contributions, depuis le plus récent jusqu'au premier. Voyez l'Annexe 2 (Liste des contributeur) pour les dates et les informations de contact.

Bart Anderson (January 2004) : Corrected conceptual errors regarding Tesla coil operation and safety.

Jason Starck (June 2000) : HTML document formatting, which led to a much better-looking second edition.

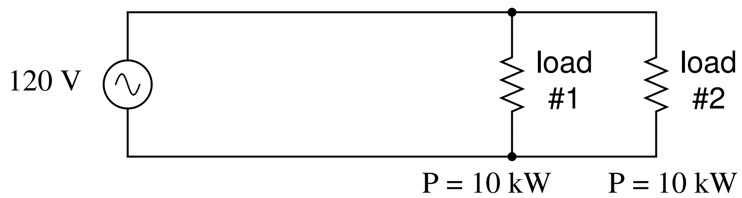
Chapter 10

POLYPHASE AC CIRCUITS

10.1 Single-phase power systems

Depicted above is a very simple AC circuit. If the load resistor's power dissipation were substantial, we might call this a "power circuit" or "power system" instead of regarding it as just a regular circuit. The distinction between a "power circuit" and a "regular circuit" may seem arbitrary, but the practical concerns are definitely not.

One such concern is the size and cost of wiring necessary to deliver power from the AC source to the load. Normally, we do not give much thought to this type of concern if we're merely analyzing a circuit for the sake of learning about the laws of electricity. However, in the real world it can be a major concern. If we give the source in the above circuit a voltage value and also give power dissipation values to the two load resistors, we can determine the wiring needs for this particular circuit :



$$I = \frac{P}{E}$$

$$I = \frac{10\ \text{kW}}{120\ \text{V}}$$

$$I = 83.33\ \text{A} \qquad \textit{(for each load resistor)}$$

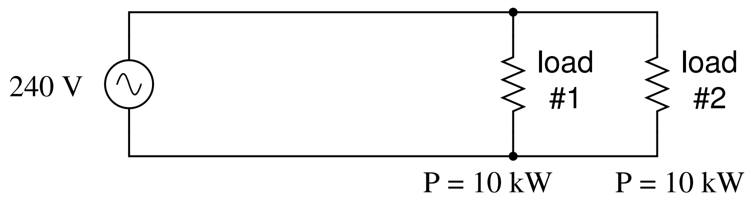
$$I_{total} = I_{load\#1} + I_{load\#2} \qquad P_{total} = (10\ \text{kW}) + (10\ \text{kW})$$

$$I_{total} = (83.33\ \text{A}) + (83.33\ \text{A}) \qquad P_{total} = 20\ \text{kW}$$

$$I_{total} = 166.67\ \text{A}$$

83.33 amps for each load resistor adds up to 166.66 amps total circuit current. This is no small amount of current, and would necessitate copper wire conductors of at least 1/0 gage. Such wire is well over 1/4 inch in diameter, weighing over 300 pounds per thousand feet. Bear in mind that copper is not cheap either! It would be in our best interest to find ways to minimize such costs if we were designing a power system with long conductor lengths.

One way to do this would be to increase the voltage of the power source and use loads built to dissipate 10 kW each at this higher voltage. The loads, of course, would have to have greater resistance values to dissipate the same power as before (10 kW each) at a greater voltage than before. The advantage would be less current required, permitting the use of smaller, lighter, and cheaper wire :



$$I = \frac{P}{E}$$

$$I = \frac{10\ \text{kW}}{240\ \text{V}}$$

$$I = 41.67\ \text{A} \quad \textit{(for each load resistor)}$$

$$I_{total} = I_{load\#1} + I_{load\#2} \qquad P_{total} = (10\ \text{kW}) + (10\ \text{kW})$$

$$I_{total} = (41.67\ \text{A}) + (41.67\ \text{A}) \qquad P_{total} = 20\ \text{kW}$$

$$I_{total} = 83.33\ \text{A}$$

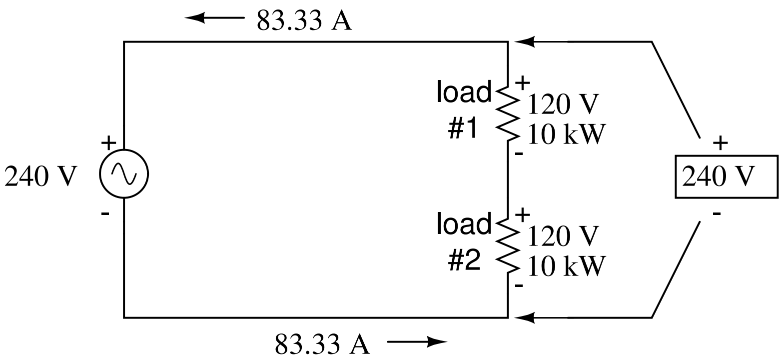
Now our *total* circuit current is 83.33 amps, half of what it was before. We can now use number 4 gage wire, which weighs less than half of what 1/0 gage wire does per unit length. This is a considerable reduction in system cost with no degradation in performance. This is why power distribution system designers elect to transmit electric power using very high voltages (many thousands of volts) : to capitalize on the savings realized by the use of smaller, lighter, cheaper wire.

However, this solution is not without disadvantages. Another practical concern with power circuits is the danger of electric shock from high voltages. Again, this is not usually the sort of thing we concentrate on while learning about the laws of electricity, but it is a very valid concern in the real world, especially when large amounts of power are being dealt with. The gain in efficiency realized by stepping up the circuit voltage presents us with increased danger of electric shock. Power distribution companies tackle this problem by stringing their power lines along high poles or towers, and insulating the lines from the supporting structures with large, porcelain insulators.

At the point of use (the electric power customer), there is still the issue of what voltage to use for powering loads. High voltage gives greater system efficiency by means of reduced conductor current, but it might not always be practical to keep power wiring out of reach at the point of use the way it can be elevated out of reach in distribution systems. This tradeoff between efficiency and danger is one that European power system designers have decided to risk, all their households and appliances operating at a nominal voltage of 240 volts instead of 120 volts as it is in North America. That is why tourists from America visiting Europe must carry small step-down transformers for their portable appliances, to step the 240 VAC (volts AC) power down to a more suitable 120 VAC.

Is there any way to realize the advantages of both increased efficiency and reduced safety hazard at the same time? One solution would be to install step-down transformers at the end-point of power use, just as the American tourist must do while in Europe. However, this would be expensive and inconvenient for anything but very small loads (where the transformers can be built cheaply) or very large loads (where the expense of thick copper wires would exceed the expense of a transformer).

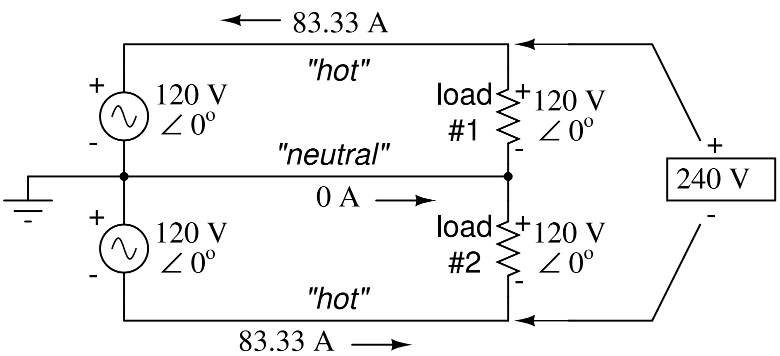
An alternative solution would be to use a higher voltage supply to provide power to two lower voltage loads in series. This approach combines the efficiency of a high-voltage system with the safety of a low-voltage system :



Notice the polarity markings (+ and -) for each voltage shown, as well as the unidirectional arrows for current. For the most part, I've avoided labeling "polarities" in the AC circuits we've been analyzing, even though the notation is valid to provide a frame of reference for phase. In later sections of this chapter, phase relationships will become very important, so I'm introducing this notation early on in the chapter for your familiarity.

The current through each load is the same as it was in the simple 120 volt circuit, but the currents are not additive because the loads are in series rather than parallel. The voltage across each load is only 120 volts, not 240, so the safety factor is better. Mind you, we still have a full 240 volts across the power system wires, but *each load* is operating at a reduced voltage. If anyone is going to get shocked, the odds are that it will be from coming into contact with the conductors of a particular load rather than from contact across the main wires of a power system.

There's only one disadvantage to this design : the consequences of one load failing open, or being turned off (assuming each load has a series on/off switch to interrupt current) are not good. Being a series circuit, if either load were to open, current would stop in the other load as well. For this reason, we need to modify the design a bit :



$$E_{total} = (120 \text{ V} \angle 0^o) + (120 \text{ V} \angle 0^o)$$

$$E_{total} = 240 \text{ V} \angle 0^o$$

$$I = \frac{P}{E}$$

$$I = \frac{10 \text{ kW}}{120 \text{ V}}$$

$$I = 83.33 \text{ A} \quad \textit{(for each load resistor)}$$

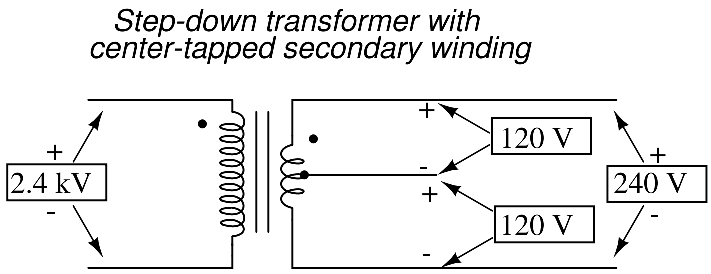
$$P_{total} = (10 \text{ kW}) + (10 \text{ kW})$$

$$P_{total} = 20 \text{ kW}$$

Instead of a single 240 volt power supply, we use two 120 volt supplies (in phase with each other!) in series to produce 240 volts, then run a third wire to the connection point between the loads to handle the eventuality of one load opening. This is called a *split-phase* power system. Three smaller wires are still cheaper than the two wires needed with the simple parallel design, so we're still ahead on efficiency. The astute observer will note that the neutral wire only has to carry the *difference* of current between the two loads back to the source. In the above case, with perfectly "balanced" loads consuming equal amounts of power, the neutral wire carries zero current.

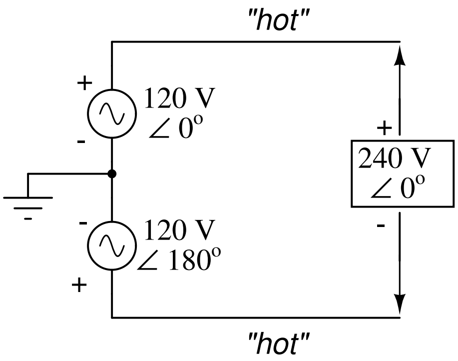
Notice how the neutral wire is connected to earth ground at the power supply end. This is a common feature in power systems containing "neutral" wires, since grounding the neutral wire ensures the least possible voltage at any given time between any "hot" wire and earth ground.

An essential component to a split-phase power system is the dual AC voltage source. Fortunately, designing and building one is not difficult. Since most AC systems receive their power from a step-down transformer anyway (stepping voltage down from high distribution levels to a user-level voltage like 120 or 240), that transformer can be built with a center-tapped secondary winding :



If the AC power comes directly from a generator (alternator), the coils can be similarly center-tapped for the same effect. The extra expense to include a center-tap connection in a transformer or alternator winding is minimal.

Here is where the (+) and (-) polarity markings really become important. This notation is often used to reference the phasings of *multiple* AC voltage sources, so it is clear whether they are aiding ("boosting") each other or opposing ("bucking") each other. If not for these polarity markings, phase relations between multiple AC sources might be very confusing. Note that the split-phase sources in the schematic (each one 120 volts $\angle$ 0^o), with polarity marks (+) to (-) just like series-aiding batteries can alternatively be represented as such :



To mathematically calculate voltage between "hot" wires, we must *subtract* voltages, because their polarity marks show them to be opposed to each other :

Polar	*Rectangular*
$120 \angle 0^o$	$120 + j0$ V
$- \; 120 \angle 180^o$	$- (-120 + j0)$ V
$240 \angle 0^o$	$240 + j0$ V

If we mark the two sources' common connection point (the neutral wire) with the same polarity mark (-), we must express their relative phase shifts as being 180^o apart. Otherwise, we'd be denoting two voltage sources in direct opposition with each other, which would give 0 volts between the two "hot" conductors. Why am I taking the time to elaborate on polarity marks and phase angles? It will make more sense in the next section!

Power systems in American households and light industry are most often of the split-phase variety, providing so-called 120/240 VAC power. The term "split-phase" merely refers to the split-voltage supply in such a system. In a more general sense, this kind of AC power supply is called *single phase* because both voltage waveforms are in phase, or in step, with each other.

The term "single phase" is a counterpoint to another kind of power system called "polyphase" which we are about to investigate in detail. Apologies for the long introduction leading up to the title-topic of this chapter. The advantages of polyphase power systems are more obvious if one first has a good understanding of single phase systems.

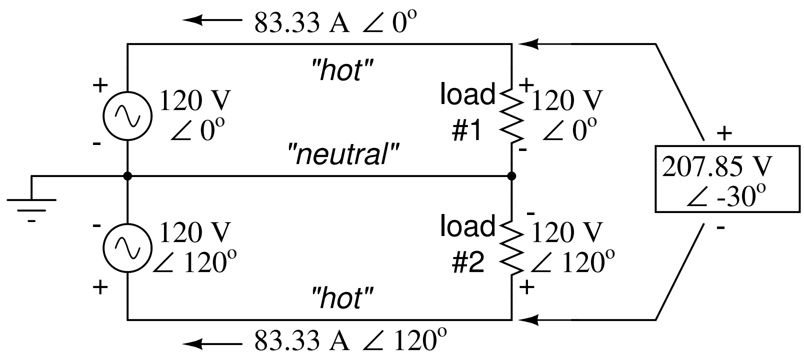
- **REVIEW :**
- *Single phase* power systems are defined by having an AC source with only one voltage waveform.
- A *split-phase* power system is one with multiple (in-phase) AC voltage sources connected in series, delivering power to loads at more than one voltage, with more than two wires. They are used primarily to achieve balance between system efficiency (low conductor currents) and safety (low load voltages).
- Split-phase AC sources can be easily created by center-tapping the coil windings of transformers or alternators.

10.2 Three-phase power systems

Split-phase power systems achieve their high conductor efficiency *and* low safety risk by splitting up the total voltage into lesser parts and powering multiple loads at those lesser voltages, while drawing currents at levels typical of a full-voltage system. This technique, by the way, works just as well for DC power systems as it does for single-phase AC systems. Such systems are usually referred to as *three-wire* systems rather than *split-phase* because "phase" is a concept restricted to AC.

But we know from our experience with vectors and complex numbers that AC voltages don't always add up as we think they would if they are out of phase with each other. This principle, applied to power systems, can be put to use to make power systems with even greater conductor efficiencies and lower shock hazard than with split-phase.

Suppose that we had two sources of AC voltage connected in series just like the split-phase system we saw before, except that each voltage source was 120^o out of phase with the other :



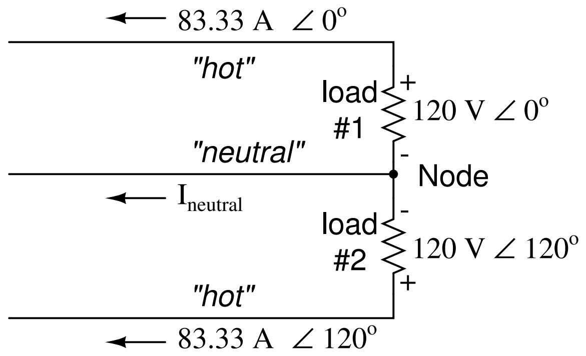
Since each voltage source is 120 volts, and each load resistor is connected directly in parallel with its respective source, the voltage across each load *must* be 120 volts as well. Given load currents of 83.33 amps, each load must still be dissipating 10 kilowatts of power. However, voltage between the two "hot" wires is not 240 volts ($120 \angle 0^o$ - $120 \angle 180^o$) because the phase difference between the two sources is not 180^o. Instead, the voltage is :

$$E_{total} = (120\text{ V} \angle 0^o) - (120\text{ V} \angle 120^o)$$

$$E_{total} = 207.85\text{ V} \angle -30^o$$

Nominally, we say that the voltage between "hot" conductors is 208 volts (rounding up), and thus the power system voltage is designated as 120/208.

If we calculate the current through the "neutral" conductor, we find that it is *not* zero, even with balanced load resistances. Kirchhoff's Current Law tells us that the currents entering and exiting the node between the two loads must be zero :



$$-I_{load\#1} - I_{load\#2} - I_{neutral} = 0$$

$$-I_{neutral} = I_{load\#1} + I_{load\#2}$$

$$I_{neutral} = -I_{load\#1} - I_{load\#2}$$

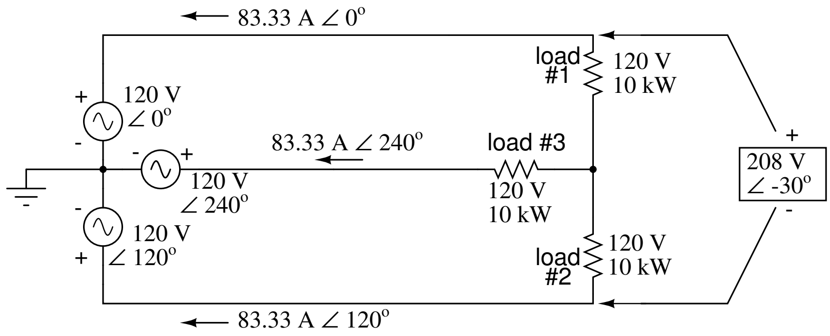
$$I_{neutral} = -(83.33\text{ A} \angle 0^o) - (83.33\text{ A} \angle 120^0)$$

$$I_{neutral} = 83.33\text{ A} \angle 240^o \quad \textit{or} \quad 83.33\text{ A} \angle -120^o$$

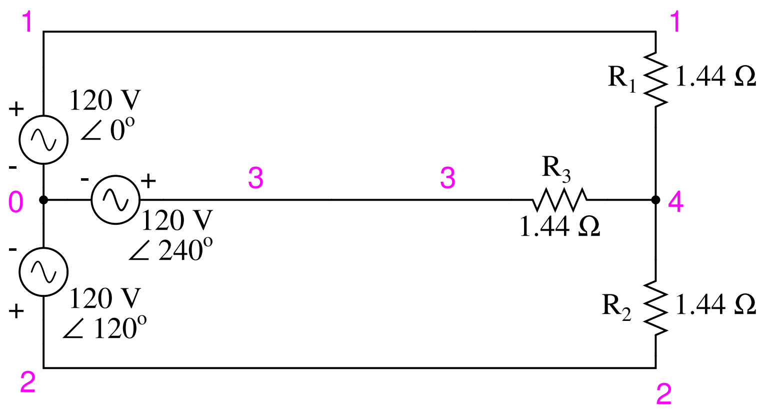
So, we find that the "neutral" wire is carrying a full 83.33 amps, just like each "hot" wire.

Note that we are still conveying 20 kW of total power to the two loads, with each load's "hot" wire carrying 83.33 amps as before. With the same amount of current through each "hot" wire, we must use the same gage copper conductors, so we haven't reduced system cost over the split-phase 120/240 system. However, we have realized a gain in safety, because the overall voltage between the two "hot" conductors is 32 volts lower than it was in the split-phase system (208 volts instead of 240 volts).

The fact that the neutral wire is carrying 83.33 amps of current raises an interesting possibility : since it's carrying current anyway, why not use that third wire as another "hot" conductor, powering another load resistor with a third 120 volt source having a phase angle of 240^o? That way, we could transmit *more* power (another 10 kW) without having to add any more conductors. Let's see how this might look :



A full mathematical analysis of all the voltages and currents in this circuit would necessitate the use of a network theorem, the easiest being the Superposition Theorem. I'll spare you the long, drawn-out calculations because you should be able to intuitively understand that the three voltage sources at three different phase angles will deliver 120 volts each to a balanced triad of load resistors. For proof of this, we can use SPICE to do the math for us :

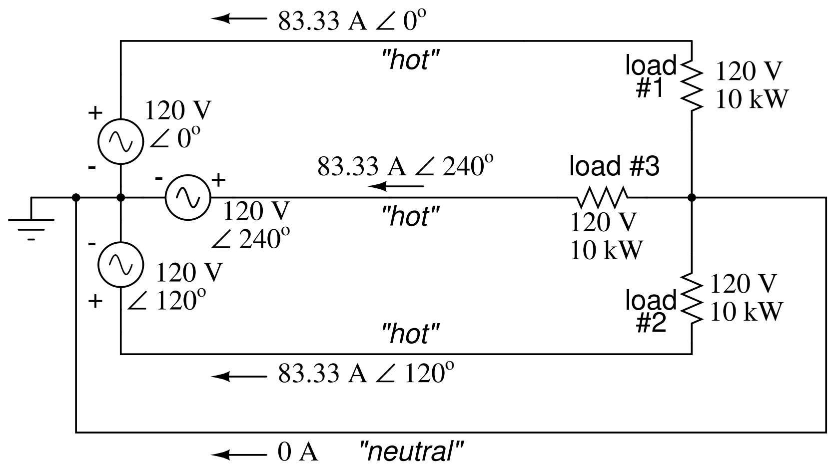



```
120/208 polyphase power system
v1 1 0 ac 120 0 sin
v2 2 0 ac 120 120 sin
v3 3 0 ac 120 240 sin
r1 1 4 1.44
r2 2 4 1.44
r3 3 4 1.44
.ac lin 1 60 60
.print ac v(1,4) v(2,4) v(3,4)
.print ac v(1,2) v(2,3) v(3,1)
.print ac i(v1) i(v2) i(v3)
.end
```

```
VOLTAGE ACROSS EACH LOAD
freq          v(1,4)       v(2,4)       v(3,4)
```

```
6.000E+01   1.200E+02   1.200E+02   1.200E+02

VOLTAGE BETWEEN "HOT" CONDUCTORS
freq        v(1,2)      v(2,3)      v(3,1)
6.000E+01   2.078E+02   2.078E+02   2.078E+02

CURRENT THROUGH EACH VOLTAGE SOURCE
freq        i(v1)       i(v2)       i(v3)
6.000E+01   8.333E+01   8.333E+01   8.333E+01
```

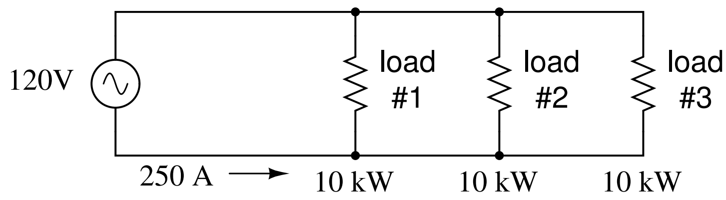
Sure enough, we get 120 volts across each load resistor, with (approximately) 208 volts between any two "hot" conductors and conductor currents equal to 83.33 amps. At that current and voltage, each load will be dissipating 10 kW of power. Notice that this circuit has no "neutral" conductor to ensure stable voltage to all loads if one should open. What we have here is a situation similar to our split-phase power circuit with no "neutral" conductor : if one load should happen to fail open, the voltage drops across the remaining load(s) will change. To ensure load voltage stability in the even of another load opening, we need a neutral wire to connect the source node and load node together :



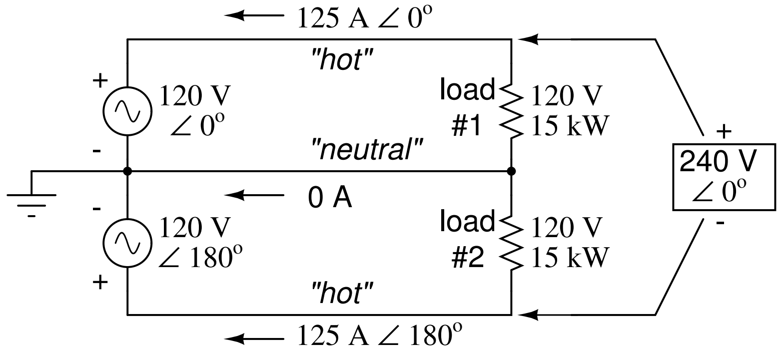
So long as the loads remain balanced (equal resistance, equal currents), the neutral wire will not have to carry any current at all. It is there just in case one or more load resistors should fail open (or be shut off through a disconnecting switch).

This circuit we've been analyzing with three voltage sources is called a *polyphase* circuit. The prefix "poly" simply means "more than one," as in "*poly*theism" (belief in more than one deity), "*poly*gon" (a geometrical shape made of multiple line segments : for example, *pentagon* and *hexagon*), and "*poly*atomic" (a substance composed of multiple types of atoms). Since the voltage sources are all at different phase angles (in this case, three different phase angles), this is a "*poly*phase" circuit. More specifically, it is a *three-phase circuit*, the kind used predominantly in large power distribution systems.

Let's survey the advantages of a three-phase power system over a single-phase system of equivalent load voltage and power capacity. A single-phase system with three loads connected directly in parallel would have a very high total current (83.33 times 3, or 250 amps :



This would necessitate 3/0 gage copper wire (*very* large!), at about 510 pounds per thousand feet, and with a considerable price tag attached. If the distance from source to load was 1000 feet, we would need over a half-ton of copper wire to do the job. On the other hand, we could build a split-phase system with two 15 kW, 120 volt loads :

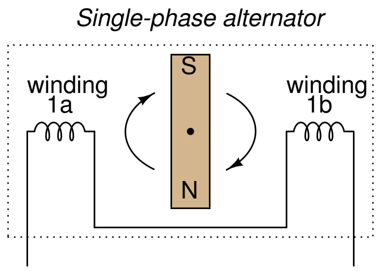


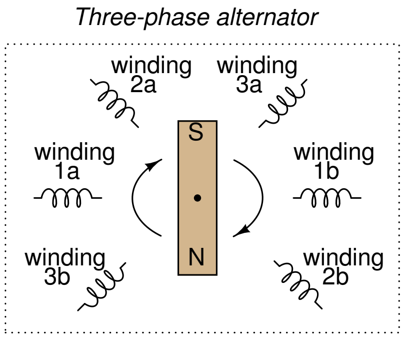
Our current is half of what it was with the simple parallel circuit, which is a great improvement. We could get away with using number 2 gage copper wire at a total mass of about 600 pounds, figuring about 200 pounds per thousand feet with three runs of 1000 feet each between source and loads. However, we also have to consider the increased safety hazard of having 240 volts present in the system, even though each load only receives 120 volts. Overall, there is greater potential for dangerous electric shock to occur.

When we contrast these two examples against our three-phase system, the advantages are quite clear. First, the conductor currents are quite a bit less (83.33 amps versus 125 or 250 amps), permitting the use of much thinner and lighter wire. We can use number 4 gage wire at about 125 pounds per thousand feet, which will total 500 pounds (four runs of 1000 feet each) for our example circuit. This represents a significant cost savings over the split-phase system, with the additional benefit that the maximum voltage in the system is lower (208 versus 240).

One question remains to be answered : how in the world do we get three AC voltage sources whose phase angles are exactly 120^o apart? Obviously we can't center-tap a transformer or alternator winding like we did in the split-phase system, since that can only give us voltage waveforms that are either in phase or 180^o out of phase. Perhaps we could figure out some way to use capacitors and inductors to create phase shifts of 120^o, but then those phase shifts would depend on the phase angles of our load impedances as well (substituting a capacitive or inductive load for a resistive load would change everything!).

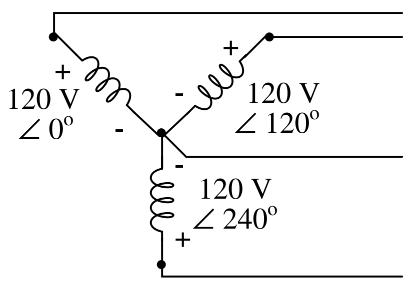
The best way to get the phase shifts we're looking for is to generate it at the source : construct the AC generator (alternator) providing the power in such a way that the rotating magnetic field passes by three sets of wire windings, each set spaced 120^o apart around the circumference of the machine :





Together, the six "pole" windings of a three-phase alternator are connected to comprise three winding pairs, each pair producing AC voltage with a phase angle 120^o shifted from either of the other two winding pairs. The interconnections between pairs of windings (as shown for the single-phase alternator : the jumper wire between windings 1a and 1b) have been omitted from the three-phase alternator drawing for simplicity.

In our example circuit, we showed the three voltage sources connected together in a "Y" configuration (sometimes called the "star" configuration), with one lead of each source tied to a common point (the node where we attached the "neutral" conductor). The common way to depict this connection scheme is to draw the windings in the shape of a "Y" like this :



The "Y" configuration is not the only option open to us, but it is probably the easiest to understand at first. More to come on this subject later in the chapter.

- **REVIEW :**

- A *single-phase* power system is one where there is only one AC voltage source (one source voltage waveform).

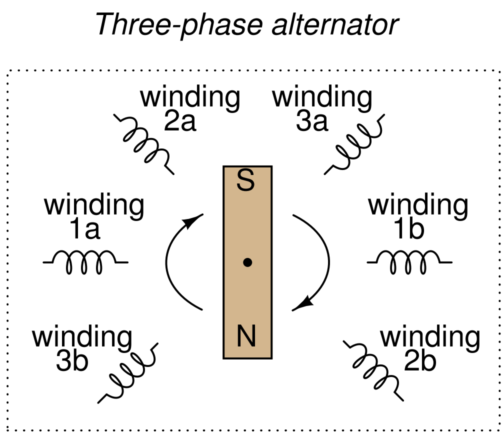
- A *split-phase* power system is one where there are two voltage sources, 180^o phase-shifted from each other, powering a two series-connected loads. The advantage of this is the ability to have lower conductor currents while maintaining low load voltages for safety reasons.

- A *polyphase* power system uses multiple voltage sources at different phase angles from each other (many "phases" of voltage waveforms at work). A polyphase power system can deliver more power at less voltage with smaller-gage conductors than single- or split-phase systems.

- The phase-shifted voltage sources necessary for a polyphase power system are created in alternators with multiple sets of wire windings. These winding sets are spaced around the circumference of the rotor's rotation at the desired angle(s).

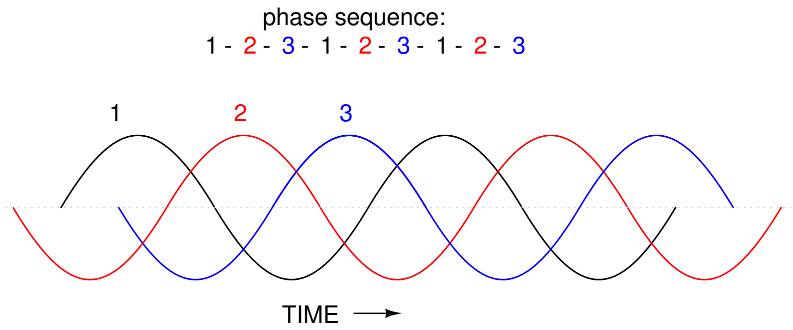
10.3 Phase rotation

Let's take the three-phase alternator design laid out earlier and watch what happens as the magnet rotates :

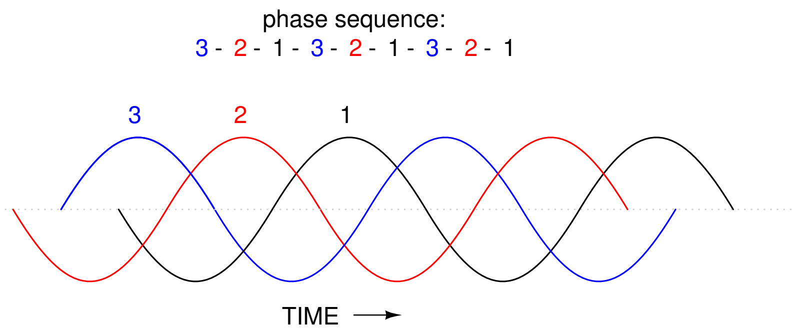


The phase angle shift of 120^o is a function of the actual rotational angle shift of the three pairs of windings. If the magnet is rotating clockwise, winding 3 will generate its peak instantaneous voltage exactly 120^o (of alternator shaft rotation) after winding 2, which will hits its peak 120^o after winding 1. The magnet passes by each pole pair at different positions in the rotational movement of the shaft. Where we decide to place the windings will dictate the amount of phase shift between the windings' AC voltage waveforms. If we make winding 1 our "reference" voltage source for phase angle (0^o), then winding 2 will have a phase angle of -120^o (120^o lagging, or 240^o leading) and winding 3 an angle of -240^o (or 120^o leading).

This sequence of phase shifts has a definite order. For clockwise rotation of the shaft, the order is 1-2-3 (winding 1 peaks first, them winding 2, then winding 3). This order keeps repeating itself as long as we continue to rotate the alternator's shaft :

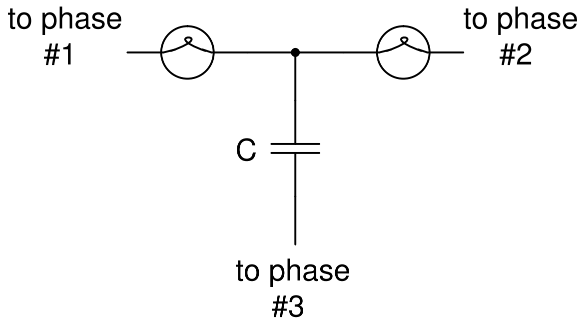


However, if we *reverse* the rotation of the alternator's shaft (turn it counter-clockwise), the magnet will pass by the pole pairs in the opposite sequence. Instead of 1-2-3, we'll have 3-2-1. Now, winding 2's waveform will be *leading* 120^o ahead of 1 instead of lagging, and 3 will be another 120^o ahead of 2 :

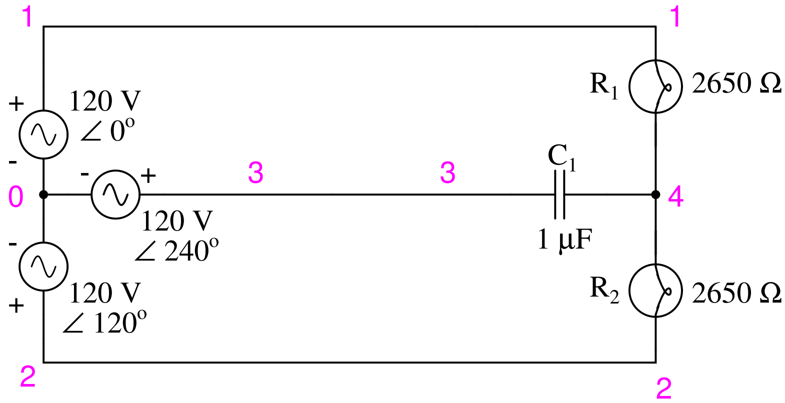


The order of voltage waveform sequences in a polyphase system is called *phase rotation* or *phase sequence*. If we're using a polyphase voltage source to power resistive loads, phase rotation will make no difference at all. Whether 1-2-3 or 3-2-1, the voltage and current magnitudes will all be the same. There are some applications of three-phase power, as we will see shortly, that depend on having phase rotation being one way or the other. Since voltmeters and ammeters would be useless in telling us what the phase rotation of an operating power system is, we need to have some other kind of instrument capable of doing the job.

One ingenious circuit design uses a capacitor to introduce a phase shift between voltage and current, which is then used to detect the sequence by way of comparison between the brightness of two indicator lamps :



The two lamps are of equal filament resistance and wattage. The capacitor is sized to have approximately the same amount of reactance at system frequency as each lamp's resistance. If the capacitor were to be replaced by a resistor of equal value to the lamps' resistance, the two lamps would glow at equal brightness, the circuit being balanced. However, the capacitor introduces a phase shift between voltage and current in the third leg of the circuit equal to 90^o. This phase shift, greater than 0^o but less than 120^o, skews the voltage and current values across the two lamps according to their phase shifts relative to phase 3. The following SPICE analysis demonstrates what will happen :




```
phase rotation detector -- sequence = v1-v2-v3
v1 1 0 ac 120 0 sin
v2 2 0 ac 120 120 sin
v3 3 0 ac 120 240 sin
r1 1 4 2650
r2 2 4 2650
c1 3 4 1u
.ac lin 1 60 60
.print ac v(1,4) v(2,4) v(3,4)
.end

freq          v(1,4)      v(2,4)      v(3,4)
6.000E+01     4.810E+01   1.795E+02   1.610E+02
```

The resulting phase shift from the capacitor causes the voltage across phase 1 lamp (between nodes 1 and 4) to fall to 48.1 volts and the voltage across phase 2 lamp (between nodes 2 and 4) to rise to 179.5 volts, making the first lamp dim and the second lamp bright. Just the opposite will happen if the phase sequence is reversed :

```
phase rotation detector -- sequence = v3-v2-v1
v1 1 0 ac 120 240 sin
v2 2 0 ac 120 120 sin
v3 3 0 ac 120 0 sin
r1 1 4 2650
r2 2 4 2650
c1 3 4 1u
.ac lin 1 60 60
.print ac v(1,4) v(2,4) v(3,4)
.end
```

```
freq          v(1,4)      v(2,4)      v(3,4)
6.000E+01     1.795E+02   4.810E+01   1.610E+02
```

Here, the first lamp receives 179.5 volts while the second receives only 48.1 volts.

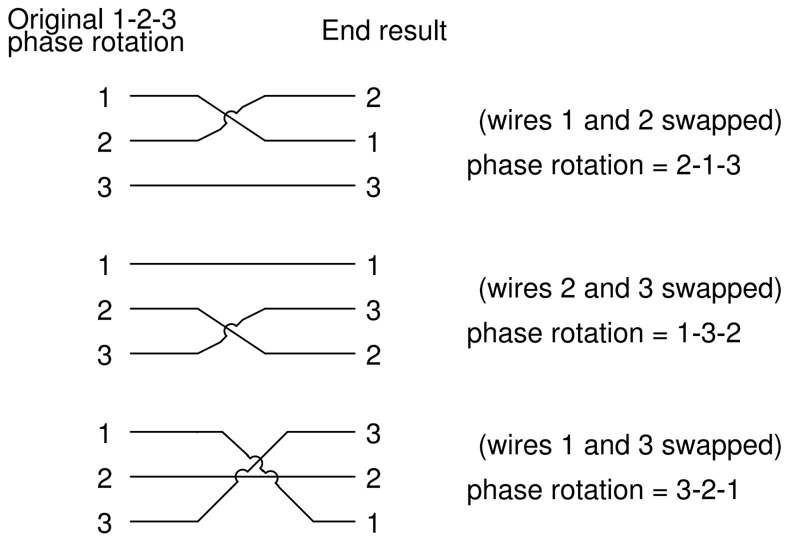
We've investigated how phase rotation is produced (the order in which pole pairs get passed by the alternator's rotating magnet) and how it can be changed by reversing the alternator's shaft rotation. However, reversal of the alternator's shaft rotation is not usually an option open to an end-user of electrical power supplied by a nationwide grid ("the" alternator actually being the combined total of all alternators in all power plants feeding the grid). There is a *much* easier way to reverse phase sequence than reversing alternator rotation : just exchange any two of the three "hot" wires going to a three-phase load.

This trick makes more sense if we take another look at a running phase sequence of a three-phase voltage source :

```
1-2-3 rotation :  1-2-3-1-2-3-1-2-3-1-2-3-1-2-3 . . .
3-2-1 rotation :  3-2-1-3-2-1-3-2-1-3-2-1-3-2-1 . . .
```

What is commonly designated as a "1-2-3" phase rotation could just as well be called "2-3-1" or "3-1-2," going from left to right in the number string above. Likewise, the opposite rotation (3-2-1) could just as easily be called "2-1-3" or "1-3-2."

Starting out with a phase rotation of 3-2-1, we can try all the possibilities for swapping any two of the wires at a time and see what happens to the resulting sequence :



No matter which pair of "hot" wires out of the three we choose to swap, the phase rotation ends up being reversed (1-2-3 gets changed to 2-1-3, 1-3-2 or 3-2-1, all equivalent).

- **REVIEW :**

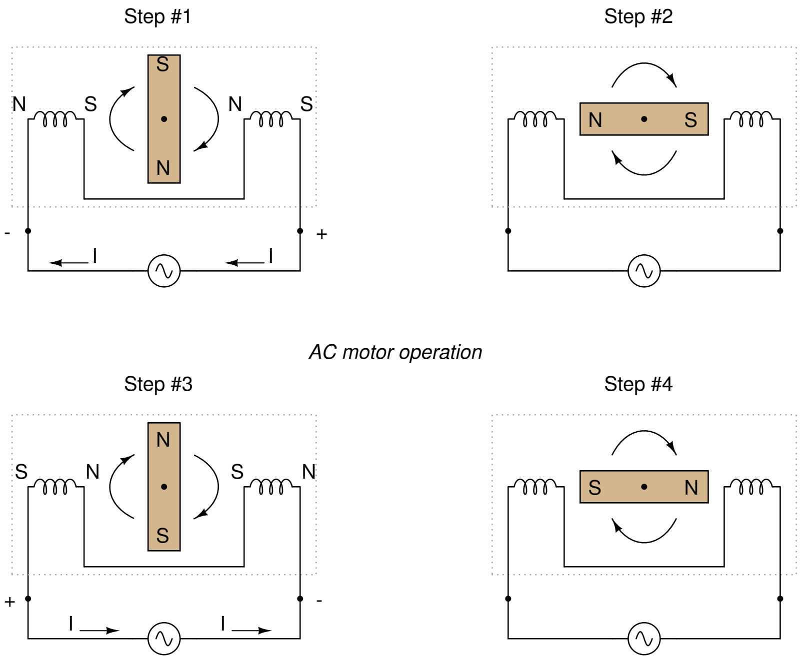
- *Phase rotation*, or *phase sequence*, is the order in which the voltage waveforms of a polyphase AC source reach their respective peaks. For a three-phase system, there are only two possible phase sequences : 1-2-3 and 3-2-1, corresponding to the two possible directions of alternator rotation.

- Phase rotation has no impact on resistive loads, but it will have impact on unbalanced reactive loads, as shown in the operation of a phase rotation detector circuit.

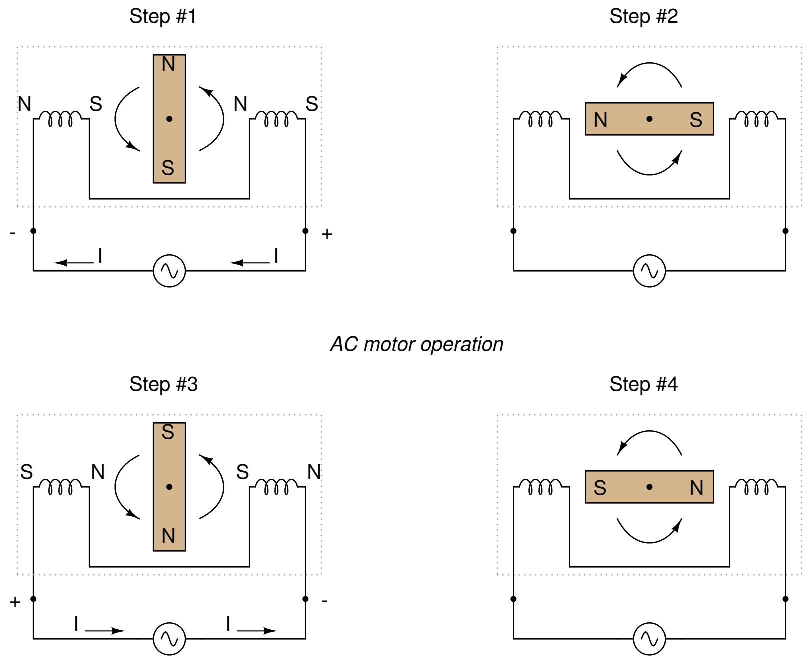
- Phase rotation can be reversed by swapping any two of the three "hot" leads supplying three-phase power to a three-phase load.

10.4 Polyphase motor design

Perhaps the most important benefit of polyphase AC power over single-phase is the design and operation of AC motors. As we studied in the first chapter of this book, some types of AC motors are virtually identical in construction to their alternator (generator) counterparts, consisting of stationary wire windings and a rotating magnet assembly. (Other AC motor designs are not quite this simple, but we will leave those details to another lesson).



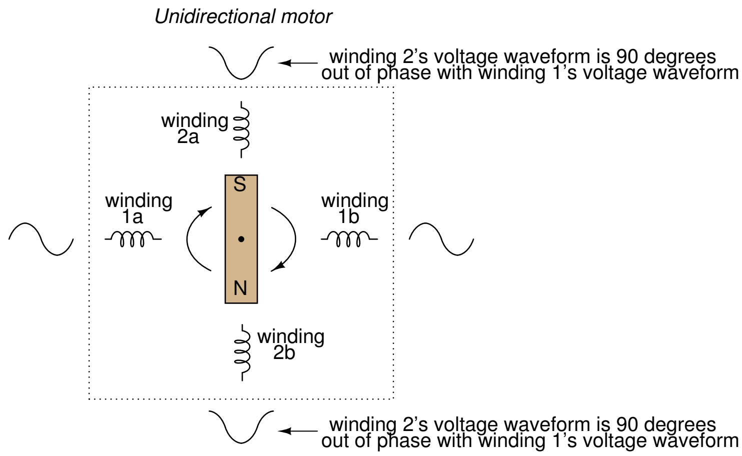
If the rotating magnet is able to keep up with the frequency of the alternating current energizing the electromagnet windings (coils), it will continue to be pulled around clockwise. However, clockwise is not the only valid direction for this motor's shaft to spin. It could just as easily be powered in a counter-clockwise direction by the same AC voltage waveform :



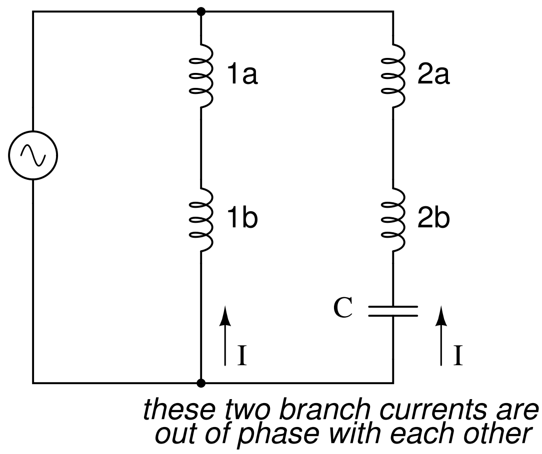
AC motor operation

Notice that with the exact same sequence of polarity cycles (voltage, current, and magnetic poles produced by the coils), the magnetic rotor can spin in either direction. This is a common trait of all single-phase AC "induction" and "synchronous" motors : they have no normal or "correct" direction of rotation. The natural question should arise at this point : how can the motor get started in the intended direction if it can run either way just as well? The answer is that these motors need a little help getting started. Once helped to spin in a particular direction. they will continue to spin that way as long as AC power is maintained to the windings.

Where that "help" comes from for a single-phase AC motor to get going in one direction can vary. Usually, it comes from an additional set of windings positioned differently from the main set, and energized with an AC voltage that is out of phase with the main power :



These supplementary coils are typically connected in series with a capacitor to introduce a phase shift in current between the two sets of windings :

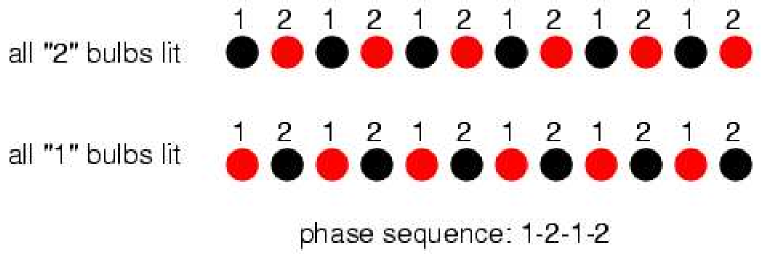


That phase shift creates magnetic fields from coils 2a and 2b that are equally out of step with the fields from coils 1a and 1b. The result is a set of magnetic fields with a definite phase rotation. It is this phase rotation that pulls the rotating magnet around in a definite direction.

Polyphase AC motors require no such trickery to spin in a definite direction. Because their supply voltage waveforms already have a definite rotation sequence, so do the respective magnetic fields generated by the motor's stationary windings. In fact, the combination of all three phase winding sets working together creates what is often called a *rotating magnetic field*. It was this concept of a rotating magnetic field that inspired Nikola Tesla to design the world's first polyphase electrical systems (simply to make simpler, more efficient motors). The line current and safety advantages of polyphase power over single phase power were discovered later.

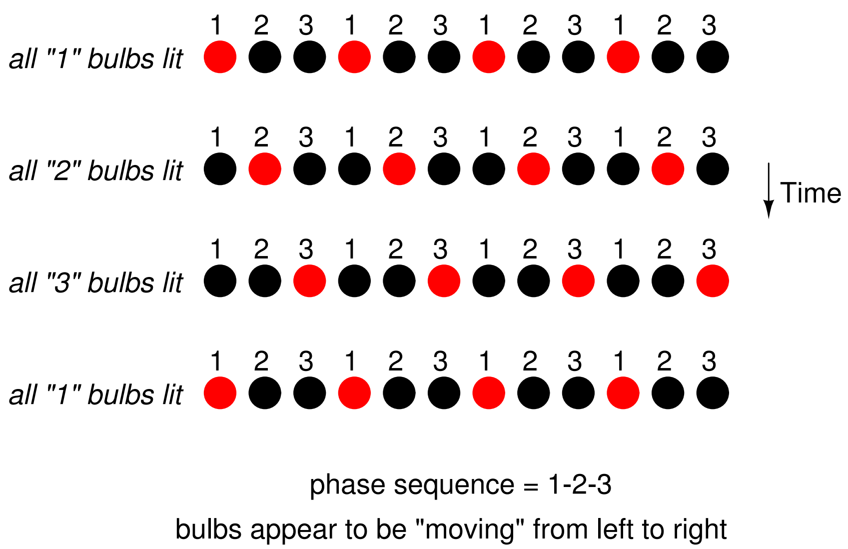
What can be a confusing concept is made much clearer through analogy. Have you ever seen a row of blinking light bulbs such as the kind used in Christmas decorations? Some strings appear to "move" in a definite direction as the bulbs alternately glow and darken in sequence. Other strings just blink on and off with no apparent motion. What makes the difference between the two types of bulb strings? Answer : phase shift!

Examine a string of lights where every other bulb is lit at any given time :

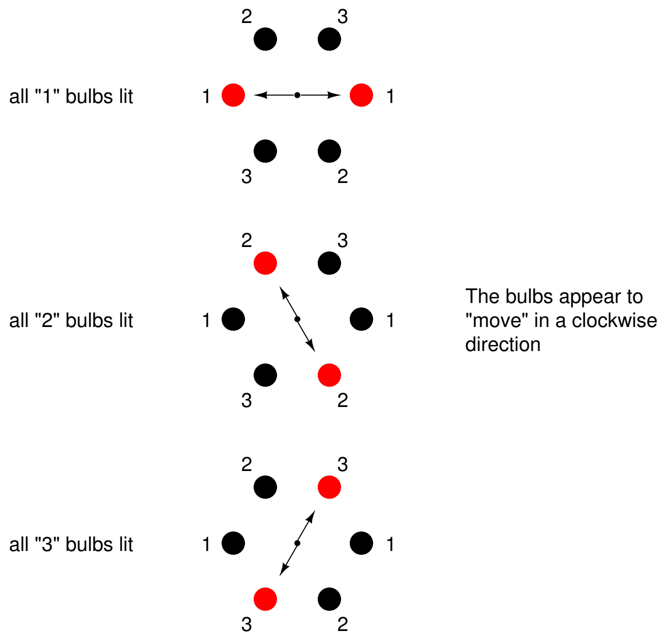


When all of the "1" bulbs are lit, the "2" bulbs are dark, and visa-versa. With this blinking sequence, there is no definite "motion" to the bulbs' light. Your eyes could follow a "motion" from left to right just as easily as from right to left. Technically, the "1" and "2" bulb blinking sequences are 180^o out of phase (exactly opposite each other). This is analogous to the single-phase AC motor, which can run just as easily in either direction, but which cannot start on its own because its magnetic field alternation lacks a definite "rotation."

Now let's examine a string of lights where there are three sets of bulbs to be sequenced instead of just two, and these three sets are equally out of phase with each other :

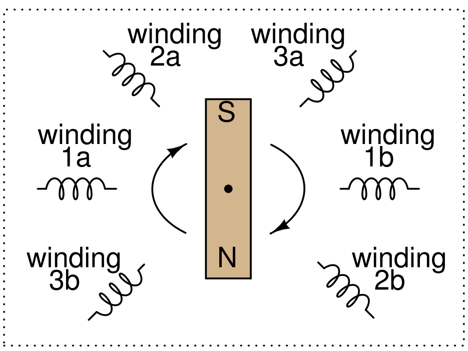


If the lighting sequence is 1-2-3 (the sequence shown), the bulbs will appear to "move" from left to right. Now imagine this blinking string of bulbs arranged into a circle :



Now the lights appear to be "moving" in a clockwise direction because they are arranged around a circle instead of a straight line. It should come as no surprise that the appearance of motion will reverse if the phase sequence of the bulbs is reversed.

The blinking pattern will either appear to move clockwise or counter-clockwise depending on the phase sequence. This is analogous to a three-phase AC motor with three sets of windings energized by voltage sources of three different phase shifts :



A phase sequence of 1-2-3 will spin the magnet in a clockwise direction. A phase sequence of 3-2-1 will spin the magnet in a counter-clockwise direction.

With phase shifts of less than 180^o we get true rotation of the magnetic field. With single-phase motors, the rotating magnetic field necessary for self-starting must to be created by way of capacitive phase shift. With polyphase motors, the necessary phase shifts are there already. Plus, the direction of shaft rotation for polyphase motors is very easily reversed : just swap any two "hot" wires going to the motor, and it will run in the opposite direction!

- **REVIEW :**

- AC "induction" and "synchronous" motors work by having a rotating magnet follow the alternating magnetic fields produced by stationary wire windings.

- Single-phase AC motors of this type need help to get started spinning in a particular direction.

- By introducing a phase shift of less than 180^o to the magnetic fields in such a motor, a definite direction of shaft rotation can be established.

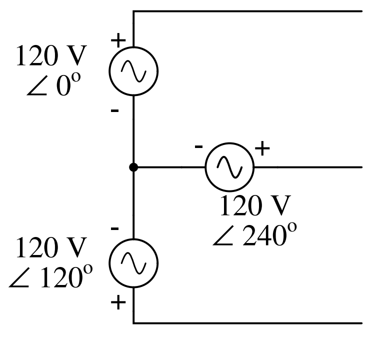
- Single-phase induction motors often use an auxiliary winding connected in series with a capacitor to create the necessary phase shift.

- Polyphase motors don't need such measures; their direction of rotation is fixed by the phase sequence of the voltage they're powered by.

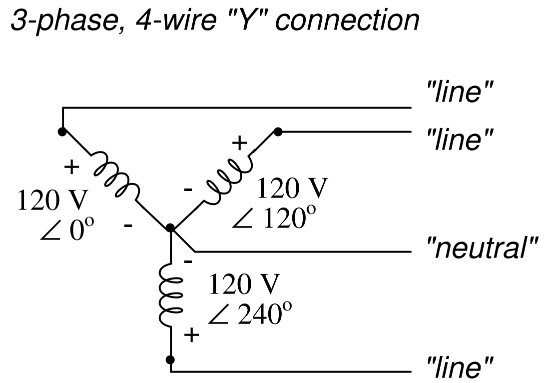
- Swapping any two "hot" wires on a polyphase AC motor will reverse its phase sequence, thus reversing its shaft rotation.

10.5 Three-phase Y and Δ configurations

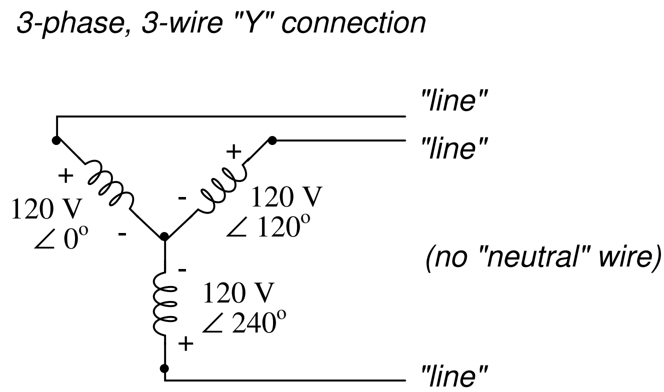
Initially we explored the idea of three-phase power systems by connecting three voltage sources together in what is commonly known as the "Y" (or "star") configuration. This configuration of voltage sources is characterized by a common connection point joining one side of each source :



If we draw a circuit showing each voltage source to be a coil of wire (alternator or transformer winding) and do some slight rearranging, the "Y" configuration becomes more obvious :



The three conductors leading away from the voltage sources (windings) toward a load are typically called *lines*, while the windings themselves are typically called *phases*. In a Y-connected system, there may or may not be a neutral wire attached at the junction point in the middle, although it certainly helps alleviate potential problems should one element of a three-phase load fail open, as discussed earlier :



When we measure voltage and current in three-phase systems, we need to be specific as to *where* we're measuring. *Line voltage* refers to the amount of voltage measured between any two line conductors in a balanced three-phase system. With the above circuit, the line voltage is roughly 208 volts. *Phase voltage* refers to the voltage measured across any one component (source winding or load impedance) in a balanced three-phase source or load. For the circuit shown above, the phase voltage is 120 volts. The terms *line current* and *phase current* follow the same logic : the former referring to current through any one line conductor, and the latter to current through any one component.

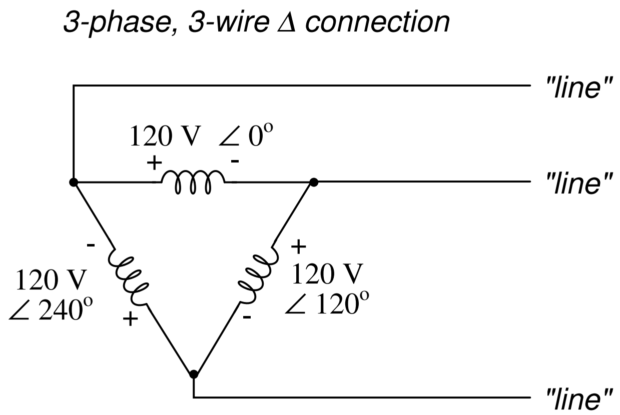
Y-connected sources and loads always have line voltages greater than phase voltages, and line currents equal to phase currents. If the Y-connected source or load is balanced, the line voltage will be equal to the phase voltage times the square root of 3 :

For "Y" circuits:

$$E_{line} = \sqrt{3}\ E_{phase}$$

$$I_{line} = I_{phase}$$

However, the "Y" configuration is not the only valid one for connecting three-phase voltage source or load elements together. Another configuration is known as the "Delta," for its geometric resemblance to the Greek letter of the same name (Δ). Take close notice of the polarity for each winding in the drawing below :



At first glance it seems as though three voltage sources like this would create a short-circuit, electrons flowing around the triangle with nothing but the internal impedance of the windings to hold them back. Due to the phase angles of these three voltage sources, however, this is not the case.

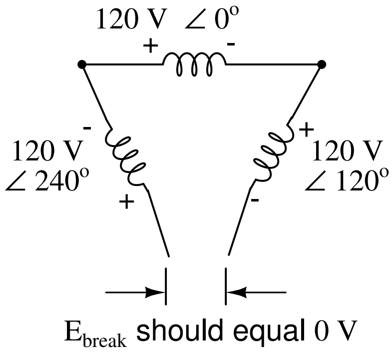
One quick check of this is to use Kirchhoff's Voltage Law to see if the three voltages around the loop add up to zero. If they do, then there will be no voltage available to push current around and around that loop, and consequently there will be no circulating current. Starting with the top winding and progressing counter-clockwise, our KVL expression looks something like this :

$$(120\ V \angle 0^{o}) + (120\ V \angle 240^{o}) + (120\ V \angle 120^{o})$$

Does it all equal 0?

Yes!

Indeed, if we add these three vector quantities together, they do add up to zero. Another way to verify the fact that these three voltage sources can be connected together in a loop without resulting in circulating currents is to open up the loop at one junction point and calculate voltage across the break :



Starting with the right winding (120 V ∠ 120^o) and progressing counter-clockwise, our KVL equation looks like this :

$$(120\text{ V} \angle 120^o) + (120 \angle 0^o) + (120\text{ V} \angle 240^o) + E_{break} = 0$$

$$0 + E_{break} = 0$$

$$E_{break} = 0$$

Sure enough, there will be zero voltage across the break, telling us that no current will circulate within the triangular loop of windings when that connection is made complete.

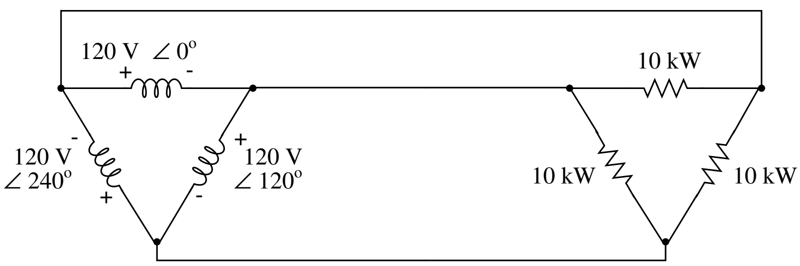
Having established that a Δ-connected three-phase voltage source will not burn itself to a crisp due to circulating currents, we turn to its practical use as a source of power in three-phase circuits. Because each pair of line conductors is connected directly across a single winding in a Δ circuit, the line voltage will be equal to the phase voltage. Conversely, because each line conductor attaches at a node between two windings, the line current will be the vector sum of the two joining phase currents. Not surprisingly, the resulting equations for a Δ configuration are as follows :

For Δ ("delta") circuits:

$$E_{line} = E_{phase}$$

$$I_{line} = \sqrt{3}\ I_{phase}$$

Let's see how this works in an example circuit :



With each load resistance receiving 120 volts from its respective phase winding at the source, the current in each phase of this circuit will be 83.33 amps :

$$I = \frac{P}{E}$$

$$I = \frac{10\ \text{kW}}{120\ \text{V}}$$

$I = 83.33$ A *(for each load resistor and source winding)*

$$I_{line} = \sqrt{3}\ I_{phase}$$

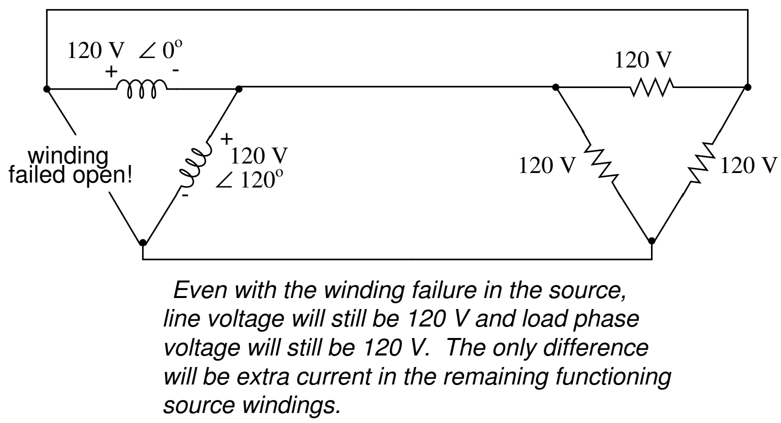
$$I_{line} = \sqrt{3}\ (83.33\ \text{A})$$

$$I_{line} = 144.34\ \text{A}$$

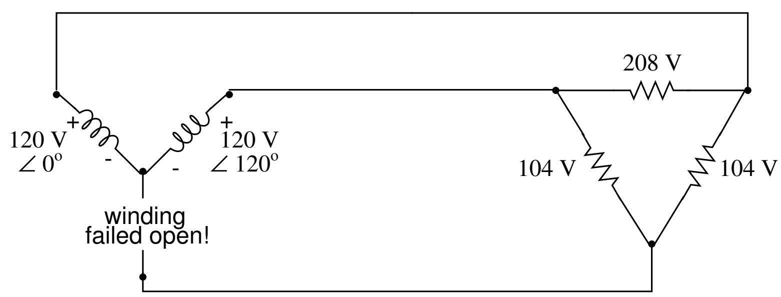
So, the each line current in this three-phase power system is equal to 144.34 amps, substantially more than the line currents in the Y-connected system we looked at earlier. One might wonder if we've lost all the advantages of three-phase power here, given the fact that we have such greater conductor currents, necessitating thicker, more costly wire. The answer is no. Although this circuit would require three number 1 gage copper conductors (at 1000 feet of distance between source and load this equates to a little over 750 pounds of copper for the whole system), it is still less than the 1000+ pounds of copper required for a single-phase system delivering the same power (30 kW) at the same voltage (120 volts conductor-to-conductor).

One distinct advantage of a Δ-connected system is its lack of a neutral wire. With a Y-connected system, a neutral wire was needed in case one of the phase loads were to fail open (or be turned off), in order to keep the phase voltages at the load from changing. This is not necessary (or even possible!) in a Δ-connected circuit. With each load phase element directly connected across a respective source phase winding, the phase voltage will be constant regardless of open failures in the load elements.

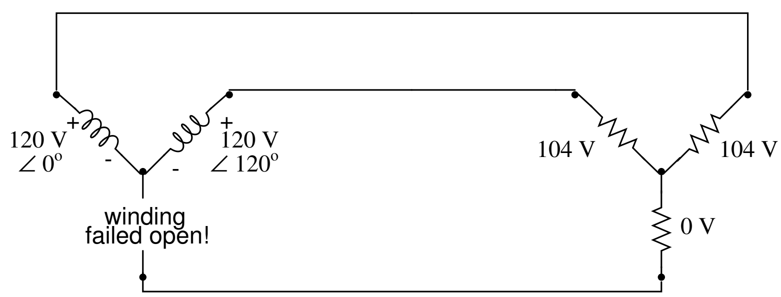
Perhaps the greatest advantage of the Δ-connected source is its fault tolerance. It is possible for one of the windings in a Δ-connected three-phase source to fail open without affecting load voltage or current!



The only consequence of a source winding failing open for a Δ-connected source is increased phase current in the remaining windings. Compare this fault tolerance with a Y-connected system suffering an open source winding :



With a Δ-connected load, two of the resistances suffer reduced voltage while one remains at the original line voltage, 208. A Y-connected load suffers an even worse fate with the same winding failure in a Y-connected source :



In this case, two load resistances suffer reduced voltage while the third loses supply voltage completely! For this reason, Δ-connected sources are preferred for reliability. However, if dual voltages are needed (e.g. 120/208) or preferred for lower line currents, Y-connected systems are the configuration of choice.

- **REVIEW :**

- The conductors connected to the three points of a three-phase source or load are called *lines*.
- The three components comprising a three-phase source or load are called *phases*.
- *Line voltage* is the voltage measured between any two lines in a three-phase circuit.
- *Phase voltage* is the voltage measured across a single component in a three-phase source or load.
- *Line current* is the current through any one line between a three-phase source and load.
- *Phase current* is the current through any one component comprising a three-phase source or load.
- In balanced "Y" circuits, line voltage is equal to phase voltage times the square root of 3, while line current is equal to phase current.

For "Y" circuits:

$$E_{line} = \sqrt{3}\ E_{phase}$$

- $I_{line} = I_{phase}$

- In balanced Δ circuits, line voltage is equal to phase voltage, while line current is equal to phase current times the square root of 3.

For Δ ("delta") circuits:

$$E_{line} = E_{phase}$$

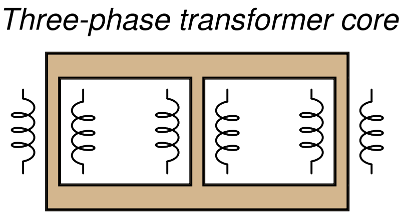
- $I_{line} = \sqrt{3}\ I_{phase}$

- Δ-connected three-phase voltage sources give greater reliability in the event of winding failure than Y-connected sources. However, Y-connected sources can deliver the same amount of power with less line current than Δ-connected sources.

10.6 Three-phase transformer circuits

Since three-phase is used so often for power distribution systems, it makes sense that we would need three-phase transformers to be able to step voltages up or down. This is only partially true, as regular single-phase transformers can be ganged together to transform power between two three-phase systems in a variety of configurations, eliminating the requirement for a special three-phase transformer. However, special three-phase transformers are built for those tasks, and are able to perform with less material requirement, less size, and less weight from their modular counterparts.

A three-phase transformer is made of three sets of primary and secondary windings, each set wound around one leg of an iron core assembly. Essentially it looks like three single-phase transformers sharing a joined core :



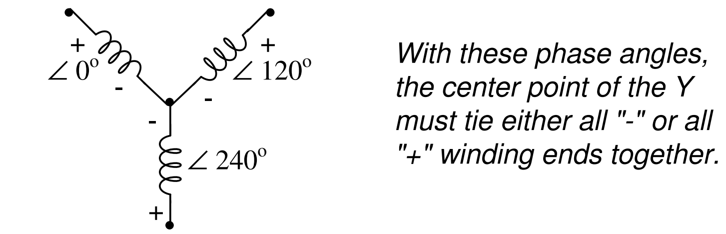
Those sets of primary and secondary windings will be connected in either Δ or Y configurations to form a complete unit. The various combinations of ways that these windings can be connected together in will be the focus of this section.

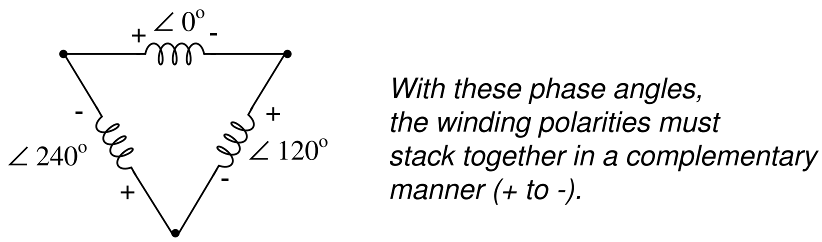
Whether the winding sets share a common core assembly or each winding pair is a separate transformer, the winding connection options are the same :

- **Primary - Secondary**
- Y - Y
- Y - Δ
- Δ - Y
- Δ - Δ

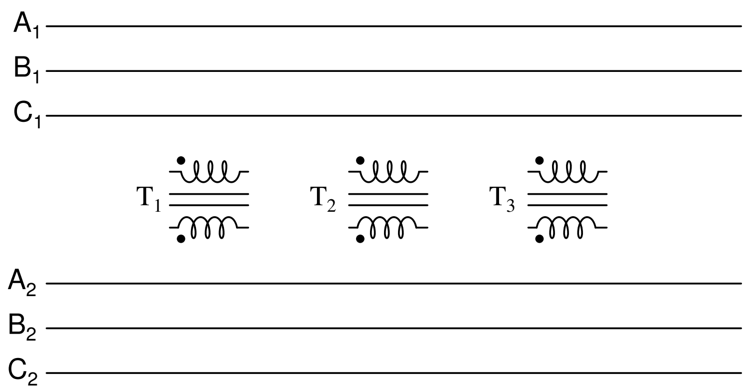
The reasons for choosing a Y or Δ configuration for transformer winding connections are the same as for any other three-phase application : Y connections provide the opportunity for multiple voltages, while Δ connections enjoy a higher level of reliability (if one winding fails open, the other two can still maintain full line voltages to the load).

Probably the most important aspect of connecting three sets of primary and secondary windings together to form a three-phase transformer bank is attention to proper winding phasing (the dots used to denote "polarity" of windings). Remember the proper phase relationships between the phase windings of Δ and Y :

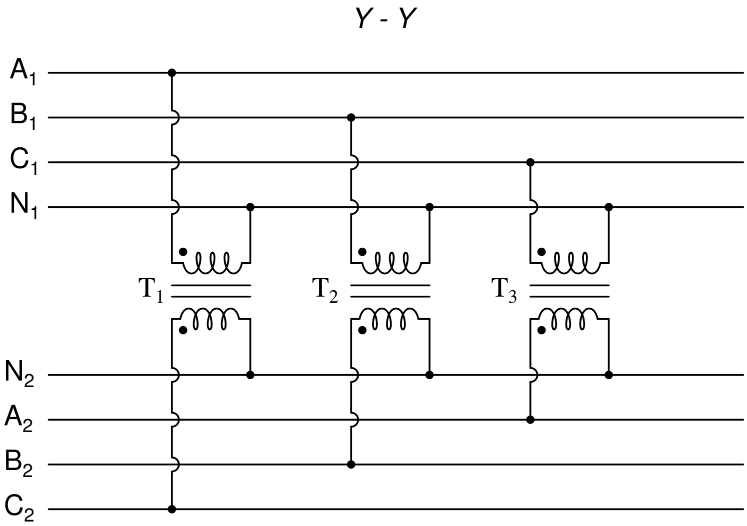




Getting this phasing correct when the windings aren't shown in regular Y or Δ configuration can be tricky. Let me illustrate :

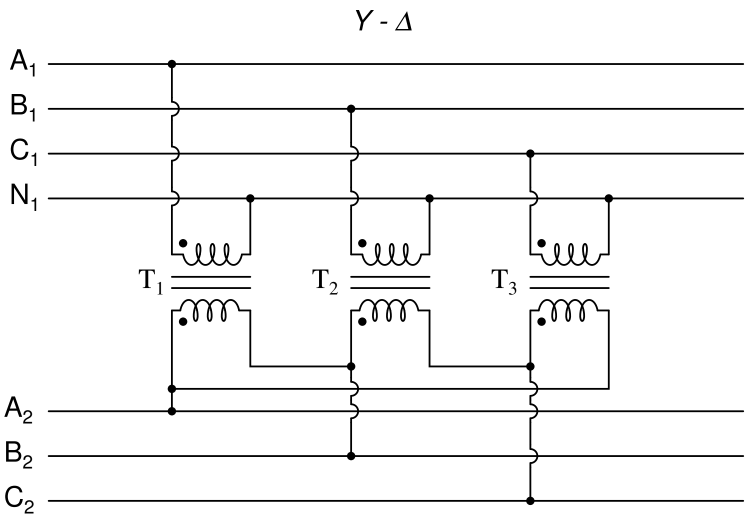


Three individual transformers are to be connected together to transform power from one three-phase system to another. First, I'll show the wiring connections for a Y-Y configuration :



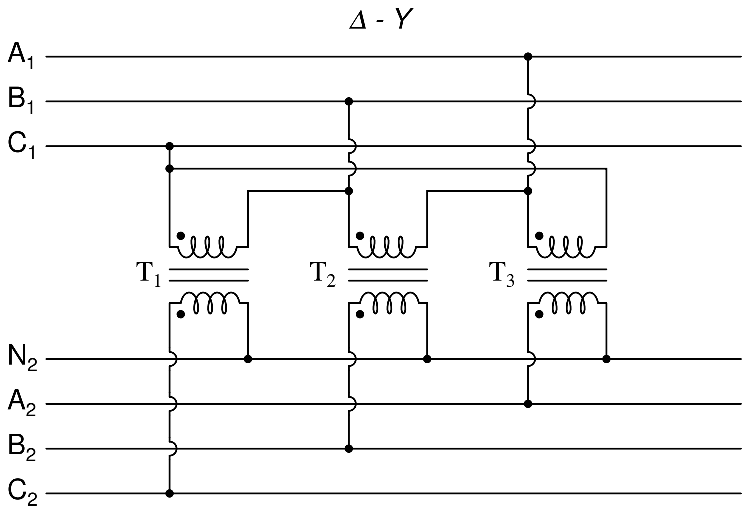
Note how all the winding ends marked with dots are connected to their respective phases A, B, and C, while the non-dot ends are connected together to form the centers of each "Y". Having both primary and secondary winding sets connected in "Y" formations allows for the use of neutral conductors (N_1 and N_2) in each power system.

Now, we'll take a look at a Y-Δ configuration :



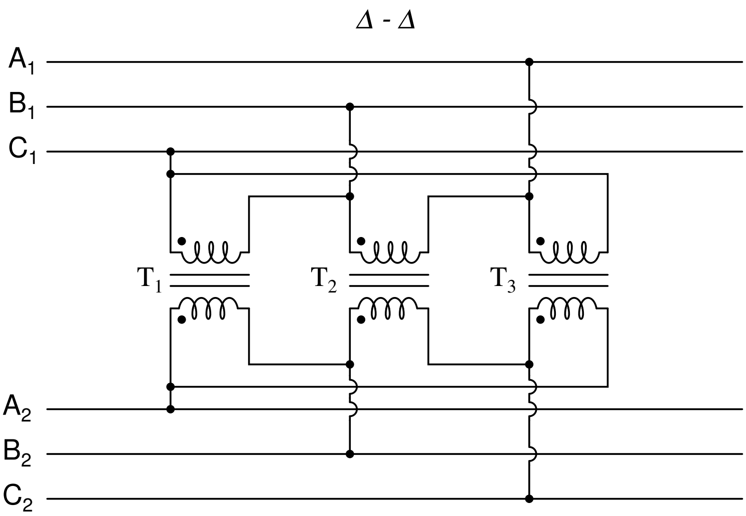
Note how the secondary windings (bottom set) are connected in a chain, the "dot"' side of one winding connected to the "non-dot" side of the next, forming the Δ loop. At every connection point between pairs of windings, a connection is made to a line of the second power system (A, B, and C).

Now, let's examine a Δ-Y system :



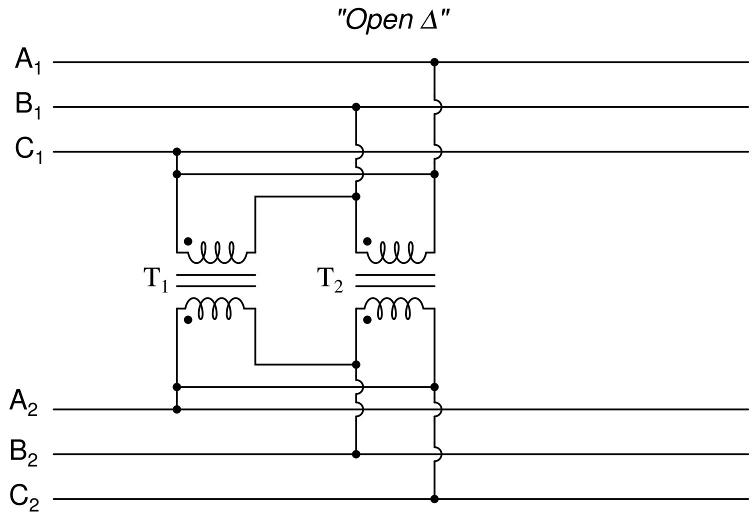
Such a configuration would allow for the provision of multiple voltages (line-to-line or line-to-neutral) in the second power system, from a source power system having no neutral.

And finally, we turn to the Δ-Δ configuration :



When there is no need for a neutral conductor in the secondary power system, Δ-Δ connection schemes are preferred because of the inherent reliability of the Δ configuration.

Considering that a Δ configuration can operate satisfactorily missing one winding, some power system designers choose to create a three-phase transformer bank with only two transformers, representing a Δ-Δ configuration with a missing winding in both the primary and secondary sides :



This configuration is called "V" or "Open-Δ." Of course, each of the two transformers have to be oversized to handle the same amount of power as three in a standard Δ configuration, but the overall size, weight, and cost advantages are often worth it. Bear in mind, however, that with one winding set missing from the Δ shape, this system no longer provides the fault tolerance of a normal Δ-Δ system. If one of the two transformers were to fail, the load voltage and current would definitely be affected.

The following photograph shows a bank of step-up transformers at the Grand Coulee hydroelectric dam in Washington state. Several transformers (green in color) may be seen from this vantage point, and they are grouped in threes : three transformers per hydroelectric generator, wired together in some form of three-phase configuration. The photograph doesn't reveal the primary winding connections, but it appears the secondaries are connected in a Y configuration, being that there is only one large high-voltage insulator protruding from each transformer. This suggests the other side of each transformer's secondary winding is at or near ground potential, which could only be true in a Y system. The building to the left is the powerhouse, where the generators and turbines are housed. On the right, the sloping concrete wall is the downstream face of the dam :

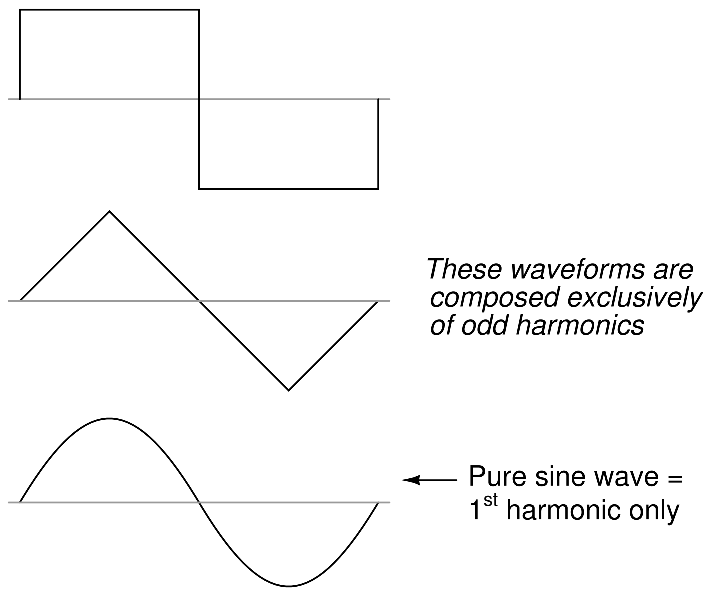
10.7 Harmonics in polyphase power systems

In the chapter on mixed-frequency signals, we explored the concept of *harmonics* in AC systems : frequencies that are integer multiples of the fundamental source frequency. With AC power systems where the source voltage waveform coming from an AC generator (alternator) is supposed to be a single-frequency sine wave, undistorted, there should be no harmonic content . . . ideally.

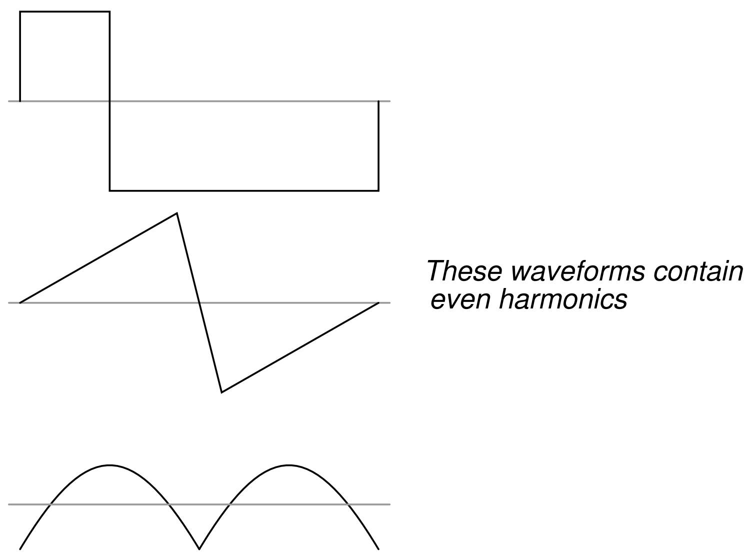
This would be true were it not for *nonlinear components*. Nonlinear components draw current disproportionately with respect to the source voltage, causing non-sinusoidal current waveforms. Examples of nonlinear components include gas-discharge lamps, semiconductor power-control devices (diodes, transistors, SCRs, TRIACs), transformers (primary winding magnetization current is usually non-sinusoidal due to the B/H saturation curve of the core), and electric motors (again, when magnetic fields within the motor's core operate near saturation levels). Even incandescent lamps generate slightly nonsinusoidal currents, as the filament resistance changes throughout the cycle due to rapid fluctuations in temperature. As we learned in the mixed-frequency chapter, *any* distortion of an otherwise sine-wave shaped waveform constitutes the presence of harmonic frequencies.

When the nonsinusoidal waveform in question is symmetrical above and below its average centerline, the harmonic frequencies will be odd integer multiples of the fundamental source frequency only, with no even integer multiples. Most nonlinear loads produce current waveforms like this, and so even-numbered harmonics (2nd, 4th, 6th, 8th, 10th, 12th, etc.) are absent or only minimally present in most AC power systems.

Examples of symmetrical waveforms – odd harmonics only :

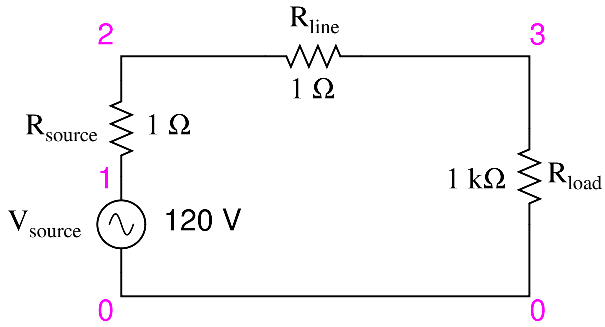


Examples of nonsymmetrical waveforms – even harmonics present :



Even though half of the possible harmonic frequencies are eliminated by the typically symmetrical distortion of nonlinear loads, the odd harmonics can still cause problems. Some of these problems are general to all power systems, single-phase or otherwise. Transformer overheating due to eddy current losses, for example, can occur in *any* AC power system where there is significant harmonic content. However, there are some problems caused by harmonic currents that are specific to polyphase power systems, and it is these problems to which this section is specifically devoted.

It is helpful to be able to simulate nonlinear loads in SPICE so as to avoid a lot of complex mathematics and obtain a more intuitive understanding of harmonic effects. First, we'll begin our simulation with a very simple AC circuit : a single sine-wave voltage source with a purely linear load and all associated resistances :



The R_{source} and R_{line} resistances in this circuit do more than just mimic the real world : they also provide convenient shunt resistances for measuring currents in the SPICE simulation : by reading voltage across a 1 Ω resistance, you obtain a direct indication of current through it, since E = IR.

A SPICE simulation of this circuit with Fourier analysis on the voltage measured across R_{line} should show us the harmonic content of this circuit's line current. Being completely linear in nature, we should expect no harmonics other than the 1st (fundamental) of 60 Hz, assuming a 60 Hz source :

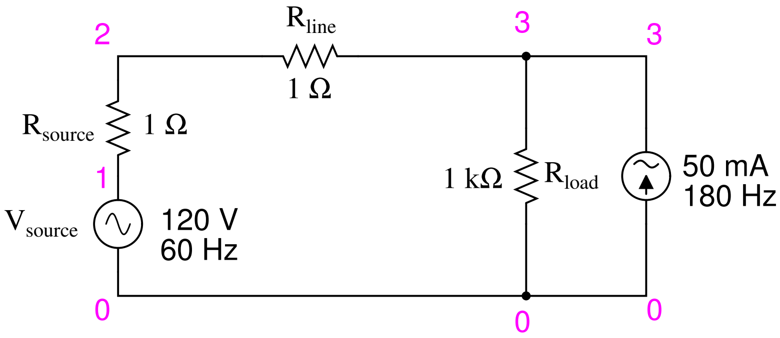
```
linear load simulation
vsource 1 0 sin(0 120 60 0 0)
rsource 1 2 1
rline 2 3 1
rload 3 0 1k
.options itl5=0
.tran 0.5m 30m 0 1u
.plot tran v(2,3)
.four 60 v(2,3)
.end
```

```
fourier components of transient response v(2,3)
dc component =   4.028E-12
harmonic frequency   fourier    normalized    phase    normalized
no          (hz)     component    component    (deg)    phase (deg)
1       6.000E+01   1.198E-01    1.000000   -72.000      0.000
2       1.200E+02   5.793E-12    0.000000    51.122    123.122
3       1.800E+02   7.407E-12    0.000000   -34.624     37.376
4       2.400E+02   9.056E-12    0.000000     4.267     76.267
5       3.000E+02   1.651E-11    0.000000   -83.461    -11.461
6       3.600E+02   3.931E-11    0.000000    36.399    108.399
7       4.200E+02   2.338E-11    0.000000   -41.343     30.657
8       4.800E+02   4.716E-11    0.000000    53.324    125.324
9       5.400E+02   3.453E-11    0.000000    21.691     93.691
total harmonic distortion =      0.000000   percent
```

A `.plot` command appears in the SPICE netlist, and normally this would result in a sine-wave graph output. In this case, however, I've purposely omitted the waveform display for brevity's sake – the `.plot` command is in the netlist simply to satisfy a quirk of SPICE's Fourier transform function.

No discrete Fourier transform is perfect, and so we see very small harmonic currents indicated (in the pico-amp range!) for all frequencies up to the 9th harmonic, which is as far as SPICE goes in performing Fourier analysis. We show 0.1198 amps (1.198E-01) for the "fourier component" of the 1st harmonic, or the fundamental frequency, which is our expected load current : about 120 mA, given a source voltage of 120 volts and a load resistance of 1 kΩ.

Next, I'd like to simulate a nonlinear load so as to generate harmonic currents. This can be done in two fundamentally different ways. One way is to design a load using nonlinear components such as diodes or other semiconductor devices which as easy to simulate with SPICE. Another is to add some AC current sources in parallel with the load resistor. The latter method is often preferred by engineers for simulating harmonics, since current sources of known value lend themselves better to mathematical network analysis than components with highly complex response characteristics. Since we're letting SPICE do all the math work, the complexity of a semiconductor component would cause no trouble for us, but since current sources can be fine-tuned to produce any arbitrary amount of current (a convenient feature), I'll choose the latter approach :




```
Nonlinear load simulation
vsource 1 0 sin(0 120 60 0 0)
rsource 1 2 1
rline 2 3 1
rload 3 0 1k
i3har 3 0 sin(0 50m 180 0 0)
.options itl5=0
.tran 0.5m 30m 0 1u
.plot tran v(2,3)
.four 60 v(2,3)
.end
```

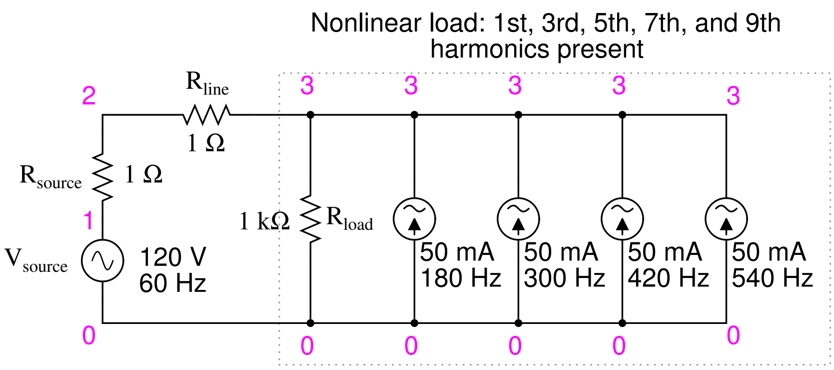
In this circuit, we have a current source of 50 mA magnitude and a frequency of 180 Hz, which is three times the source frequency of 60 Hz. Connected in parallel with the 1 kΩ load resistor, its current will add with the resistor's to make a nonsinusoidal total line current. I'll show the waveform plot here just so you can see the effects of this 3rd-harmonic current on the total current, which would ordinarily be a plain sine wave :

```
time        v(2,3)
- - - - - - - - - - -  - - - - - - - - - - - - - - - - - - - - - - -
0.000E+00  0.000E+00 .        .              *             .        .
5.000E-04  4.918E-02 .        .              .      *      .        .
1.000E-03  8.924E-02 .        .              .           * .        .
1.500E-03  1.137E-01 .        .              .             . *      .
2.000E-03  1.204E-01 .        .              .             .  *     .
2.500E-03  1.123E-01 .        .              .             . *      .
3.000E-03  9.595E-02 .        .              .            *.        .
3.500E-03  7.962E-02 .        .              .          *  .        .
4.000E-03  7.051E-02 .        .              .         *   .        .
4.500E-03  7.242E-02 .        .              .         *   .        .
5.000E-03  8.457E-02 .        .              .           * .        .
5.500E-03  1.018E-01 .        .              .             *        .
6.000E-03  1.163E-01 .        .              .             . *      .
6.500E-03  1.201E-01 .        .              .             .  *     .
7.000E-03  1.075E-01 .        .              .             .*       .
7.500E-03  7.738E-02 .        .              .          *  .        .
8.000E-03  3.338E-02 .        .              .    *        .        .
8.500E-03 -1.687E-02 .        .            * .             .        .
9.000E-03 -6.394E-02 .        .    *         .             .        .
9.500E-03 -9.932E-02 .        *              .             .        .
1.000E-02 -1.179E-01 .      * .              .             .        .
1.050E-02 -1.191E-01 .     *  .              .             .        .
1.100E-02 -1.074E-01 .       *.              .             .        .
1.150E-02 -9.010E-02 .        .*             .             .        .
1.200E-02 -7.551E-02 .        .  *           .             .        .
1.250E-02 -6.986E-02 .        .   *          .             .        .
1.300E-02 -7.551E-02 .        .  *           .             .        .
1.350E-02 -9.010E-02 .        .*             .             .        .
1.400E-02 -1.074E-01 .       *.              .             .        .
1.450E-02 -1.191E-01 .     *  .              .             .        .
1.500E-02 -1.179E-01 .      * .              .             .        .
1.550E-02 -9.932E-02 .        *              .             .        .
1.600E-02 -6.394E-02 .        .    *         .             .        .
1.650E-02 -1.687E-02 .        .            * .             .        .
- - - - - - - - - - -  - - - - - - - - - - - - - - - - - - - - - - -

fourier components of transient response v(2,3)
dc component =   1.349E-11
harmonic  frequency  fourier    normalized    phase    normalized
no          (hz)     component   component    (deg)    phase (deg)
1       6.000E+01   1.198E-01    1.000000   -72.000       0.000
2       1.200E+02   1.609E-11    0.000000    67.570     139.570
3       1.800E+02   4.990E-02    0.416667   144.000     216.000
4       2.400E+02   1.074E-10    0.000000  -169.546     -97.546
```

```
5       3.000E+02   3.871E-11     0.000000   169.582      241.582
6       3.600E+02   5.736E-11     0.000000   140.845      212.845
7       4.200E+02   8.407E-11     0.000000   177.071      249.071
8       4.800E+02   1.329E-10     0.000000   156.772      228.772
9       5.400E+02   2.619E-10     0.000000   160.498      232.498
total harmonic distortion =     41.666663   percent
```

In the Fourier analysis, the mixed frequencies are unmixed and presented separately. Here we see the same 0.1198 amps of 60 Hz (fundamental) current as we did in the first simulation, but appearing in the 3rd harmonic row we see 49.9 mA : our 50 mA, 180 Hz current source at work. Why don't we see the entire 50 mA through the line? Because that current source is connected across the 1 kΩ load resistor, so some of its current is shunted through the load and never goes through the line back to the source. It's an inevitable consequence of this type of simulation, where one part of the load is "normal" (a resistor) and the other part is imitated by a current source.

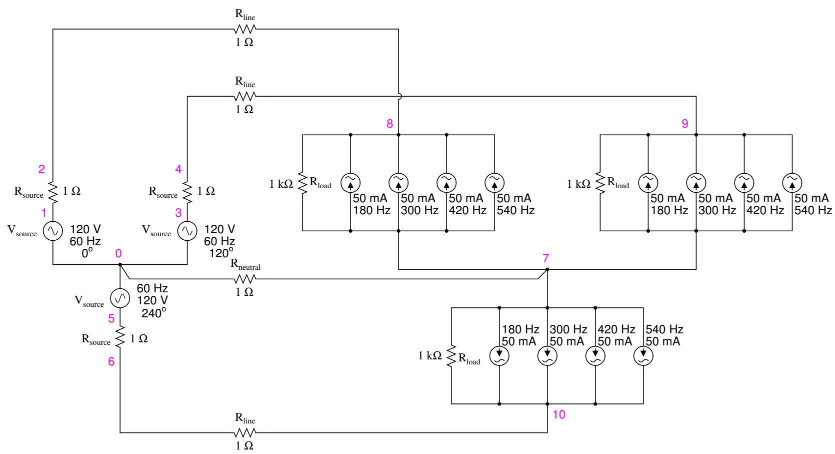
If we were to add more current sources to the "load," we would see further distortion of the line current waveform from the ideal sine-wave shape, and each of those harmonic currents would appear in the Fourier analysis breakdown :




```
Nonlinear load simulation
vsource 1 0 sin(0 120 60 0 0)
rsource 1 2 1
rline 2 3 1
rload 3 0 1k
i3har 3 0 sin(0 50m 180 0 0)
i5har 3 0 sin(0 50m 300 0 0)
i7har 3 0 sin(0 50m 420 0 0)
i9har 3 0 sin(0 50m 540 0 0)
.options itl5=0
.tran 0.5m 30m 0 1u
.plot tran v(2,3)
.four 60 v(2,3)
.end

fourier components of transient response v(2,3)
```

```
dc component =   6.299E-11
harmonic  frequency  fourier    normalized    phase    normalized
no          (hz)      component   component    (deg)    phase (deg)
1       6.000E+01   1.198E-01    1.000000   -72.000      0.000
2       1.200E+02   1.900E-09    0.000000   -93.908    -21.908
3       1.800E+02   4.990E-02    0.416667   144.000    216.000
4       2.400E+02   5.469E-09    0.000000  -116.873    -44.873
5       3.000E+02   4.990E-02    0.416667     0.000     72.000
6       3.600E+02   6.271E-09    0.000000    85.062    157.062
7       4.200E+02   4.990E-02    0.416666  -144.000    -72.000
8       4.800E+02   2.742E-09    0.000000   -38.781     33.219
9       5.400E+02   4.990E-02    0.416666    72.000    144.000
total harmonic distortion =    83.333296  percent
```

As you can see from the Fourier analysis, every harmonic current source is equally represented in the line current, at 49.9 mA each. So far, this is just a single-phase power system simulation. Things get more interesting when we make it a three-phase simulation. Two Fourier analyses will be performed : one for the voltage across a line resistor, and one for the voltage across the neutral resistor. As before, reading voltages across fixed resistances of 1 Ω each gives direct indications of current through those resistors :




```
Y-Y source/load 4-wire system with harmonics
*
* phase1 voltage source and r (120 v /_ 0 deg)
vsource1 1 0 sin(0 120 60 0 0)
rsource1 1 2 1
*
* phase2 voltage source and r (120 v /_ 120 deg)
vsource2 3 0 sin(0 120 60 5.55555m 0)
rsource2 3 4 1
*
```

```
* phase3 voltage source and r (120 v /_ 240 deg)
vsource3 5 0 sin(0 120 60 11.1111m 0)
rsource3 5 6 1
*
* line and neutral wire resistances
rline1 2 8 1
rline2 4 9 1
rline3 6 10 1
rneutral 0 7 1
*
* phase 1 of load
rload1 8 7 1k
i3har1 8 7 sin(0 50m 180 0 0)
i5har1 8 7 sin(0 50m 300 0 0)
i7har1 8 7 sin(0 50m 420 0 0)
i9har1 8 7 sin(0 50m 540 0 0)
*
* phase 2 of load
rload2 9 7 1k
i3har2 9 7 sin(0 50m 180 5.55555m 0)
i5har2 9 7 sin(0 50m 300 5.55555m 0)
i7har2 9 7 sin(0 50m 420 5.55555m 0)
i9har2 9 7 sin(0 50m 540 5.55555m 0)
*
* phase 3 of load
rload3 10 7 1k
i3har3 10 7 sin(0 50m 180 11.1111m 0)
i5har3 10 7 sin(0 50m 300 11.1111m 0)
i7har3 10 7 sin(0 50m 420 11.1111m 0)
i9har3 10 7 sin(0 50m 540 11.1111m 0)
*
* analysis stuff
.options itl5=0
.tran 0.5m 100m 12m 1u
.plot tran v(2,8)
.four 60 v(2,8)
.plot tran v(0,7)
.four 60 v(0,7)
.end
```

Fourier analysis of line current :

```
fourier components of transient response v(2,8)
dc component =  -6.404E-12
harmonic  frequency  fourier    normalized    phase    normalized
no          (hz)     component    component    (deg)    phase (deg)
1      6.000E+01   1.198E-01     1.000000     0.000       0.000
```

```
2       1.200E+02   2.218E-10     0.000000   172.985      172.985
3       1.800E+02   4.975E-02     0.415423     0.000        0.000
4       2.400E+02   4.236E-10     0.000000   166.990      166.990
5       3.000E+02   4.990E-02     0.416667     0.000        0.000
6       3.600E+02   1.877E-10     0.000000  -147.146     -147.146
7       4.200E+02   4.990E-02     0.416666     0.000        0.000
8       4.800E+02   2.784E-10     0.000000  -148.811     -148.811
9       5.400E+02   4.975E-02     0.415422     0.000        0.000
total harmonic distortion =     83.209009  percent
```

Fourier analysis of neutral current :

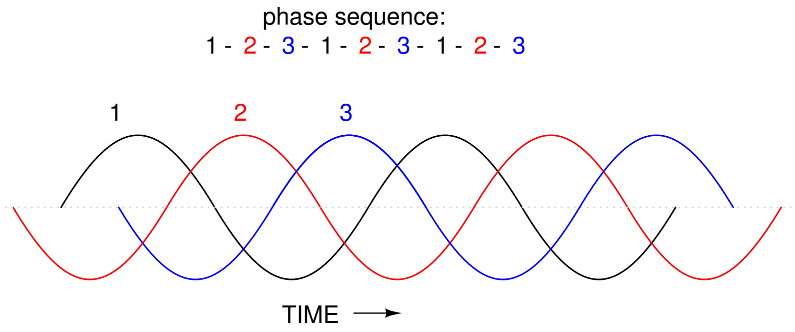
```
fourier components of transient response v(0,7)
dc component =   1.819E-10
harmonic  frequency  fourier    normalized    phase    normalized
no          (hz)     component   component    (deg)    phase (deg)
1       6.000E+01   4.337E-07     1.000000    60.018        0.000
2       1.200E+02   1.869E-10     0.000431    91.206       31.188
3       1.800E+02   1.493E-01  344147.7638  -180.000     -240.018
4       2.400E+02   1.257E-09     0.002898   -21.103      -81.121
5       3.000E+02   9.023E-07     2.080596   119.981       59.963
6       3.600E+02   3.396E-10     0.000783    15.882      -44.136
7       4.200E+02   1.264E-06     2.913955    59.993       -0.025
8       4.800E+02   5.975E-10     0.001378    35.584      -24.434
9       5.400E+02   1.493E-01  344147.4889  -179.999     -240.017
```

This is a balanced Y-Y power system, each phase identical to the single-phase AC system simulated earlier. Consequently, it should come as no surprise that the Fourier analysis for line current in one phase of the 3-phase system is nearly identical to the Fourier analysis for line current in the single-phase system : a fundamental (60 Hz) line current of 0.1198 amps, and odd harmonic currents of approximately 50 mA each.

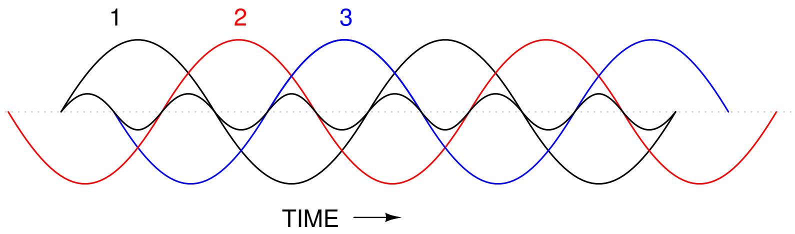
What should be surprising here is the analysis for the neutral conductor's current, as determined by the voltage drop across the $R_{neutral}$ resistor between SPICE nodes 0 and 7. In a balanced 3-phase Y load, we would expect the neutral current to be zero. Each phase current – which by itself would go through the neutral wire back to the supplying phase on the source Y – should cancel each other in regard to the neutral conductor because they're all the same magnitude and all shifted 120^o apart. In a system with no harmonic currents, this *is* what happens, leaving zero current through the neutral conductor. However, we cannot say the same for *harmonic* currents in the same system.

Note that the fundamental frequency (60 Hz, or the 1st harmonic) current is virtually absent from the neutral conductor. Our Fourier analysis shows only 0.4337 μA of 1st harmonic when reading voltage across $R_{neutral}$. The same may be said about the 5th and 7th harmonics, both of those currents having negligible magnitude. In contrast, the 3rd and 9th harmonics are strongly represented within the neutral conductor, with 149.3 mA (1.493E-01 volts across 1 Ω) each! This is very nearly 150 mA, or three times the current sources' values, individually. With three sources per harmonic frequency in the load, it appears our 3rd and 9th harmonic currents in each phase are *adding* to form the neutral current.

This is exactly what's happening, though it might not be apparent why this is so. The key to understanding this is made clear in a time-domain graph of phase currents. Examine this plot of balanced phase currents over time, with a phase sequence of 1-2-3 :

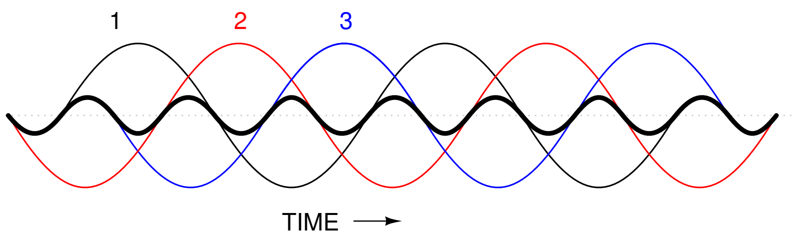


With the three fundamental waveforms equally shifted across the time axis of the graph, it is easy to see how they would cancel each other to give a resultant current of zero in the neutral conductor. Let's consider, though, what a 3rd harmonic waveform for phase 1 would look like superimposed on the graph :



Observe how this harmonic waveform has the same phase relationship to the 2nd and 3rd fundamental waveforms as it does with the 1st : in each positive half-cycle of *any* of the fundamental waveforms, you will find exactly two positive half-cycles and one negative half-cycle of the harmonic waveform. What this means is that the 3rd-harmonic waveforms of three 120^o phase-shifted fundamental-frequency waveforms are actually *in phase* with each other. The phase shift figure of 120^o generally assumed in three-phase AC systems applies only to the fundamental frequencies, not to their harmonic multiples!

If we were to plot all three 3rd-harmonic waveforms on the same graph, we would see them precisely overlap and appear as a single, unified waveform (shown here in bold) :



For the more mathematically inclined, this principle may be expressed symbolically. Suppose that **A** represents one waveform and **B** another, both at the same frequency, but shifted 120^o from each other in terms of phase. Let's call the 3rd harmonic of each waveform **A'** and **B'**, respectively. The phase shift between **A'** and **B'** is not 120^o (that is the phase shift between **A** and **B**), but 3 times that, because the **A'** and **B'** waveforms alternate three times as fast as **A** and **B**. The shift between waveforms is only accurately expressed in terms of *phase angle* when the same angular velocity is assumed. When relating waveforms of different frequency, the most accurate way to represent phase shift is in terms of *time*; and the *time-shift* between **A'** and **B'** is equivalent to 120^o at a frequency three times lower, or 360^o at the frequency of **A'** and **B'**. A phase shift of 360^o is the same as a phase shift of 0^o, which is to say no phase shift at all. Thus, **A'** and **B'** must be in phase with each other :

Phase sequence = ***A-B-C***

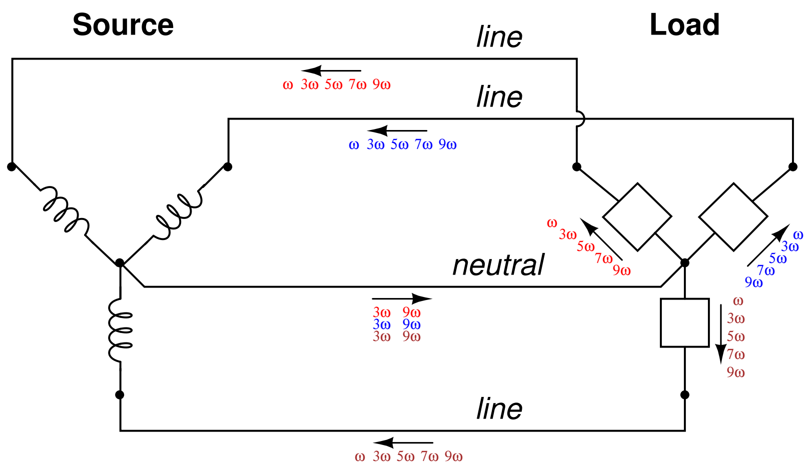
Fundamental	**A** 0°	**B** 120°	**C** 240°
3rd harmonic	**A'** 3 x 0° (0°)	**B'** 3 x 120° (360° = 0°)	**C'** 3 x 240° (720° = 0°)

This characteristic of the 3rd harmonic in a three-phase system also holds true for any integer multiples of the 3rd harmonic. So, not only are the 3rd harmonic waveforms of each fundamental waveform in phase with each other, but so are the 6th harmonics, the 9th harmonics, the 12th harmonics, the 15th harmonics, the 18th harmonics, the 21st harmonics, and so on. Since only odd harmonics appear in systems where waveform distortion is symmetrical about the centerline – and most nonlinear loads create symmetrical distortion – even-numbered multiples of the 3rd harmonic (6th, 12th, 18th, etc.) are generally not significant, leaving only the odd-numbered multiples (3rd, 9th, 21st, etc.) to significantly contribute to neutral currents.

In polyphase power systems with some number of phases other than three, this effect occurs with harmonics of the same multiple. For instance, the harmonic currents that add in the neutral conductor of a star-connected 4-phase system where the phase shift between fundamental waveforms is 90^o would be the 4th, 8th, 12th, 16th, 20th, and so on.

Due to their abundance and significance in three-phase power systems, the 3rd harmonic and its multiples have their own special name : *triplen harmonics*. All triplen harmonics add with each other in the neutral conductor of a 4-wire Y-connected load. In power systems containing substantial nonlinear loading, the triplen harmonic currents may be of great enough magnitude to cause neutral conductors to overheat. This is very problematic, as other safety concerns prohibit neutral conductors from having overcurrent protection, and thus there is no provision for automatic interruption of these high currents.

The following illustration shows how triplen harmonic currents created at the load add within the neutral conductor. The symbol "ω" is used to represent angular velocity, and is mathematically equivalent to 2πf. So, "ω" represents the fundamental frequency, "3ω " represents the 3rd harmonic, "5ω" represents the 5th harmonic, and so on :



Triplen harmonic currents add in neutral conductor

In an effort to mitigate these additive triplen currents, one might be tempted to remove the neutral wire entirely. If there is no neutral wire in which triplen currents can flow together, then they won't, right? Unfortunately, doing so just causes a different problem : the load's "Y" center-point will no longer be at the same potential as the source's, meaning that each phase of the load will receive a different voltage than what is produced by the source. We'll re-run the last SPICE simulation without the 1 Ω $R_{neutral}$ resistor and see what happens :

```
Y-Y source/load (no neutral) with harmonics
*
* phase1 voltage source and r (120 v /_ 0 deg)
vsource1 1 0 sin(0 120 60 0 0)
rsource1 1 2 1
*
* phase2 voltage source and r (120 v /_ 120 deg)
vsource2 3 0 sin(0 120 60 5.55555m 0)
rsource2 3 4 1
*
* phase3 voltage source and r (120 v /_ 240 deg)
vsource3 5 0 sin(0 120 60 11.1111m 0)
rsource3 5 6 1
*
* line resistances
rline1 2 8 1
rline2 4 9 1
rline3 6 10 1
*
* phase 1 of load
rload1 8 7 1k
i3har1 8 7 sin(0 50m 180 0 0)
i5har1 8 7 sin(0 50m 300 0 0)
```

```
i7har1 8 7 sin(0 50m 420 0 0)
i9har1 8 7 sin(0 50m 540 0 0)
*
* phase 2 of load
rload2 9 7 1k
i3har2 9 7 sin(0 50m 180 5.55555m 0)
i5har2 9 7 sin(0 50m 300 5.55555m 0)
i7har2 9 7 sin(0 50m 420 5.55555m 0)
i9har2 9 7 sin(0 50m 540 5.55555m 0)
*
* phase 3 of load
rload3 10 7 1k
i3har3 10 7 sin(0 50m 180 11.1111m 0)
i5har3 10 7 sin(0 50m 300 11.1111m 0)
i7har3 10 7 sin(0 50m 420 11.1111m 0)
i9har3 10 7 sin(0 50m 540 11.1111m 0)
*
* analysis stuff
.options itl5=0
.tran 0.5m 100m 12m 1u
.plot tran v(2,8)
.four 60 v(2,8)
.plot tran v(0,7)
.four 60 v(0,7)
.plot tran v(8,7)
.four 60 v(8,7)
.end
```

Fourier analysis of line current :

```
fourier components of transient response v(2,8)
dc component =   5.423E-11
harmonic  frequency  fourier    normalized    phase    normalized
no          (hz)      component   component    (deg)    phase (deg)
1      6.000E+01   1.198E-01    1.000000     0.000       0.000
2      1.200E+02   2.388E-10    0.000000   158.016     158.016
3      1.800E+02   3.136E-07    0.000003   -90.009     -90.009
4      2.400E+02   5.963E-11    0.000000  -111.510    -111.510
5      3.000E+02   4.990E-02    0.416665     0.000       0.000
6      3.600E+02   8.606E-11    0.000000  -124.565    -124.565
7      4.200E+02   4.990E-02    0.416668     0.000       0.000
8      4.800E+02   8.126E-11    0.000000  -159.638    -159.638
9      5.400E+02   9.406E-07    0.000008   -90.005     -90.005
total harmonic distortion =     58.925539  percent
```

Fourier analysis of voltage between the two "Y" center-points :

```
fourier components of transient response v(0,7)
```

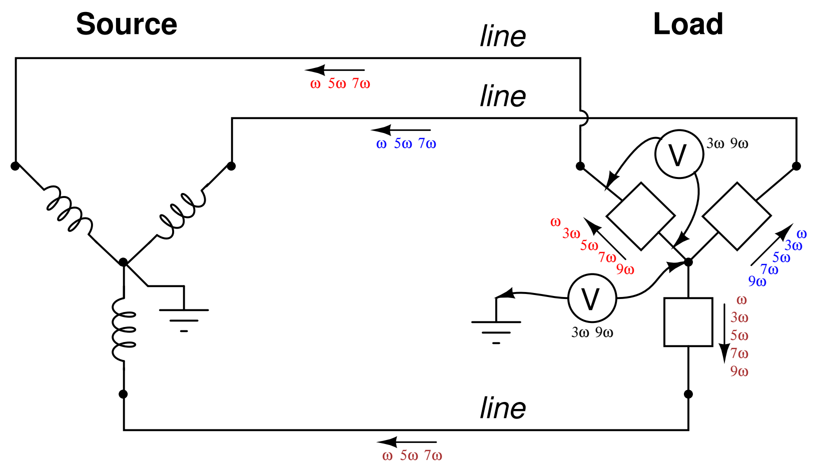
```
dc component =   6.093E-08
harmonic  frequency  fourier    normalized    phase    normalized
no         (hz)      component    component    (deg)    phase (deg)
1      6.000E+01   1.453E-04     1.000000    60.018       0.000
2      1.200E+02   6.263E-08     0.000431    91.206      31.188
3      1.800E+02   5.000E+01  344147.7879  -180.000    -240.018
4      2.400E+02   4.210E-07     0.002898   -21.103     -81.121
5      3.000E+02   3.023E-04     2.080596   119.981      59.963
6      3.600E+02   1.138E-07     0.000783    15.882     -44.136
7      4.200E+02   4.234E-04     2.913955    59.993      -0.025
8      4.800E+02   2.001E-07     0.001378    35.584     -24.434
9      5.400E+02   5.000E+01  344147.4728  -179.999    -240.017
total harmonic distortion =  ************  percent
```

Fourier analysis of load phase voltage :

```
fourier components of transient response v(8,7)
dc component =   6.070E-08
harmonic  frequency  fourier    normalized    phase    normalized
no         (hz)      component    component    (deg)    phase (deg)
1      6.000E+01   1.198E+02     1.000000     0.000       0.000
2      1.200E+02   6.231E-08     0.000000    90.473      90.473
3      1.800E+02   5.000E+01     0.417500  -180.000    -180.000
4      2.400E+02   4.278E-07     0.000000   -19.747     -19.747
5      3.000E+02   9.995E-02     0.000835   179.850     179.850
6      3.600E+02   1.023E-07     0.000000    13.485      13.485
7      4.200E+02   9.959E-02     0.000832   179.790     179.789
8      4.800E+02   1.991E-07     0.000000    35.462      35.462
9      5.400E+02   5.000E+01     0.417499  -179.999    -179.999
total harmonic distortion =     59.043467  percent
```

Strange things are happening, indeed. First, we see that the triplen harmonic currents (3rd and 9th) all but disappear in the lines connecting load to source. The 5th and 7th harmonic currents are present at their normal levels (approximately 50 mA), but the 3rd and 9th harmonic currents are of negligible magnitude. Second, we see that there is substantial harmonic voltage between the two "Y" center-points, between which the neutral conductor used to connect. According to SPICE, there is 50 volts of both 3rd and 9th harmonic frequency between these two points, which is definitely not normal in a linear (no harmonics), balanced Y system. Finally, the voltage as measured across one of the load's phases (between nodes 8 and 7 in the SPICE analysis) likewise shows strong triplen harmonic voltages of 50 volts each.

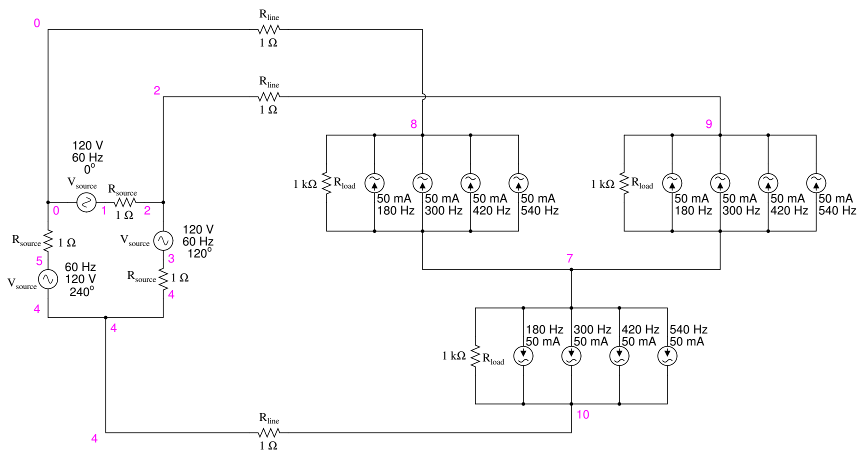
The following illustration is a graphical summary of the aforementioned effects :



Triplen voltages appear between Y centers.
Triplen voltages appear across load phases.
Non-triplen currents appear in line conductors.

In summary, removal of the neutral conductor leads to a "hot" center-point on the load "Y", and also to harmonic load phase voltages of equal magnitude, all comprised of triplen frequencies. In the previous simulation where we had a 4-wire, Y-connected system, the undesirable effect from harmonics was excessive neutral *current*, but at least each phase of the load received voltage nearly free of harmonics.

Since removing the neutral wire didn't seem to work in eliminating the problems caused by harmonics, perhaps switching to a Δ configuration will. Let's try a Δ source instead of a Y, keeping the load in its present Y configuration, and see what happens. The measured parameters will be line current (voltage across R_{line}, nodes 0 and 8), load phase voltage (nodes 8 and 7), and source phase current (voltage across R_{source}, nodes 1 and 2) :




```
Delta-Y source/load with harmonics
*
```

```
* phase1 voltage source and r (120 v /_ 0 deg)
vsource1 1 0 sin(0 207.846 60 0 0)
rsource1 1 2 1
*
* phase2 voltage source and r (120 v /_ 120 deg)
vsource2 3 2 sin(0 207.846 60 5.55555m 0)
rsource2 3 4 1
*
* phase3 voltage source and r (120 v /_ 240 deg)
vsource3 5 4 sin(0 207.846 60 11.1111m 0)
rsource3 5 0 1
*
* line resistances
rline1 0 8 1
rline2 2 9 1
rline3 4 10 1
*
* phase 1 of load
rload1 8 7 1k
i3har1 8 7 sin(0 50m 180 9.72222m 0)
i5har1 8 7 sin(0 50m 300 9.72222m 0)
i7har1 8 7 sin(0 50m 420 9.72222m 0)
i9har1 8 7 sin(0 50m 540 9.72222m 0)
*
* phase 2 of load
rload2 9 7 1k
i3har2 9 7 sin(0 50m 180 15.2777m 0)
i5har2 9 7 sin(0 50m 300 15.2777m 0)
i7har2 9 7 sin(0 50m 420 15.2777m 0)
i9har2 9 7 sin(0 50m 540 15.2777m 0)
*
* phase 3 of load
rload3 10 7 1k
i3har3 10 7 sin(0 50m 180 4.16666m 0)
i5har3 10 7 sin(0 50m 300 4.16666m 0)
i7har3 10 7 sin(0 50m 420 4.16666m 0)
i9har3 10 7 sin(0 50m 540 4.16666m 0)
*
* analysis stuff
.options itl5=0
.tran 0.5m 100m 16m 1u
.plot tran v(0,8) v(8,7) v(1,2)
.four 60 v(0,8) v(8,7) v(1,2)
.end
```

Note : the following paragraph is for those curious readers who follow every detail of my SPICE *netlists. If you just want to find out what happens in the circuit, skip this paragraph!* When simulating circuits having AC sources of differing frequency and differing phase, the only way to do it in SPICE is to set up the sources with a *delay time* or *phase offset* specified in seconds. Thus, the 0^o source has these five specifying figures : "(0 207.846 60 0 0)", which means 0 volts DC offset, 207.846 volts peak amplitude (120 times the square root of three, to ensure the load phase voltages remain at 120 volts each), 60 Hz, 0 time delay, and 0 damping factor. The 120^o phase-shifted source has these figures : "(0 207.846 60 5.55555m 0)", all the same as the first except for the time delay factor of 5.55555 milliseconds, or 1/3 of the full period of 16.6667 milliseconds for a 60 Hz waveform. The 240^o source must be time-delayed twice that amount, equivalent to a fraction of 240/360 of 16.6667 milliseconds, or 11.1111 milliseconds. This is for the Δ-connected source. The Y-connected load, on the other hand, requires a different set of time-delay figures for its harmonic current sources, because the phase voltages in a Y load are not in phase with the phase voltages of a Δ source. If Δ source voltages V_{AC}, V_{BA}, and V_{CB} are referenced at 0^o, 120^o, and 240^o, respectively, then "Y" load voltages V_A, V_B, and V_C will have phase angles of -30^o, 90^o, and 210^o, respectively. This is an intrinsic property of all Δ-Y circuits and not a quirk of SPICE. Therefore, when I specified the delay times for the harmonic sources, I had to set them at 15.2777 milliseconds (-30^o, or +330^o), 4.16666 milliseconds (90^o), and 9.72222 milliseconds (210^o). One final note : when delaying AC sources in SPICE, they don't "turn on" until their delay time has elapsed, which means any mathematical analysis up to that point in time will be in error. Consequently, I set the `.tran` transient analysis line to hold off analysis until 16 milliseconds after start, which gives all sources in the netlist time to engage before any analysis takes place.

The result of this analysis is almost as disappointing as the last. Line currents remain unchanged (the only substantial harmonic content being the 5th and 7th harmonics), and load phase voltages remain unchanged as well, with a full 50 volts of triplen harmonic (3rd and 9th) frequencies across each load component. Source phase current is a fraction of the line current, which should come as no surprise. Both 5th and 7th harmonics are represented there, with negligible triplen harmonics :

Fourier analysis of line current :

```
fourier components of transient response v(0,8)
dc component =  -6.850E-11
harmonic  frequency  fourier    normalized    phase    normalized
no          (hz)     component   component    (deg)    phase (deg)
1       6.000E+01   1.198E-01    1.000000   150.000       0.000
2       1.200E+02   2.491E-11    0.000000   159.723       9.722
3       1.800E+02   1.506E-06    0.000013     0.005    -149.996
4       2.400E+02   2.033E-11    0.000000    52.772     -97.228
5       3.000E+02   4.994E-02    0.416682    30.002    -119.998
6       3.600E+02   1.234E-11    0.000000    57.802     -92.198
7       4.200E+02   4.993E-02    0.416644   -29.998    -179.998
8       4.800E+02   8.024E-11    0.000000  -174.200    -324.200
9       5.400E+02   4.518E-06    0.000038  -179.995    -329.995
total harmonic distortion =     58.925038  percent
```

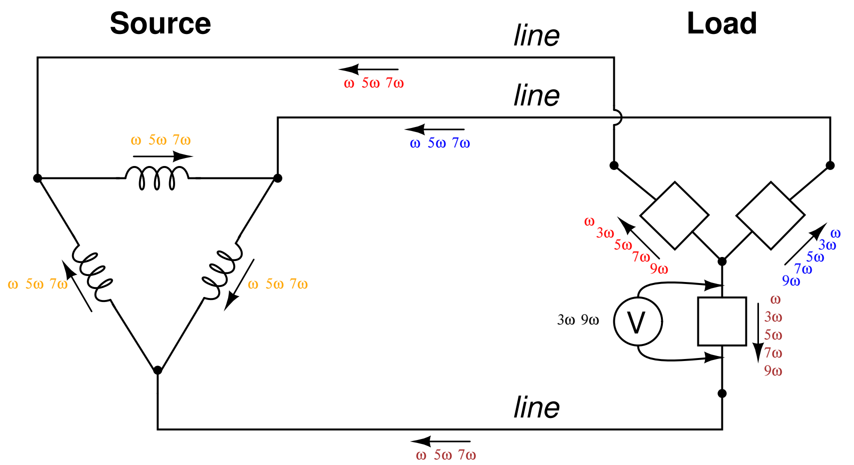
Fourier analysis of load phase voltage :

```
fourier components of transient response v(8,7)
dc component =   1.259E-08
```

```
harmonic  frequency  fourier    normalized    phase    normalized
no          (hz)     component   component    (deg)    phase (deg)
1       6.000E+01   1.198E+02    1.000000   150.000       0.000
2       1.200E+02   1.941E-07    0.000000    49.693    -100.307
3       1.800E+02   5.000E+01    0.417222   -89.998    -239.998
4       2.400E+02   1.519E-07    0.000000    66.397     -83.603
5       3.000E+02   6.466E-02    0.000540  -151.112    -301.112
6       3.600E+02   2.433E-07    0.000000    68.162     -81.838
7       4.200E+02   6.931E-02    0.000578   148.548      -1.453
8       4.800E+02   2.398E-07    0.000000  -174.897    -324.897
9       5.400E+02   5.000E+01    0.417221    90.006     -59.995
total harmonic distortion =     59.004109  percent
```

Fourier analysis of source phase current :

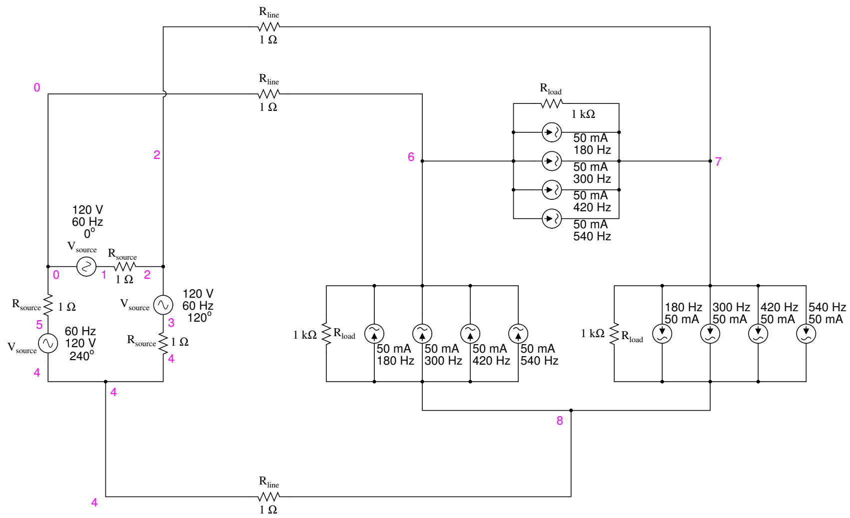
```
fourier components of transient response v(1,2)
dc component =   3.564E-11
harmonic  frequency  fourier    normalized    phase    normalized
no          (hz)     component   component    (deg)    phase (deg)
1       6.000E+01   6.906E-02    1.000000    -0.181       0.000
2       1.200E+02   1.525E-11    0.000000  -156.674    -156.493
3       1.800E+02   1.422E-06    0.000021  -179.996    -179.815
4       2.400E+02   2.949E-11    0.000000  -110.570    -110.390
5       3.000E+02   2.883E-02    0.417440  -179.996    -179.815
6       3.600E+02   2.324E-11    0.000000   -91.926     -91.745
7       4.200E+02   2.883E-02    0.417398  -179.994    -179.813
8       4.800E+02   4.140E-11    0.000000   -39.875     -39.694
9       5.400E+02   4.267E-06    0.000062     0.006       0.186
total harmonic distortion =     59.031969  percent
```



Triplen voltages appear across load phases.
Non-triplen currents appear in line conductors
and in source phase windings.

Really, the only advantage of the Δ-Y configuration from the standpoint of harmonics is that there is no longer a center-point at the load posing a shock hazard. Otherwise, the load components receive the same harmonically-rich voltages and the lines see the same currents as in a three-wire Y system.

If we were to reconfigure the system into a Δ-Δ arrangement, that should guarantee that each load component receives non-harmonic voltage, since each load phase would be directly connected in parallel with each source phase. The complete lack of any neutral wires or "center points" in a Δ-Δ system prevents strange voltages or additive currents from occurring. It would seem to be the ideal solution. Let's simulate and observe, analyzing line current, load phase voltage, and source phase current :



```
Delta-Delta source/load with harmonics
*
* phase1 voltage source and r (120 v /_ 0 deg)
vsource1 1 0 sin(0 120 60 0 0)
rsource1 1 2 1
*
* phase2 voltage source and r (120 v /_ 120 deg)
vsource2 3 2 sin(0 120 60 5.55555m 0)
rsource2 3 4 1
*
* phase3 voltage source and r (120 v /_ 240 deg)
vsource3 5 4 sin(0 120 60 11.1111m 0)
rsource3 5 0 1
*
* line resistances
rline1 0 6 1
rline2 2 7 1
rline3 4 8 1
*
* phase 1 of load
rload1 7 6 1k
i3har1 7 6 sin(0 50m 180 0 0)
i5har1 7 6 sin(0 50m 300 0 0)
i7har1 7 6 sin(0 50m 420 0 0)
i9har1 7 6 sin(0 50m 540 0 0)
*
* phase 2 of load
rload2 8 7 1k
i3har2 8 7 sin(0 50m 180 5.55555m 0)
i5har2 8 7 sin(0 50m 300 5.55555m 0)
i7har2 8 7 sin(0 50m 420 5.55555m 0)
i9har2 8 7 sin(0 50m 540 5.55555m 0)
*
* phase 3 of load
rload3 6 8 1k
i3har3 6 8 sin(0 50m 180 11.1111m 0)
i5har3 6 8 sin(0 50m 300 11.1111m 0)
i7har3 6 8 sin(0 50m 420 11.1111m 0)
i9har3 6 8 sin(0 50m 540 11.1111m 0)
*
* analysis stuff
.options itl5=0
.tran 0.5m 100m 16m 1u
.plot tran v(0,6) v(7,6) v(2,1) i(3har1)
.four 60 v(0,6) v(7,6) v(2,1)
.end
```

Fourier analysis of line current :

```
fourier components of transient response v(0,6)
dc component =  -6.007E-11
harmonic  frequency  fourier    normalized    phase    normalized
no         (hz)     component    component    (deg)    phase (deg)
1      6.000E+01   2.070E-01    1.000000   150.000       0.000
2      1.200E+02   5.480E-11    0.000000   156.666       6.666
3      1.800E+02   6.257E-07    0.000003    89.990     -60.010
4      2.400E+02   4.911E-11    0.000000     8.187    -141.813
5      3.000E+02   8.626E-02    0.416664  -149.999    -300.000
6      3.600E+02   1.089E-10    0.000000   -31.997    -181.997
7      4.200E+02   8.626E-02    0.416669   150.001       0.001
8      4.800E+02   1.578E-10    0.000000   -63.940    -213.940
9      5.400E+02   1.877E-06    0.000009    89.987     -60.013
total harmonic distortion =     58.925538  percent
```

Fourier analysis of load phase voltage :

```
fourier components of transient response v(7,6)
dc component =  -5.680E-10
harmonic  frequency  fourier    normalized    phase    normalized
no         (hz)     component    component    (deg)    phase (deg)
1      6.000E+01   1.195E+02    1.000000     0.000       0.000
2      1.200E+02   1.039E-09    0.000000   144.749     144.749
3      1.800E+02   1.251E-06    0.000000    89.974      89.974
4      2.400E+02   4.215E-10    0.000000    36.127      36.127
5      3.000E+02   1.992E-01    0.001667  -180.000    -180.000
6      3.600E+02   2.499E-09    0.000000    -4.760      -4.760
7      4.200E+02   1.992E-01    0.001667  -180.000    -180.000
8      4.800E+02   2.951E-09    0.000000  -151.385    -151.385
9      5.400E+02   3.752E-06    0.000000    89.905      89.905
total harmonic distortion =      0.235702  percent
```

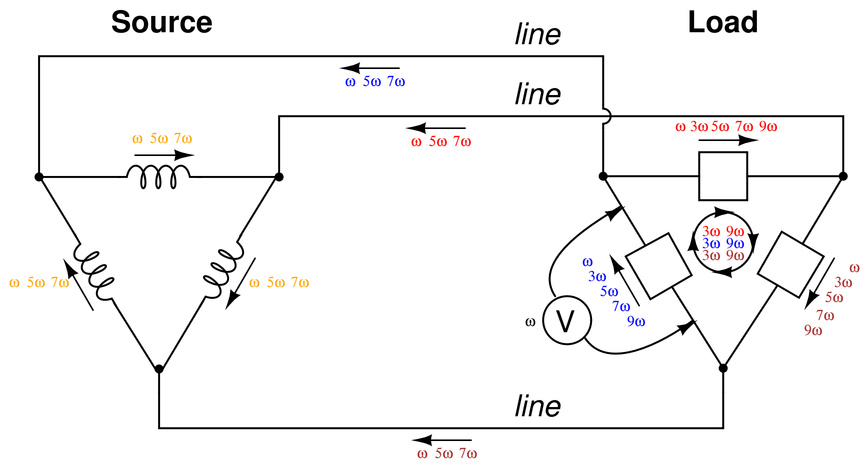
Fourier analysis of source phase current :

```
fourier components of transient response v(2,1)
dc component =  -1.923E-12
harmonic  frequency  fourier    normalized    phase    normalized
no         (hz)     component    component    (deg)    phase (deg)
1      6.000E+01   1.194E-01    1.000000   179.940       0.000
2      1.200E+02   2.569E-11    0.000000   133.491     -46.449
3      1.800E+02   3.129E-07    0.000003    89.985     -89.955
4      2.400E+02   2.657E-11    0.000000    23.368    -156.571
5      3.000E+02   4.980E-02    0.416918  -180.000    -359.939
6      3.600E+02   4.595E-11    0.000000   -22.475    -202.415
7      4.200E+02   4.980E-02    0.416921  -180.000    -359.939
8      4.800E+02   7.385E-11    0.000000   -63.759    -243.699
9      5.400E+02   9.385E-07    0.000008    89.991     -89.949
```

```
total harmonic distortion =     58.961298   percent
```

As predicted earlier, the load phase voltage is almost a pure sine-wave, with negligible harmonic content, thanks to the direct connection with the source phases in a Δ-Δ system. But what happened to the triplen harmonics? The 3rd and 9th harmonic frequencies don't appear in any substantial amount in the line current, nor in the load phase voltage, nor in the source phase current! We know that triplen currents exist, because the 3rd and 9th harmonic current sources are intentionally placed in the phases of the load, but where did those currents go?

Remember that the triplen harmonics of 120^o phase-shifted fundamental frequencies are in phase with each other. Note the directions that the arrows of the current sources within the load phases are pointing, and think about what would happen if the 3rd and 9th harmonic sources were DC sources instead. What we would have is current *circulating within the loop formed by the Δ-connected phases*. This is where the triplen harmonic currents have gone : they stay within the Δ of the load, never reaching the line conductors or the windings of the source. These results may be graphically summarized as such :



Load phases receive undistorted sine-wave voltage.
Triplen currents are confined to circulate within load phases.
Non-triplen currents appear in line conductors
and in source phase windings.

This is a major benefit of the Δ-Δ system configuration : triplen harmonic currents remain confined in whatever set of components create them, and do not "spread" to other parts of the system.

- **REVIEW :**
- *Nonlinear* components are those that draw a non-sinusoidal (non-sine-wave) current waveform when energized by a sinusoidal (sine-wave) voltage. Since any distortion of an originally pure sine-wave constitutes harmonic frequencies, we can say that nonlinear components generate harmonic currents.
- When the sine-wave distortion is symmetrical above and below the average centerline of the waveform, the only harmonics present will be *odd-numbered*, not even-numbered.

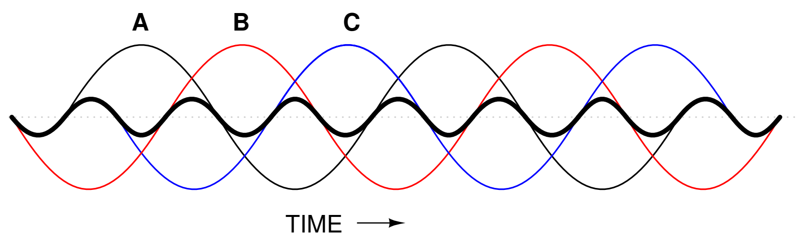
- The 3rd harmonic, and integer multiples of it (6th, 9th, 12th, 15th) are known as *triplen* harmonics. They are in phase with each other, despite the fact that their respective fundamental waveforms are 120^o out of phase with each other.

- In a 4-wire Y-Y system, triplen harmonic currents add within the neutral conductor.

- Triplen harmonic currents in a Δ-connected set of components circulate within the loop formed by the Δ.

10.8 Harmonic phase sequences

In the last section, we saw how the 3rd harmonic and all of its integer multiples (collectively called *triplen* harmonics) generated by 120^o phase-shifted fundamental waveforms are actually in phase with each other. In a 60 Hz three-phase power system, where phases **A**, **B**, and **C** are 120^o apart, the third-harmonic multiples of those frequencies (180 Hz) fall perfectly into phase with each other. This can be thought of in graphical terms, and/or in mathematical terms :



*Phase sequence = **A-B-C***

Fundamental	**A** 0^o	**B** 120^o	**C** 240^o
3rd harmonic	**A'** 3×0^o (0^o)	**B'** 3×120^o $(360^o = 0^o)$	**C'** 3×240^o $(720^o = 0^o)$

If we extend the mathematical table to include higher odd-numbered harmonics, we will notice an interesting pattern develop with regard to the rotation or sequence of the harmonic frequencies :

Fundamental	**A** 0^o	**B** 120^o	**C** 240^o	**A-B-C**
3rd harmonic	**A'** 3×0^o (0^o)	**B'** 3×120^o $(360^o = 0^o)$	**C'** 3×240^o $(720^o = 0^o)$	*no rotation*
5th harmonic	**A''** 5×0^o (0^o)	**B''** 5×120^o $(600^o = 720^o - 120^o)$ (-120^o)	**C''** 5×240^o $(1200^o = 1440^o - 240^o)$ (-240^o)	**C-B-A**
7th harmonic	**A'''** 7×0^o (0^o)	**B'''** 7×120^o $(840^o = 720^o + 120^o)$ (120^o)	**C'''** 7×240^o $(1680^o = 1440^o + 240^o)$ (240^o)	**A-B-C**
9th harmonic	**A''''** 9×0^o (0^o)	**B''''** 9×120^o $(1080^o = 0^o)$	**C''''** 9×240^o $(2160^o = 0^o)$	*no rotation*

Harmonics such as the 7th, which "rotate" with the same sequence as the fundamental, are called *positive sequence*. Harmonics such as the 5th, which "rotate" in the opposite sequence as the fundamental, are called *negative sequence*. Triplen harmonics (3rd and 9th shown in this table) which don't "rotate" at all because they're in phase with each other, are called *zero sequence*.

This pattern of positive-zero-negative-positive continues indefinitely for all odd-numbered harmonics, lending itself to expression in a table like this :

Rotation sequences according to harmonic number

+	1st	7th	13th	19th	← Rotates with fundamental
0	3rd	9th	15th	21st	← Does not rotate
-	5th	11th	17th	23rd	← Rotates against fundamental

Sequence especially matters when we're dealing with AC motors, since the mechanical rotation of the rotor depends on the torque produced by the sequential "rotation" of the applied 3-phase power. Positive-sequence frequencies work to push the rotor in the proper direction, whereas negative-sequence frequencies actually work *against* the direction of the rotor's rotation. Zero-sequence frequencies neither contribute to nor detract from the rotor's torque. An excess of negative-sequence harmonics (5th, 11th, 17th, and/or 23rd) in the power supplied to a three-phase AC motor will result in a degradation of performance and possible overheating. Since the higher-order harmonics tend to be attenuated more by system inductances and magnetic core losses, and generally originate with less amplitude anyway, the primary harmonic of concern is the 5th, which is 300 Hz in 60 Hz power systems and 250 Hz in 50 Hz power systems.

10.9 Contributors

Les contributeurs de ce chapitre sont listés dans l'ordre chronologique de leurs contributions, depuis le plus récent jusqu'au premier. Voyez l'Annexe 2 (Liste des contributeur) pour les dates et les informations de contact.

Ed Beroset (May 6, 2002) : Suggested better ways to illustrate the meaning of the prefix "poly-".

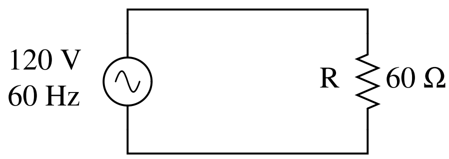
Jason Starck (June 2000) : HTML document formatting, which led to a much better-looking second edition.

Chapter 11

FACTEUR DE PUISSANCE

11.1 La puissance dans les circuits AC résistifs et réactifs

Considérons un circuit pour un système de puissance AC simple phase, où une source de tension AC de 120 volts, 60 Hz délivre une puissance dans une charge résistive :



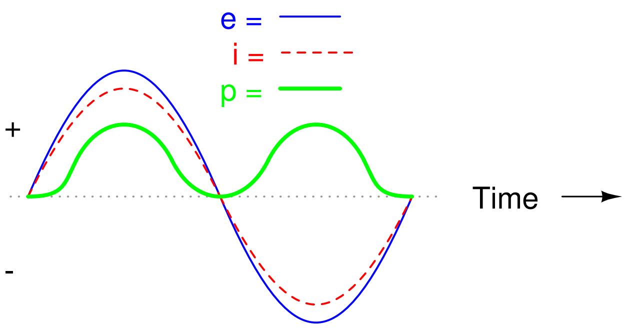
$$Z_R = 60 + j0\ \Omega \quad \textit{or} \quad 60\ \Omega \angle 0^o$$

$$I = \frac{E}{Z}$$

$$I = \frac{120\ V}{60\ \Omega}$$

$$I = 2\ A$$

Dans cet exemple, le courant dans la charge sera de 2 A, RMS. La puissance dissipée dans la charge sera de 240 watts. Comme cette charge est purement résistive (pas de réactance), le courant est en phase avec la tension et les calculs sont similaires à ceux dans un circuit DC équivalent. Si nous dessinions les formes d'onde de la tension, du courant et de la puissance pour ce circuit, cela ressemblerait à ceci :

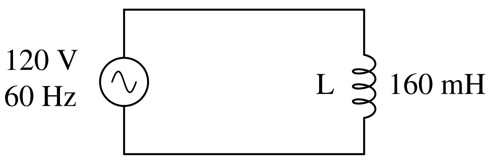


Notez que la forme d'onde pour la puissance est toujours positive, jamais négative pour ce circuit résistif. Cela signifie aur la puissance est toujours dissipée par la charge résistive et ne retourne jamais à la source comme c'est le cas pour les sources réactives. Si la source est un générateur mécanique, il faudra 240 watts d'énergie mécanique (à peu près 1/3 de cheval) pour faire le tour.

Notez aussi que la forme d'onde pour la puissance n'est pas à la même fréquence que la tension ou le courant! Elle est au *double* de la forme d'onde de la tension ou du courant. Cette fréquence differente interdit notre expression de puissance dans une circuit AC en utilisant la même notation complexe (rectangulaire ou polaire) que celle utilisée pour la tension, le courant et l'impédance car cette forme de symbolisme implique des relations de phases inchangées. Lorsque les fréquences ne sont pas identiques, les relations de phase changent constamment.

Aussi étrange que cela puisse paraître, la meilleure manière d'effectuer les calculs de puissance AC est d'utiliser la notation *scalaire* et de traiter toutes les relations liées à la phase avec la trigonométrie.

Par comparaison, considérons un circuit AC simple avec une charge purement résistive :



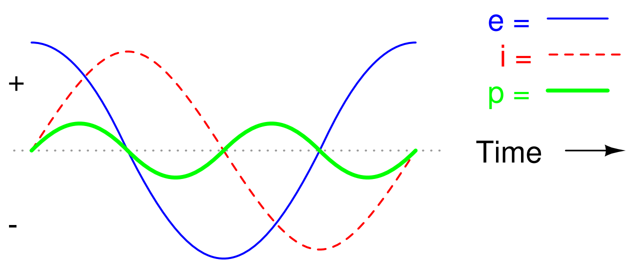
$$X_L = 60.319\ \Omega$$

$$Z_L = 0 + j60.319\ \Omega \quad \textit{or} \quad 60.319\ \Omega \angle 90^{\circ}$$

$$I = \frac{E}{Z}$$

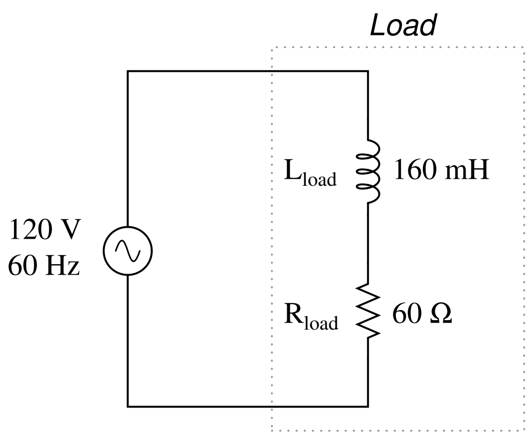
$$I = \frac{120\ \text{V}}{60.319\ \Omega}$$

$$I = 1.989\ \text{A}$$



Notez que la source de puissance alterne également entre les cycles positifs et négatifs. Cela signifie que la puissance est alternativement absorbée ou renvoyée à la source. Si la source était un générateur mécanique, il n'y aura quasiment pas besoin (pratiquement) d'énergie mécanique pour faire tourner l'axe car aucune puissance n'est consommée par la charge. L'arbre du générateur sera facile à tourner et l'inductance ne deviendra pas aussi chaude que le sera la résistance.

Considérons maintenant un circuit AC avec une charge constituée d'une inductance et d'une résistance :



$$X_L = 60.319\ \Omega$$

$$Z_L = 0 + j60.319\ \Omega \quad or \quad 60.319\ \Omega \angle 90^o$$

$$Z_R = 60 + j0\ \Omega \quad or \quad 60\ \Omega \angle 0^o$$

$$Z_{total} = 60 + j60.319\ \Omega \quad or \quad 85.078\ \Omega \angle 45.152^o$$

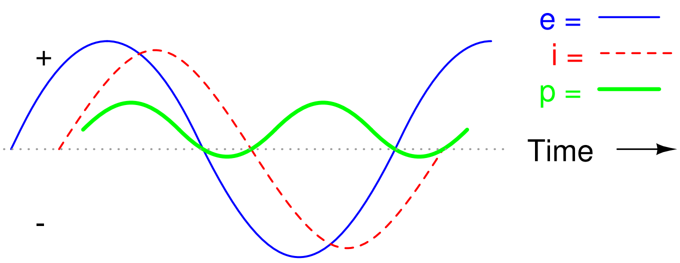
$$I = \frac{E}{Z}$$

$$I = \frac{120\ V}{85.078\ \Omega}$$

$$I = 1.410\ A$$

At a frequency of 60 Hz, the 160 millihenrys of inductance gives us 60.319 Ω of inductive reactance. This reactance combines with the 60 Ω of resistance to form a total load impedance of 60 + j60.319 Ω, or 85.078 Ω ∠ 45.152^o. If we're not concerned with phase angles (which we're not at this point), we may calculate current in the circuit by taking the polar magnitude of the voltage source (120 volts) and dividing it my the polar magnitude of the impedance (85.078 Ω). With a power supply voltage of 120 volts RMS, our load current is 1.410 amps. This is the figure an RMS ammeter would indicate if connected in series with the resistor and inductor.

We already know that reactive components dissipate zero power, as they equally absorb power from, and return power to, the rest of the circuit. Therefore, any inductive reactance in this load will likewise dissipate zero power. The only thing left to dissipate power here is the resistive portion of the load impedance. If we look at the waveform plot of voltage, current, and total power for this circuit, we see how this combination works :



As with any reactive circuit, the power alternates between positive and negative instantaneous values over time. In a purely reactive circuit that alternation between positive and negative power is equally divided, resulting in a net power dissipation of zero. However, in circuits with mixed resistance and reactance like this one, the power waveform will still alternate between positive and negative, but the amount of positive power will exceed the amount of negative power. In other words, the combined inductive/resistive load will consume more power than it returns back to the source.

Looking at the waveform plot for power, it should be evident that the wave spends more time on the positive side of the center line than on the negative, indicating that there is more power absorbed by the load than there is returned to the circuit. What little returning of power that occurs is due to the reactance; the imbalance of positive versus negative power is due to the resistance as it dissipates energy outside of the circuit (usually in the form of heat). If the source were a mechanical generator, the amount of mechanical energy needed to turn the shaft would be the amount of power averaged between the positive and negative power cycles.

Mathematically representing power in an AC circuit is a challenge, because the power wave isn't at the same frequency as voltage or current. Furthermore, the phase angle for power means something quite different from the phase angle for either voltage or current. Whereas the angle for voltage or current represents a relative *shift in timing* between two waves, the phase angle for power represents a *ratio* between power dissipated and power returned. Because of this way in which AC power differs from AC voltage or current, it is actually easier to arrive at figures for power by calculating with *scalar* quantities of voltage, current, resistance, and reactance than it is to try to derive it from *vector*, or *complex* quantities of voltage, current, and impedance that we've worked with so far.

- **REVIEW :**
- In a purely resistive circuit, all circuit power is dissipated by the resistor(s). Voltage and current are in phase with each other.
- In a purely reactive circuit, no circuit power is dissipated by the load(s). Rather, power is alternately absorbed from and returned to the AC source. Voltage and current are 90^o out of phase with each other.
- In a circuit consisting of resistance and reactance mixed, there will be more power dissipated by the load(s) than returned, but some power will definitely be dissipated and some will merely be absorbed and returned. Voltage and current in such a circuit will be out of phase by a value somewhere between 0^o and 90^o.

11.2 True, Reactive, and Apparent power

We know that reactive loads such as inductors and capacitors dissipate zero power, yet the fact that they drop voltage and draw current gives the deceptive impression that they actually *do* dissipate power. This "phantom power" is called *reactive power*, and it is measured in a unit called *Volt-Amps-Reactive* (VAR), rather than watts. The mathematical symbol for reactive power is (unfortunately) the capital letter Q. The actual amount of power being used, or dissipated, in a circuit is called *true power*, and it is measured in watts (symbolized by the capital letter P, as always). The combination of reactive power and true power is called *apparent power*, and it is the product of a circuit's voltage and current, without reference to phase angle. Apparent power is measured in the unit of *Volt-Amps* (VA) and is symbolized by the capital letter S.

As a rule, true power is a function of a circuit's dissipative elements, usually resistances (R). Reactive power is a function of a circuit's reactance (X). Apparent power is a function of a circuit's total impedance (Z). Since we're dealing with scalar quantities for power calculation, any complex starting quantities such as voltage, current, and impedance must be represented by their *polar magnitudes*, not by real or imaginary rectangular components. For instance, if I'm calculating true power from current and resistance, I must use the polar magnitude for current, and not merely the "real" or "imaginary" portion of the current. If I'm calculating apparent power from voltage and impedance, both of these formerly complex quantities must be reduced to their polar magnitudes for the scalar arithmetic.

There are several power equations relating the three types of power to resistance, reactance, and impedance (all using scalar quantities) :

P = true power $\quad P = I^2R \quad P = \frac{E^2}{R}$

Measured in units of ***Watts***

Q = reactive power $\quad Q = I^2X \quad Q = \frac{E^2}{X}$

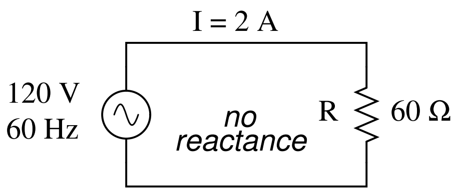
Measured in units of ***Volt-Amps-Reactive (VAR)***

S = apparent power $\quad S = I^2Z \quad S = \frac{E^2}{Z} \quad S = IE$

Measured in units of ***Volt-Amps (VA)***

Please note that there are two equations each for the calculation of true and reactive power. There are three equations available for the calculation of apparent power, P=IE being useful *only* for that purpose. Examine the following circuits and see how these three types of power interrelate :

Resistive load only :

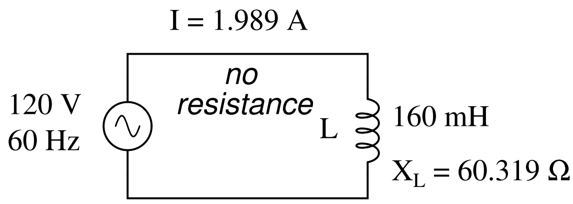


P = true power = I^2R = 240 W

Q = reactive power = I^2X = 0 VAR

S = apparent power = I^2Z = 240 VA

Reactive load only :

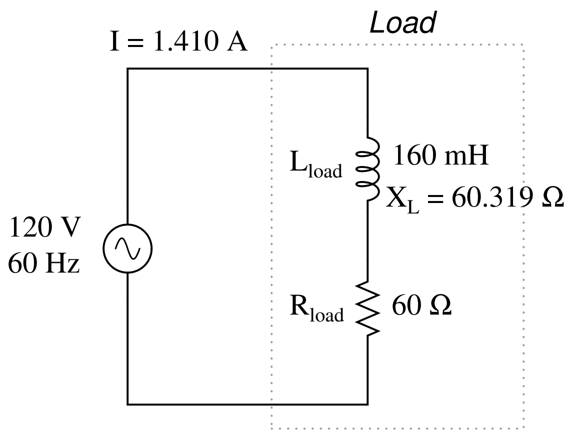


P = true power = I^2R = 0 W

Q = reactive power = I^2X = 238.73 VAR

S = apparent power = I^2Z = 238.73 VA

Resistive/reactive load :

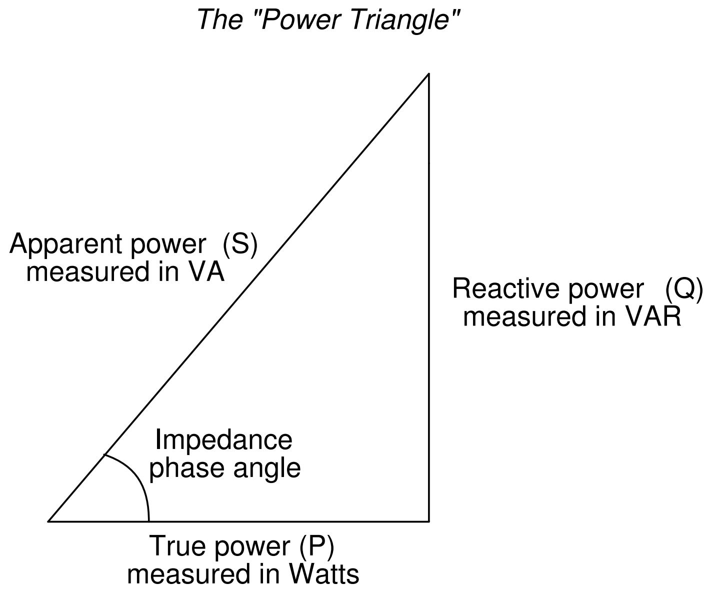


P = true power = I^2R = 119.365 W

Q = reactive power = I^2X = 119.998 VAR

S = apparent power = I^2Z = 169.256 VA

These three types of power – true, reactive, and apparent – relate to one another in trigonometric form. We call this the *power triangle* :



Using the laws of trigonometry, we can solve for the length of any side (amount of any type of power), given the lengths of the other two sides, or the length of one side and an angle.

- **REVIEW :**
- Power dissipated by a load is referred to as *true power*. True power is symbolized by the letter P and is measured in the unit of Watts (W).
- Power merely absorbed and returned in load due to its reactive properties is referred to as *reactive power*. Reactive power is symbolized by the letter Q and is measured in the unit of Volt-Amps-Reactive (VAR).
- Total power in an AC circuit, both dissipated and absorbed/returned is referred to as *apparent power*. Apparent power is symbolized by the letter S and is measured in the unit of Volt-Amps (VA).
- These three types of power are trigonometrically related to one another. In a right triangle, P = adjacent length, Q = opposite length, and S = hypotenuse length. The opposite angle is equal to the circuit's impedance (Z) phase angle.

11.3 Calculating power factor

As was mentioned before, the angle of this "power triangle" graphically indicates the ratio between the amount of dissipated (or *consumed*) power and the amount of absorbed/returned power. It also happens to be the same angle as that of the circuit's impedance in polar form. When expressed as a fraction, this ratio between true power and apparent power is called the *power factor* for this circuit. Because true power and apparent power form the adjacent and hypotenuse sides of a right triangle, respectively, the power factor ratio is also equal to the cosine of that phase angle. Using values from the last example circuit :

$$\text{Power factor} = \frac{\text{True power}}{\text{Apparent power}}$$

$$\text{Power factor} = \frac{119.365 \text{ W}}{169.256 \text{ VA}}$$

$$\text{Power factor} = 0.705$$

$$\cos 45.152^{o} = 0.705$$

It should be noted that power factor, like all ratio measurements, is a *unitless* quantity.

For the purely resistive circuit, the power factor is 1 (perfect), because the reactive power equals zero. Here, the power triangle would look like a horizontal line, because the opposite (reactive power) side would have zero length.

For the purely inductive circuit, the power factor is zero, because true power equals zero. Here, the power triangle would look like a vertical line, because the adjacent (true power) side would have zero length.

The same could be said for a purely capacitive circuit. If there are no dissipative (resistive) components in the circuit, then the true power must be equal to zero, making any power in the circuit purely reactive. The power triangle for a purely capacitive circuit would again be a vertical line (pointing down instead of up as it was for the purely inductive circuit).

Power factor can be an important aspect to consider in an AC circuit, because any power factor less than 1 means that the circuit's wiring has to carry more current than what would be necessary with zero reactance in the circuit to deliver the same amount of (true) power to the resistive load. If our last example circuit had been purely resistive, we would have been able to deliver a full 169.256 watts to the load with the same 1.410 amps of current, rather than the mere 119.365 watts that it is presently dissipating with that same current quantity. The poor power factor makes for an inefficient power delivery system.

Poor power factor can be corrected, paradoxically, by adding another load to the circuit drawing an equal and opposite amount of reactive power, to cancel out the effects of the load's inductive reactance. Inductive reactance can only be canceled by capacitive reactance, so we have to add a *capacitor* in parallel to our example circuit as the additional load. The effect of these two opposing reactances in parallel is to bring the circuit's total impedance equal to its total resistance (to make the impedance phase angle equal, or at least closer, to zero).

Since we know that the (uncorrected) reactive power is 119.998 VAR (inductive), we need to calculate the correct capacitor size to produce the same quantity of (capacitive) reactive power. Since this capacitor will be directly in parallel with the source (of known voltage), we'll use the power formula which starts from voltage and reactance :

$$Q = \frac{E^2}{X}$$

. . . solving for X . . .

$$X = \frac{E^2}{Q}$$

$$X = \frac{(120\ V)^2}{119.998\ VAR}$$

$$X = 120.002\ \Omega$$

$$X_C = \frac{1}{2\pi f C}$$

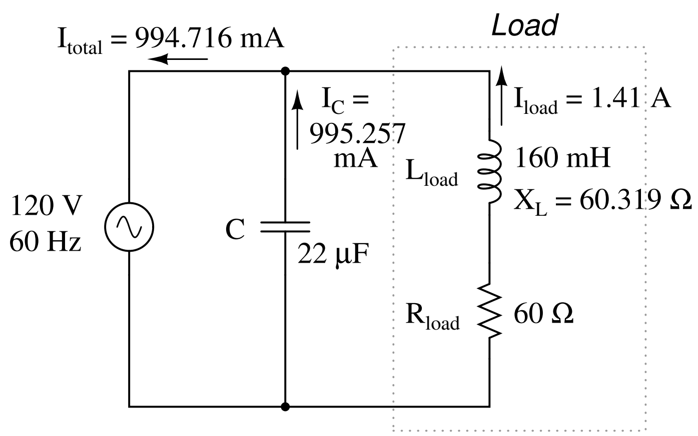
. . . solving for C . . .

$$C = \frac{1}{2\pi f X_C}$$

$$C = \frac{1}{2\pi(60\ Hz)(120.002\ \Omega)}$$

$$C = 22.105\ \mu F$$

Let's use a rounded capacitor value of 22 μF and see what happens to our circuit :



$$Z_{total} = Z_C \; // \; (Z_L \text{ -- } Z_R)$$

$$Z_{total} = (120.57\ \Omega \angle -90^o) \; // \; (60.319\ \Omega \angle 90^o \text{ -- } 60\ \Omega \angle 0^o)$$

$$Z_{total} = 120.64 - j573.58m\ \Omega \quad \textit{or} \quad 120.64\ \Omega \angle 0.2724^o$$

$$P = \text{true power} = I^2R = 119.365\ W$$

$$S = \text{apparent power} = I^2Z = 119.366\ VA$$

The power factor for the circuit, overall, has been substantially improved. The main current has been decreased from 1.41 amps to 994.7 milliamps, while the power dissipated at the load resistor remains unchanged at 119.365 watts. The power factor is much closer to being 1 :

$$\text{Power factor} = \frac{\text{True power}}{\text{Apparent power}}$$

$$\text{Power factor} = \frac{119.365\ W}{119.366\ VA}$$

$$\text{Power factor} = 0.9999887$$

$$\text{Impedance (polar) angle} = 0.272^o$$

$$\cos 0.272^o = 0.9999887$$

Since the impedance angle is still a positive number, we know that the circuit, overall, is still more inductive than it is capacitive. If our power factor correction efforts had been perfectly on-target, we would have arrived at an impedance angle of exactly zero, or purely resistive. If we had added too large of a capacitor in parallel, we would have ended up with an impedance angle that was negative, indicating that the circuit was more capacitive than inductive.

It should be noted that too much capacitance in an AC circuit will result in a low power factor just as well as too much inductance. You must be careful not to over-correct when adding capacitance to an AC circuit. You must also be *very* careful to use the proper capacitors for the job (rated adequately for power system voltages and the occasional voltage spike from lightning strikes, for continuous AC service, and capable of handling the expected levels of current).

If a circuit is predominantly inductive, we say that its power factor is *lagging* (because the current wave for the circuit lags behind the applied voltage wave). Conversely, if a circuit is predominantly capacitive, we say that its power factor is *leading*. Thus, our example circuit started out with a power factor of 0.705 lagging, and was corrected to a power factor of 0.999 lagging.

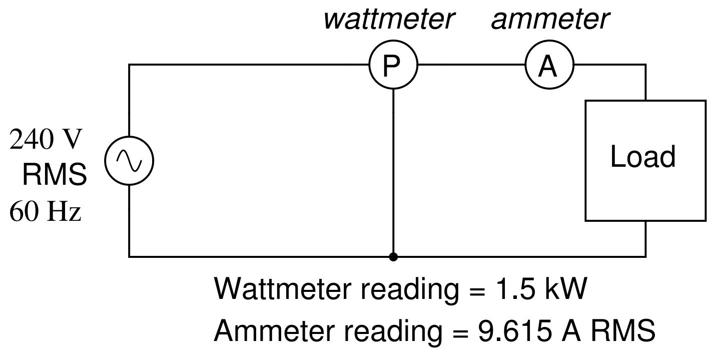
- **REVIEW :**
- Poor power factor in an AC circuit may be "corrected," or re-established at a value close to 1, by adding a parallel reactance opposite the effect of the load's reactance. If the load's

reactance is inductive in nature (which is almost always will be), parallel *capacitance* is what is needed to correct poor power factor.

11.4 Practical power factor correction

When the need arises to correct for poor power factor in an AC power system, you probably won't have the luxury of knowing the load's exact inductance in henrys to use for your calculations. You may be fortunate enough to have an instrument called a *power factor meter* to tell you what the power factor is (a number between 0 and 1), and the apparent power (which can be figured by taking a voltmeter reading in volts and multiplying by an ammeter reading in amps). In less favorable circumstances you may have to use an oscilloscope to compare voltage and current waveforms, measuring phase shift in *degrees* and calculating power factor by the cosine of that phase shift.

Most likely, you will have access to a wattmeter for measuring true power, whose reading you can compare against a calculation of apparent power (from multiplying total voltage and total current measurements). From the values of true and apparent power, you can determine reactive power and power factor. Let's do an example problem to see how this works :



First, we need to calculate the apparent power in kVA. We can do this by multiplying load voltage by load current :

$$S = IE$$

$$S = (9.615\ A)(240\ V)$$

$$S = 2.308\ kVA$$

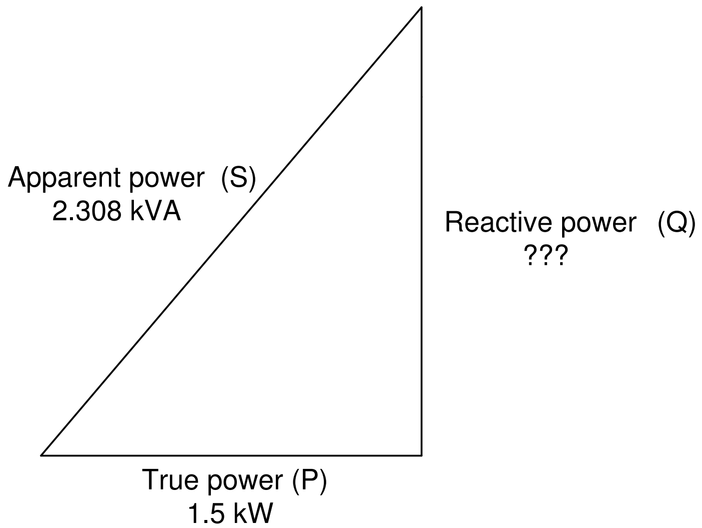
As we can see, 2.308 kVA is a much larger figure than 1.5 kW, which tells us that the power factor in this circuit is rather poor (substantially less than 1). Now, we figure the power factor of this load by dividing the true power by the apparent power :

$$\text{Power factor} = \frac{P}{S}$$

$$\text{Power factor} = \frac{1.5 \text{ kW}}{2.308 \text{ kVA}}$$

$$\text{Power factor} = 0.65$$

Using this value for power factor, we can draw a power triangle, and from that determine the reactive power of this load :



To determine the unknown (reactive power) triangle quantity, we use the Pythagorean Theorem "backwards," given the length of the hypotenuse (apparent power) and the length of the adjacent side (true power) :

$$\text{Reactive power} = \sqrt{(\text{Apparent power})^2 - (\text{True power})^2}$$

$$Q = 1.754 \text{ kVAR}$$

If this load is an electric motor, or most any other industrial AC load, it will have a lagging (inductive) power factor, which means that we'll have to correct for it with a *capacitor* of appropriate size, wired in parallel. Now that we know the amount of reactive power (1.754 kVAR), we can calculate the size of capacitor needed to counteract its effects :

$$Q = \frac{E^2}{X}$$

. . . solving for X . . .

$$X = \frac{E^2}{Q}$$

$$X = \frac{(240)^2}{1.754\ \text{kVAR}}$$

$$X = 32.845\ \Omega$$

$$X_C = \frac{1}{2\pi f C}$$

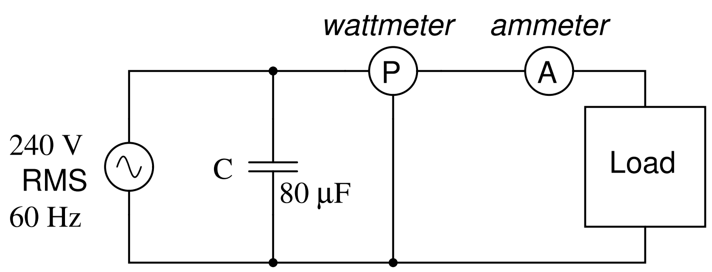
. . . solving for C . . .

$$C = \frac{1}{2\pi f X_C}$$

$$C = \frac{1}{2\pi(60\ \text{Hz})(32.845\ \Omega)}$$

$$C = 80.761\ \mu\text{F}$$

Rounding this answer off to 80 μF, we can place that size of capacitor in the circuit and calculate the results :

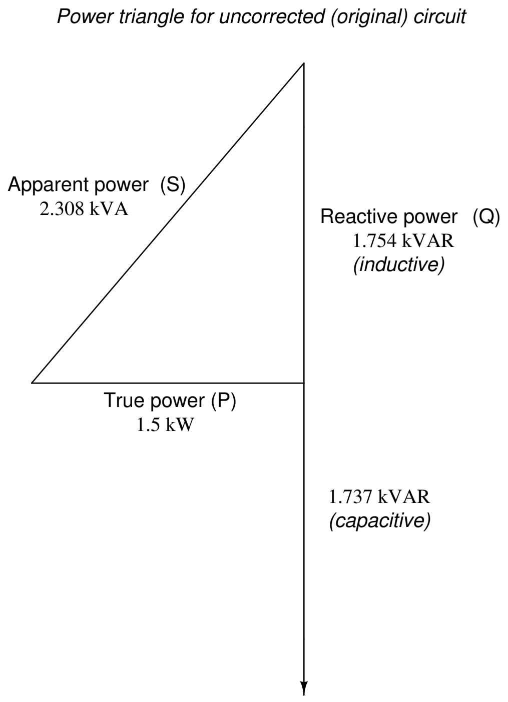


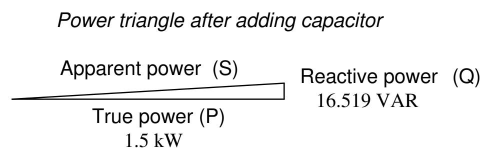
An 80 μF capacitor will have a capacitive reactance of 33.157 Ω, giving a current of 7.238 amps, and a corresponding reactive power of 1.737 kVAR (for the capacitor *only*). Since the capacitor's current is 180^o out of phase from the the load's inductive contribution to current draw, the capacitor's reactive power will directly subtract from the load's reactive power, resulting in :

Inductive kVAR - Capacitive kVAR = Total kVAR

1.754 kVAR - 1.737 kVAR = 16.519 VAR

This correction, of course, will not change the amount of true power consumed by the load, but it will result in a substantial reduction of apparent power, and of the total current drawn from the 240 Volt source :





The new apparent power can be found from the true and new reactive power values, using the standard form of the Pythagorean Theorem :

$$\text{Apparent power} = \sqrt{(\text{Reactive power})^2 + (\text{True power})^2}$$

$$\text{Apparent power} = 1.50009 \text{ kVA}$$

This gives a corrected power factor of (1.5kW / 1.5009 kVA), or 0.99994, and a new total current of (1.50009 kVA / 240 Volts), or 6.25 amps, a substantial improvement over the uncorrected value of 9.615 amps! This lower total current will translate to less heat losses in the circuit wiring, meaning greater system efficiency (less power wasted).

11.5 Contributors

Les contributeurs de ce chapitre sont listés dans l'ordre chronologique de leurs contributions, depuis le plus récent jusqu'au premier. Voyez l'Annexe 2 (Liste des contributeur) pour les dates et les informations de contact.

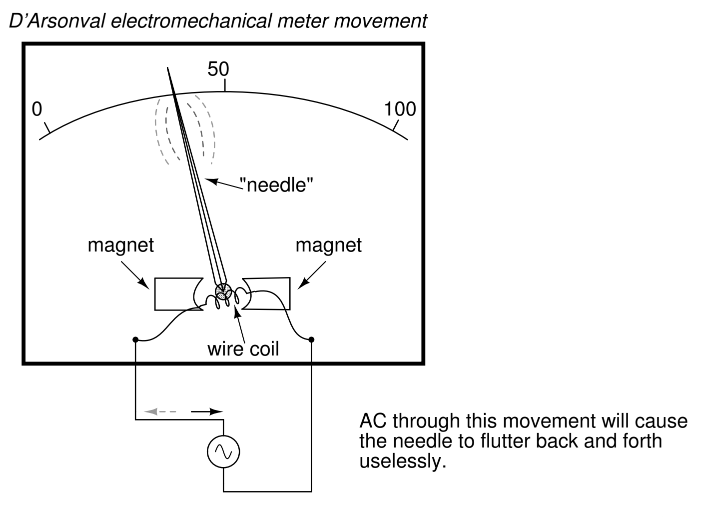
Jason Starck (June 2000) : HTML document formatting, which led to a much better-looking second edition.

Chapter 12

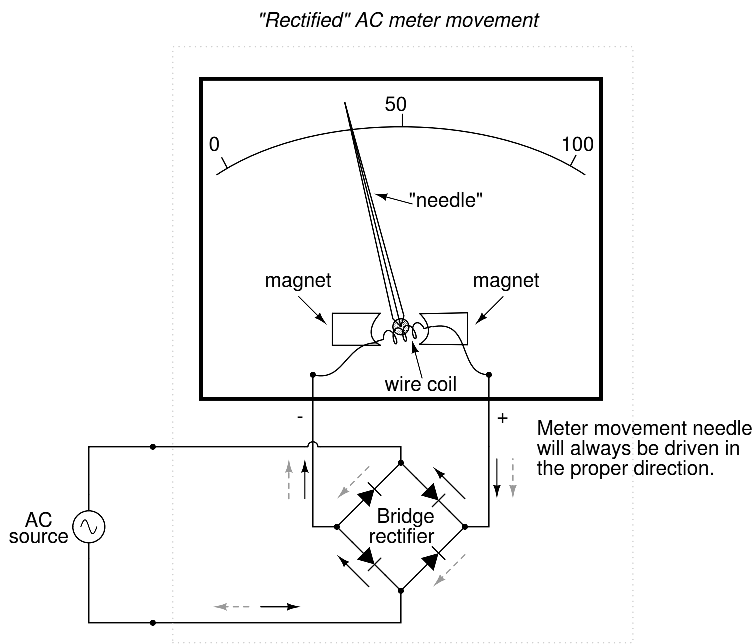
CIRCUITS DE MESURE AC

12.1 Voltmètres et ampèremètres AC

AC electromechanical meter movements come in two basic arrangements : those based on DC movement designs, and those engineered specifically for AC use. Permanent-magnet moving coil (PMMC) meter movements will not work correctly if directly connected to alternating current, because the direction of needle movement will change with each half-cycle of the AC. Permanent-magnet meter movements, like permanent-magnet motors, are devices whose motion depends on the polarity of the applied voltage (or, you can think of it in terms of the direction of the current).

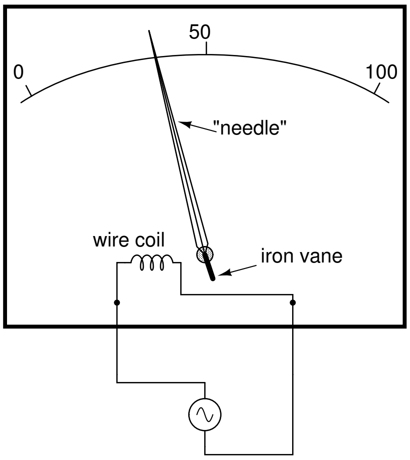


In order to use a DC-style meter movement such as the D'Arsonval design, the alternating current must be *rectified* into DC. Cela est facilité par l'utilisation de composants appelés *diodes.* Nous avons vu des diodes utilisées dans un circuit d'exemple démontrant la création de fréquences harmoniques depuis une onde sinus distortue (ou rectifiée). Dans entrer dans des détails élaborés sur le pourquoi et le comment du fonctionnement des duides, rappelez-vous simplement qu'elles agissent comme une soupape à sens unique pour le flux des électrons : en agissant comme un conducteur pour une polarisation et comme isolant pour l'autre. Bizarrement, la tête de la flèche de chaque symbole de diode pointe à *l'opposé* de la direction du flux des électrons contrairement à ce qui pourrait être attendu. Mis sous forme de pont, les quatre diodes serviront à orienter le courant alternatif vers la mesure de mouvement dans une direction constante de toutes les portions du cycle AC :



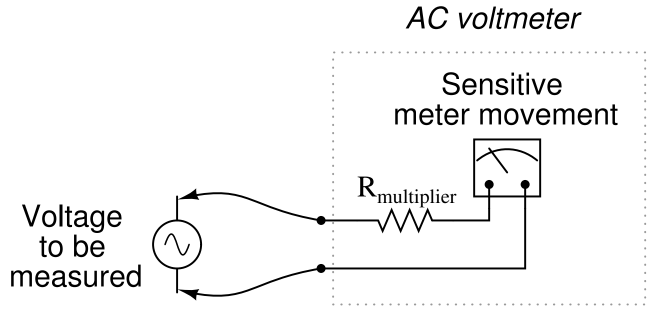
Another strategy for a practical AC meter movement is to redesign the movement without the inherent polarity sensitivity of the DC types. This means avoiding the use of permanent magnets. Probably the simplest design is to use a nonmagnetized iron vane to move the needle against spring tension, the vane being attracted toward a stationary coil of wire energized by the AC quantity to be measured.

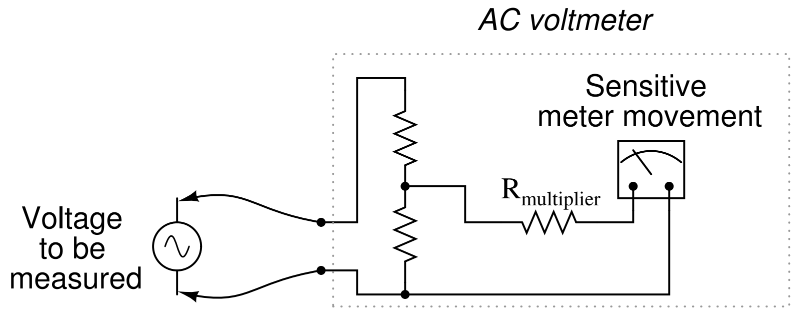
Iron-vane electromechanical meter movement



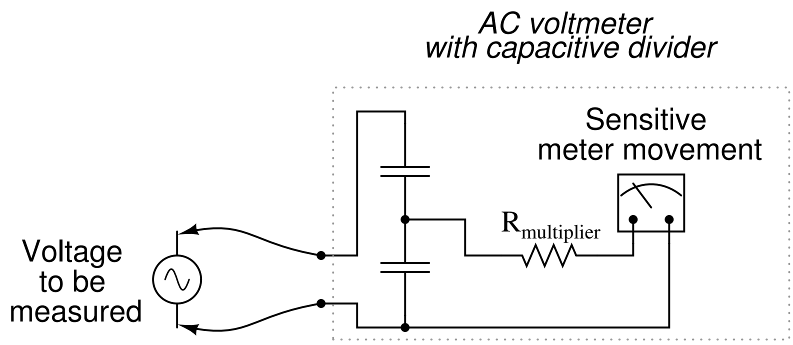
Electrostatic attraction between two metal plates separated by an air gap is an alternative mechanism for generating a needle-moving force proportional to applied voltage. This works just as well for AC as it does for DC, or should I say, just as poorly! The forces involved are very small, much smaller than the magnetic attraction between an energized coil and an iron vane, and as such these "electrostatic" meter movements tend to be fragile and easily disturbed by physical movement. But, for some high-voltage AC applications, the electrostatic movement is an elegant technology. If nothing else, this technology possesses the advantage of extremely high input impedance, meaning that no current need be drawn from the circuit under test. Also, electrostatic meter movements are capable of measuring very high voltages without need for range resistors or other, external apparatus.

When a sensitive meter movement needs to be re-ranged to function as an AC voltmeter, series-connected "multiplier" resistors and/or resistive voltage dividers may be employed just as in DC meter design :

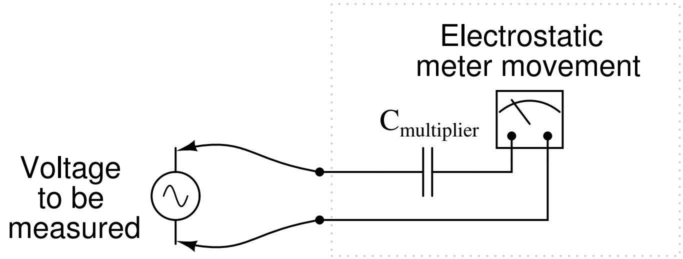




Capacitors may be used instead of resistors, though, to make voltmeter divider circuits. This strategy has the advantage of being non-dissipative (no true power consumed and no heat produced) :

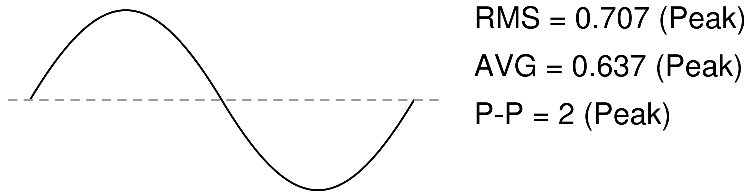


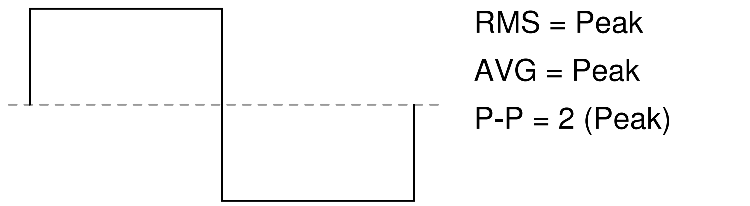
If the meter movement is electrostatic, and thus inherently capacitive in nature, a single "multiplier" capacitor may be connected in series to give it a greater voltage measuring range, just as a series-connected multiplier resistor gives a moving-coil (inherently resistive) meter movement a greater voltage range :

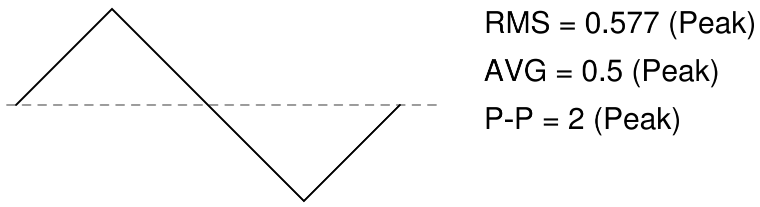


The Cathode Ray Tube (CRT) mentioned in the DC metering chapter is ideally suited for measuring AC voltages, especially if the electron beam is swept side-to-side across the screen of the tube while the measured AC voltage drives the beam up and down. A graphical representation of the AC wave shape and not just a measurement of magnitude can easily be had with such a device. However, CRT's have the disadvantages of weight, size, significant power consumption, and fragility (being made of evacuated glass) working against them. For these reasons, electromechanical AC meter movements still have a place in practical usage.

With some of the advantages and disadvantages of these meter movement technologies having been discussed already, there is another factor crucially important for the designer and user of AC metering instruments to be aware of. This is the issue of RMS measurement. As we already know, AC measurements are often cast in a scale of DC power equivalence, called *RMS* (**R**oot-**M**ean-**S**quare) for the sake of meaningful comparisons with DC and with other AC waveforms of varying shape. None of the meter movement technologies so far discussed inherently measure the RMS value of an AC quantity. Meter movements relying on the motion of a mechanical needle ("rectified" D'Arsonval, iron-vane, and electrostatic) all tend to mechanically average the instantaneous values into an overall average value for the waveform. This average value is not necessarily the same as RMS, although many times it is mistaken as such. Average and RMS values rate against each other as such for these three common waveform shapes :







Since RMS seems to be the kind of measurement most people are interested in obtaining with an instrument, and electromechanical meter movements naturally deliver *average* measurements rather than RMS, what are AC meter designers to do? Cheat, of course! Typically the assumption is made that the waveform shape to be measured is going to be sine (by far the most common, especially for power systems), and then the meter movement scale is altered by the appropriate multiplication factor. For sine waves we see that RMS is equal to 0.707 times the peak value while Average is 0.637 times the peak, so we can divide one figure by the other to obtain an average-to-RMS conversion factor of 1.109 :

$$\frac{0.707}{0.637} = 1.1099$$

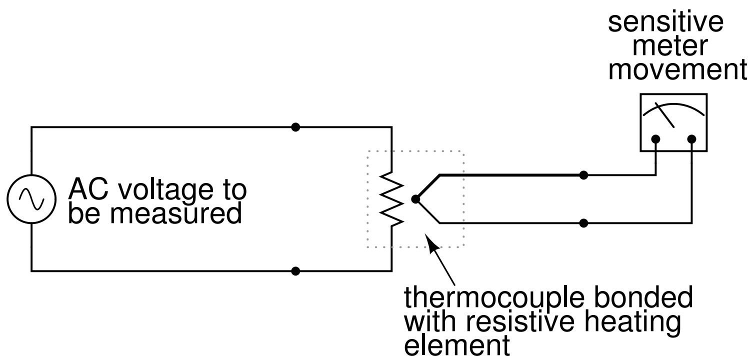
In other words, the meter movement will be calibrated to indicate approximately 1.11 times higher than it would ordinarily (naturally) indicate with no special accommodations. It must be stressed that this "cheat" only works well when the meter is used to measure pure sine wave sources. Note that for triangle waves, the ratio between RMS and Average is not the same as for sine waves :

$$\frac{0.577}{0.5} = 1.154$$

With square waves, the RMS and Average values are identical! An AC meter calibrated to accurately read RMS voltage or current on a pure sine wave will *not* give the proper value while indicating the magnitude of anything other than a perfect sine wave. This includes triangle waves, square waves, or any kind of distorted sine wave. With harmonics becoming an ever-present phenomenon in large AC power systems, this matter of accurate RMS measurement is no small matter.

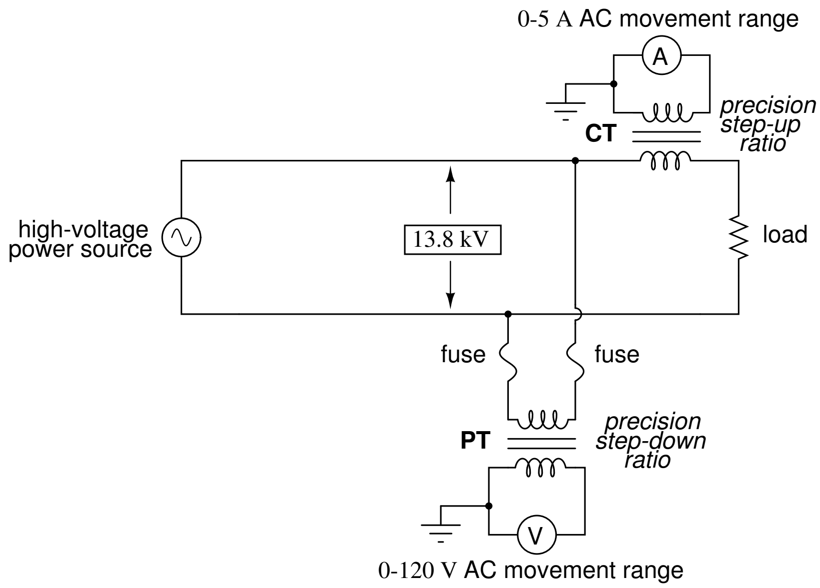
The astute reader will note that I have omitted the CRT "movement" from the RMS/Average discussion. This is because a CRT with its practically weightless electron beam "movement" displays the Peak (or Peak-to-Peak if you wish) of an AC waveform rather than Average or RMS. Still, a similar problem arises : how do you determine the RMS value of a waveform from it? Conversion factors between Peak and RMS only hold so long as the waveform falls neatly into a known category of shape (sine, triangle, and square are the only examples with Peak/RMS/Average conversion factors given here!).

One answer is to design the meter movement around the very definition of RMS : the effective heating value of an AC voltage/current as it powers a resistive load. Suppose that the AC source to be measured is connected across a resistor of known value, and the heat output of that resistor is measured with a device like a thermocouple. This would provide a far more direct measurement means of RMS than any conversion factor could, for it will work with ANY waveform shape whatsoever :

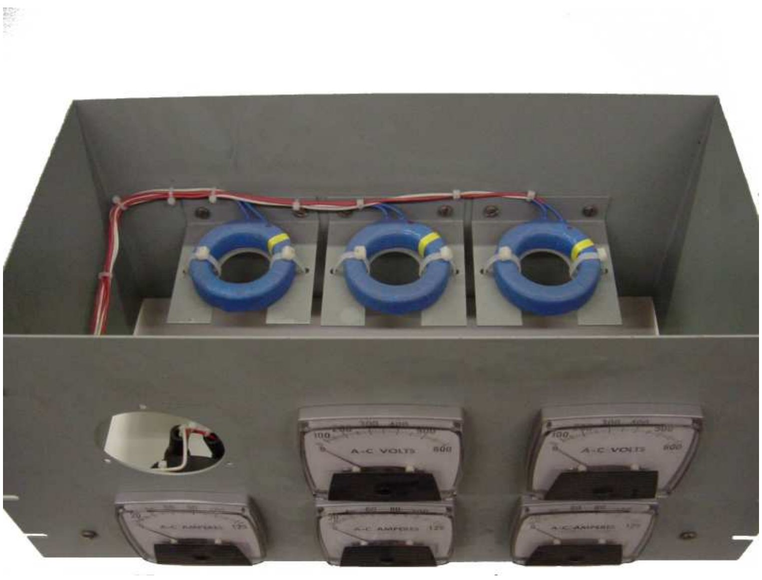


While the device shown above is somewhat crude and would suffer from unique engineering problems of its own, the concept illustrated is very sound. The resistor converts the AC voltage or current quantity into a thermal (heat) quantity, effectively squaring the values in real-time. The system's mass works to average these values by the principle of thermal inertia, and then the meter scale itself is calibrated to give an indication based on the square-root of the thermal measurement : perfect Root-Mean-Square indication all in one device! In fact, one major instrument manufacturer has implemented this technique into its high-end line of handheld electronic multimeters for "true-RMS" capability.

Calibrating AC voltmeters and ammeters for different full-scale ranges of operation is much the same as with DC instruments : series "multiplier" resistors are used to give voltmeter movements higher range, and parallel "shunt" resistors are used to allow ammeter movements to measure currents beyond their natural range. However, we are not limited to these techniques as we were with DC : because we can to use transformers with AC, meter ranges can be electromagnetically rather than resistively "stepped up" or "stepped down," sometimes far beyond what resistors would have practically allowed for. Potential Transformers (PT's) and Current Transformers (CT's) are precision instrument devices manufactured to produce very precise ratios of transformation between primary and secondary windings. They can allow small, simple AC meter movements to indicate extremely high voltages and currents in power systems with accuracy and complete electrical isolation (something multiplier and shunt resistors could never do) :



Shown here is a voltage and current meter panel from a three-phase AC system. The three "donut" current transformers (CTs) can be seen in the rear of the panel. Three AC ammeters (rated 5 amps full-scale deflection each) on the front of the panel indicate current through each conductor going through a CT. As this panel has been removed from service, there are no current-carrying conductors threaded through the center of the CT "donuts" anymore :



Because of the expense (and often large size) of instrument transformers, they are not used to scale AC meters for any applications other than high voltage and high current. For scaling a milliamp or microamp movement to a range of 120 volts or 5 amps, normal precision resistors (multipliers and shunts) are used, just as with DC.

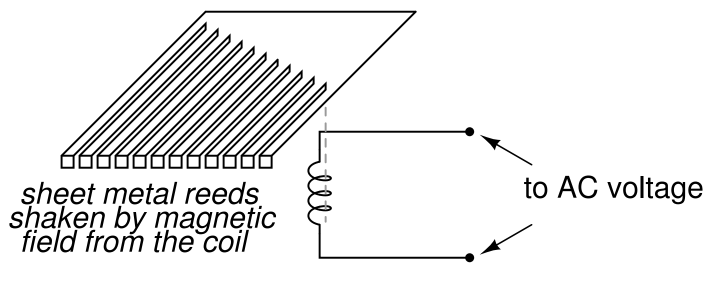
- **REVIEW :**
- Polarized (DC) meter movements must use devices called *diodes* to be able to indicate AC quantities.
- Electromechanical meter movements, whether electromagnetic or electrostatic, naturally provide the *average* value of a measured AC quantity. These instruments may be ranged to indicate RMS value, but only if the shape of the AC waveform is precisely known beforehand!
- So-called *true RMS* meters use different technology to provide indications representing the actual RMS (rather than skewed average or peak) of an AC waveform.

12.2 Frequency and phase measurement

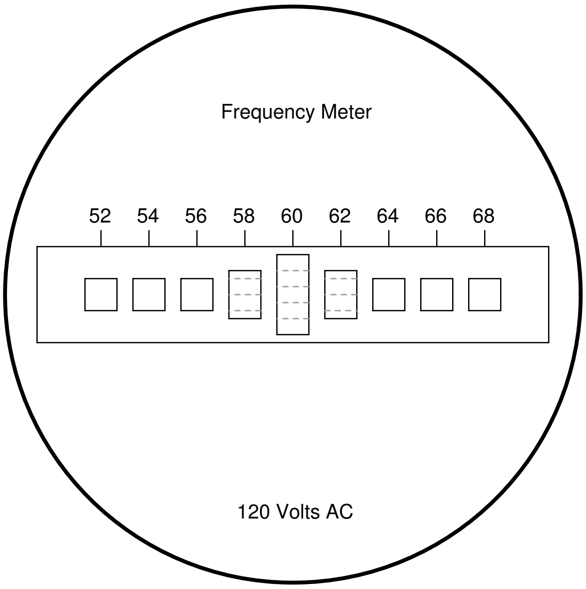
An important electrical quantity with no equivalent in DC circuits is *frequency.* Frequency measurement is very important in many applications of alternating current, especially in AC power systems designed to run efficiently at one frequency and one frequency only. If the AC is being generated by an electromechanical alternator, the frequency will be directly proportional to the shaft speed of the machine, and frequency could be measured simply by measuring the speed of the shaft. If frequency needs to be measured at some distance from the alternator, though, other means of measurement will be necessary.

One simple but crude method of frequency measurement in power systems utilizes the principle of mechanical resonance. Every physical object possessing the property of elasticity (springiness) has an inherent frequency at which it will prefer to vibrate. The tuning fork is a great example of this : strike it once and it will continue to vibrate at a tone specific to its length. Longer tuning forks have lower resonant frequencies : their tones will be lower on the musical scale than shorter forks.

Imagine a row of progressively-sized tuning forks arranged side-by-side. They are all mounted on a common base, and that base is vibrated at the frequency of the measured AC voltage (or current) by means of an electromagnet. Whichever tuning fork is closest in resonant frequency to the frequency of that vibration will tend to shake the most (or the loudest). If the forks' tines were flimsy enough, we could see the relative motion of each by the length of the blur we would see as we inspected each one from an end-view perspective. Well, make a collection of "tuning forks" out of a strip of sheet metal cut in a pattern akin to a rake, and you have the *vibrating reed* frequency meter :

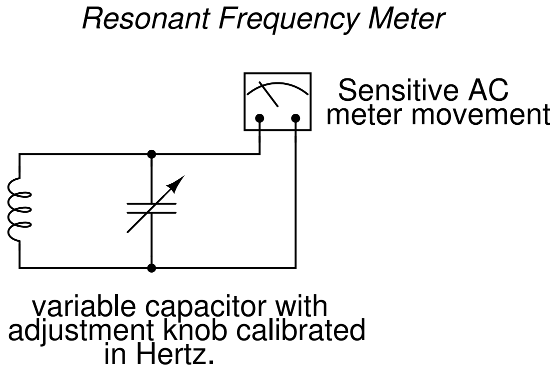


The user of this meter views the ends of all those unequal length reeds as they are collectively shaken at the frequency of the applied AC voltage to the coil. The one closest in resonant frequency to the applied AC will vibrate the most, looking something like this :



Vibrating reed meters, obviously, are not precision instruments, but they are very simple and therefore easy to manufacture to be rugged. They are often found on small engine-driven generator sets for the purpose of setting engine speed so that the frequency is somewhat close to 60 (50 in Europe) Hertz.

While reed-type meters are imprecise, their operational principle is not. In lieu of mechanical resonance, we may substitute electrical resonance and design a frequency meter using an inductor and capacitor in the form of a tank circuit (parallel inductor and capacitor). One or both components are made adjustable, and a meter is placed in the circuit to indicate maximum amplitude of voltage across the two components. The adjustment knob(s) are calibrated to show resonant frequency for any given setting, and the frequency is read from them after the device has been adjusted for maximum indication on the meter. Essentially, this is a tunable filter circuit which is adjusted and then read in a manner similar to a bridge circuit (which must be balanced for a "null" condition and then read).



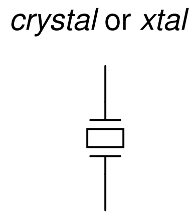
This technique is a popular one for amateur radio operators (or at least it was before the advent of inexpensive digital frequency instruments called *counters*), especially because it doesn't require direct connection to the circuit. So long as the inductor and/or capacitor can intercept enough stray field (magnetic or electric, respectively) from the circuit under test to cause the meter to indicate, it will work.

In frequency as in other types of electrical measurement, the most accurate means of measurement are usually those where an unknown quantity is compared against a known *standard*, the basic instrument doing nothing more than indicating when the two quantities are equal to each other. This is the basic principle behind the DC (Wheatstone) bridge circuit and it is a sound metrological principle applied throughout the sciences. If we have access to an accurate frequency standard (a source of AC voltage holding very precisely to a single frequency), then measurement of any unknown frequency by comparison should be relatively easy.

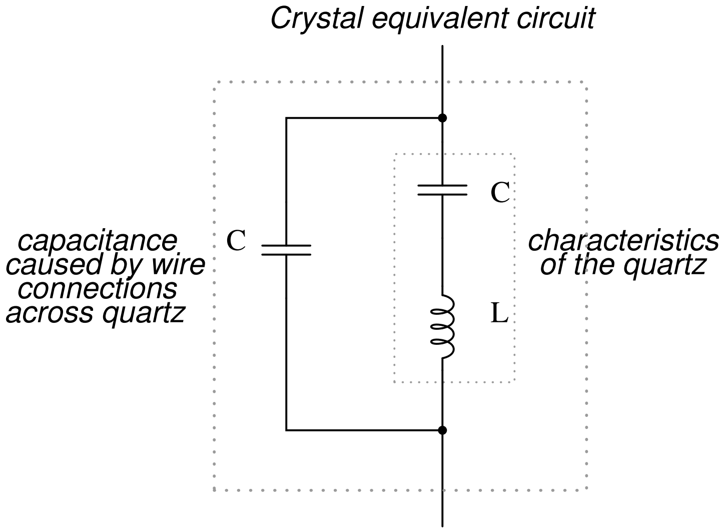
For that frequency standard, we turn our attention back to the tuning fork, or at least a more modern variation of it called the *quartz crystal*. Quartz is a naturally occurring mineral possessing a very interesting property called *piezoelectricity*. Piezoelectric materials produce a voltage across their length when physically stressed, and will physically deform when an external voltage is applied across their lengths. This deformation is very, very slight in most cases, but it does exist.

Quartz rock is elastic (springy) within that small range of bending which an external voltage would produce, which means that it will have a mechanical resonant frequency of its own capable of being manifested as an electrical voltage signal. In other words, if a chip of quartz is struck, it will "ring" with its own unique frequency determined by the length of the chip, and that resonant oscillation will produce an equivalent voltage across multiple points of the quartz chip which can be tapped into by wires fixed to the surface of the chip. In reciprocal manner, the quartz chip will tend to vibrate most when it is "excited" by an applied AC voltage at precisely the right frequency, just like the reeds on a vibrating-reed frequency meter.

Chips of quartz rock can be precisely cut for desired resonant frequencies, and that chip mounted securely inside a protective shell with wires extending for connection to an external electric circuit. When packaged as such, the resulting device is simply called a *crystal* (or sometimes "*xtal*"), and its schematic symbol looks like this :



Electrically, that quartz chip is equivalent to a series LC resonant circuit. The dielectric properties of quartz contribute an additional capacitive element to the equivalent circuit, and in the end it looks something like this :



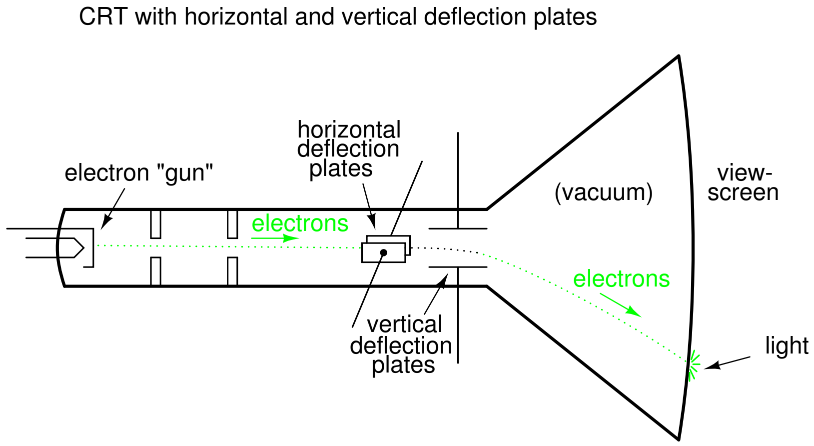
The "capacitance" and "inductance" shown in series are merely electrical equivalents of the quartz's mechanical resonance properties : they do not exist as discrete components within the crystal. The capacitance shown in parallel due to the wire connections across the dielectric (insulating) quartz body is real, and it has an effect on the resonant response of the whole system. A full discussion on crystal dynamics is not necessary here, but what needs to be understood about crystals is this resonant circuit equivalence and how it can be exploited within an oscillator circuit to achieve an output voltage with a stable, known frequency.

Crystals, as resonant elements, typically have much higher "Q" (*quality*) values than tank circuits built from inductors and capacitors, principally due to the relative absence of stray resistance, making their resonant frequencies very definite and precise. Because the resonant frequency is solely dependent on the physical properties of quartz (a very stable substance, mechanically), the resonant frequency variation over time with a quartz crystal is very, very low. This is how *quartz movement* watches obtain their high accuracy : by means of an electronic oscillator stabilized by the resonant action of a quartz crystal.

For laboratory applications, though, even greater frequency stability may be desired. To achieve this, the crystal in question may be placed in a temperature stabilized environment (usually an oven), thus eliminating frequency errors due to thermal expansion and contraction of the quartz.

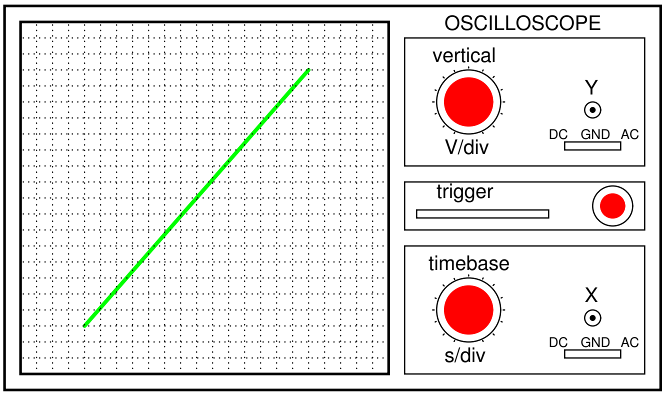
For the ultimate in a frequency standard though, nothing discovered thus far surpasses the accuracy of a single resonating atom. This is the principle of the so-called *atomic clock*, which uses an atom of mercury (or cesium) suspended in a vacuum, excited by outside energy to resonate at its own unique frequency. The resulting frequency is detected as a radio-wave signal and that forms the basis for the most accurate clocks known to humanity. National standards laboratories around the world maintain a few of these hyper-accurate clocks, and broadcast frequency signals based on those atoms' vibrations for scientists and technicians to tune in and use for frequency calibration purposes.

Now we get to the practical part : once we have a *source* of accurate frequency, how do we compare that against an unknown frequency to obtain a measurement? One way is to use a CRT as a frequency-comparison device. Cathode Ray Tubes typically have means of deflecting the electron beam in the horizontal as well as the vertical axis. If metal plates are used to electrostatically deflect the electrons, there will be a pair of plates to the left and right of the beam as well as a pair of plates above and below the beam.



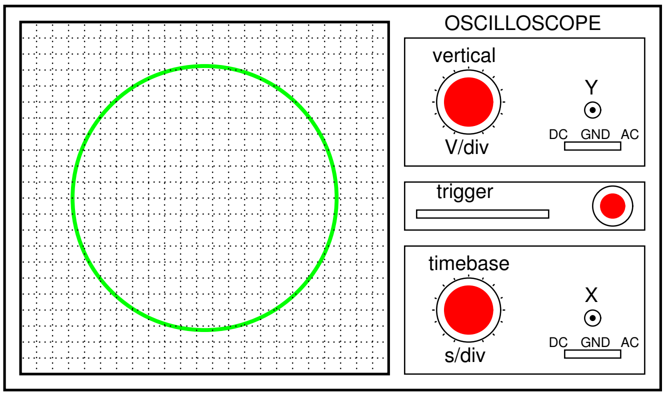
If we allow one AC signal to deflect the beam up and down (connect that AC voltage source to the "vertical" deflection plates) and another AC signal to deflect the beam left and right (using the other pair of deflection plates), patterns will be produced on the screen of the CRT indicative of the *ratio* of these two AC frequencies. These patterns are called *Lissajous figures* and are a common means of comparative frequency measurement in electronics.

If the two frequencies are the same, we will obtain a simple figure on the screen of the CRT, the shape of that figure being dependent upon the phase shift between the two AC signals. Here is a sampling of Lissajous figures for two sine-wave signals of equal frequency, shown as they would appear on the face of an oscilloscope (an AC voltage-measuring instrument using a CRT as its "movement"). The first picture is of the Lissajous figure formed by two AC voltages perfectly in phase with each other :



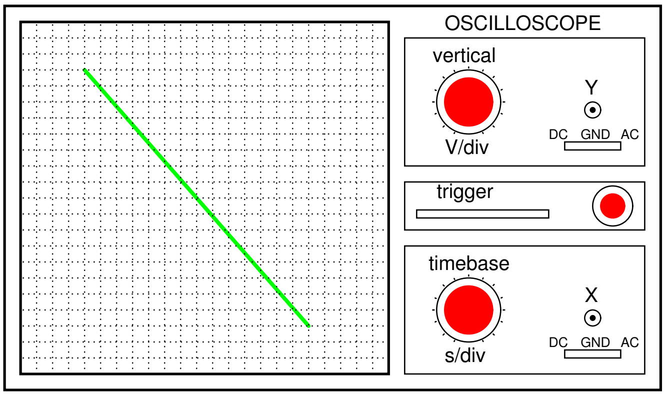
Lissajous figure: same frequency, 0 degrees phase shift

If the two AC voltages are not in phase with each other, a straight line will not be formed. Rather, the Lissajous figure will take on the appearance of an oval, becoming perfectly circular if the phase shift is exactly 90^o between the two signals, and if their amplitudes are equal :



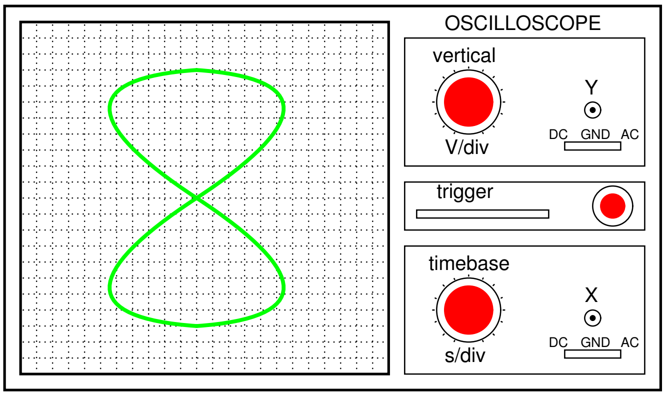
Lissajous figure: same frequency, 90 or 270 degrees phase shift

Finally, if the two AC signals are directly opposing one another in phase (180^o shift), we will end up with a line again, only this time it will be oriented in the opposite direction :



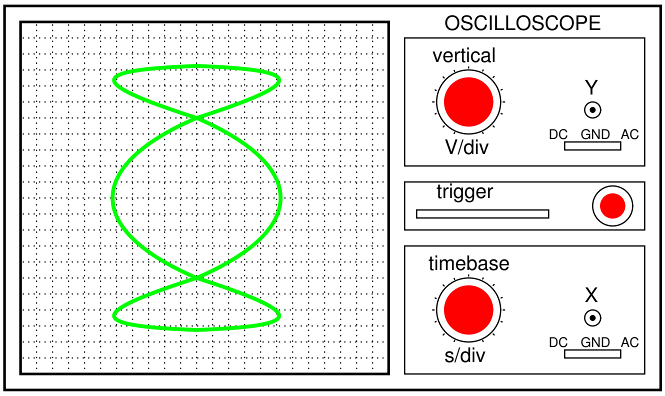
Lissajous figure: same frequency, 180 degrees phase shift

When we are faced with signal frequencies that are not the same, Lissajous figures get quite a bit more complex. Consider the following examples and their given vertical/horizontal frequency ratios :



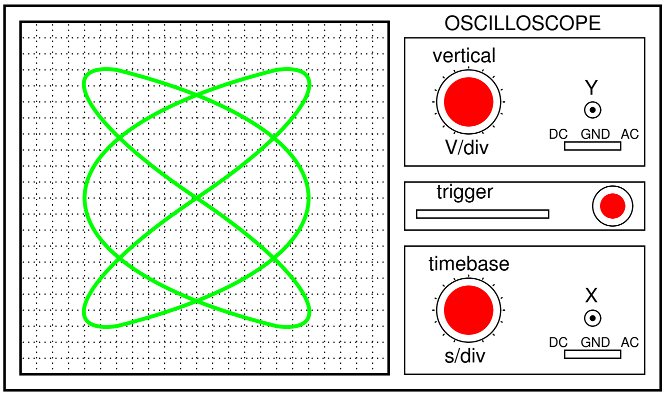
Lissajous figure: Horizontal frequency is twice that of vertical

The more complex the ratio between horizontal and vertical frequencies, the more complex the Lissajous figure. Consider the following illustration of a 3 :1 frequency ratio between horizontal and vertical :



Lissajous figure: Horizontal frequency is three times that of vertical

. . . and a 3 :2 frequency ratio (horizontal = 3, vertical = 2) :



Lissajous figure: Horizontal/Vertical frequency ratio is 3:2

In cases where the frequencies of the two AC signals are not exactly a simple ratio of each other (but close), the Lissajous figure will appear to "move," slowly changing orientation as the phase angle between the two waveforms rolls between 0^o and 180^o. If the two frequencies are locked in an exact integer ratio between each other, the Lissajous figure will be stable on the viewscreen of the CRT.

The physics of Lissajous figures limits their usefulness as a frequency-comparison technique to cases where the frequency ratios are simple integer values (1 :1, 1 :2, 1 :3, 2 :3, 3 :4, etc.). Despite this limitation, Lissajous figures are a popular means of frequency comparison wherever an accessible frequency standard (signal generator) exists.

- **REVIEW :**
- Some frequency meters work on the principle of mechanical resonance, indicating frequency by relative oscillation among a set of uniquely tuned "reeds" shaken at the measured frequency.
- Other frequency meters use electric resonant circuits (LC tank circuits, usually) to indicate frequency. One or both components is made to be adjustable, with an accurately calibrated

adjustment knob, and a sensitive meter is read for maximum voltage or current at the point of resonance.

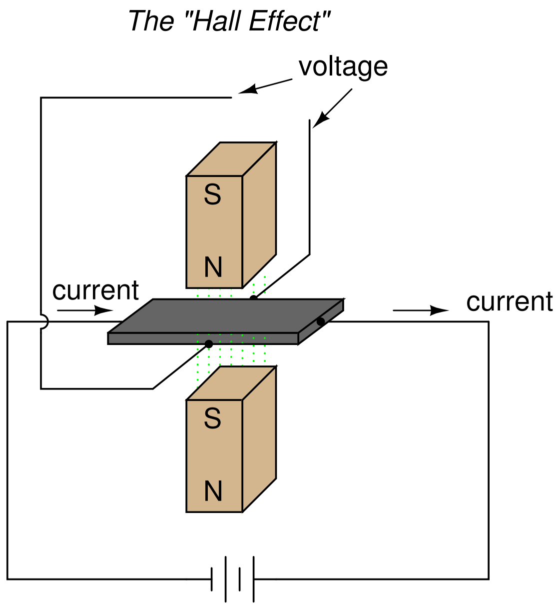
- Frequency can be measured in a comparative fashion, as is the case when using a CRT to generate *Lissajous figures*. Reference frequency signals can be made with a high degree of accuracy by oscillator circuits using quartz crystals as resonant devices. For ultra precision, atomic clock signal standards (based on the resonant frequencies of individual atoms) can be used.

12.3 Power measurement

Power measurement in AC circuits can be quite a bit more complex than with DC circuits for the simple reason that phase shift makes complicates the matter beyond multiplying voltage by current figures obtained with meters. What is needed is an instrument able to determine the product (multiplication) of *instantaneous* voltage and current. Fortunately, the common electrodynamometer movement with its stationary and moving coil does a fine job of this.

Three phase power measurement can be accomplished using two dynamometer movements with a common shaft linking the two moving coils together so that a single pointer registers power on a meter movement scale. This, obviously, makes for a rather expensive and complex movement mechanism, but it is a workable solution.

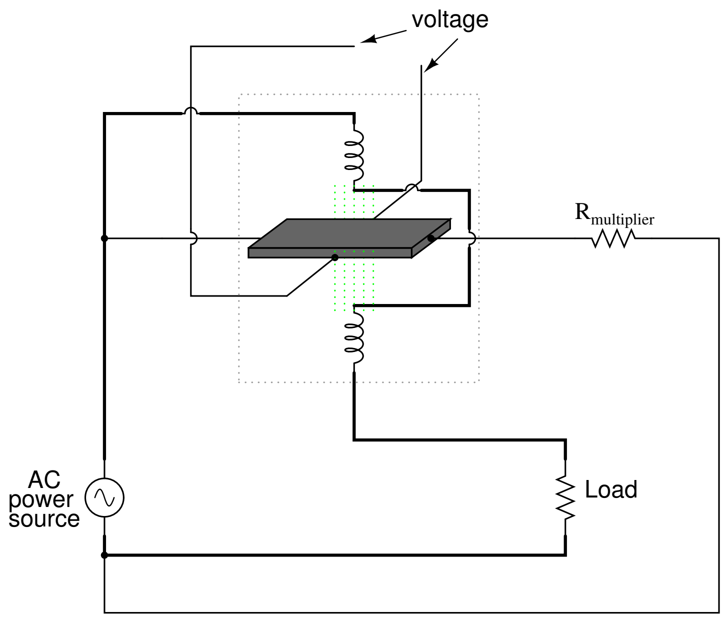
An ingenious method of deriving an electronic power meter (one that generates an electric signal representing power in the system rather than merely move a pointer) is based on the Hall effect. The Hall effect is an unusual effect first noticed by E. H. Hall in 1879, whereby a voltage is generated along the width of a current-carrying conductor exposed to a perpendicular magnetic field :



The voltage generated across the width of the flat, rectangular conductor is directly proportional to both the magnitude of the current through it and the strength of the magnetic field. Mathematically, it is a product (multiplication) of these two variables. The amount of "Hall Voltage" produced for any given set of conditions also depends on the type of material used for the flat, rectangular conductor. It has been found that specially prepared "semiconductor" materials produce a greater Hall voltage than do metals, and so modern Hall Effect devices are made of these.

It makes sense then that if we were to build a device using a Hall-effect sensor where the current through the conductor was pushed by AC voltage from an external circuit and the magnetic field was set up by a pair or wire coils energized by the current of the AC power circuit, the Hall voltage would be in direct proportion to the multiple of circuit current and voltage. Having no mass to move (unlike an electromechanical movement), this device is able to provide *instantaneous* power measurement :

A Hall effect power sensor



Not only will the output voltage of the Hall effect device be the representation of instantaneous power at any point in time, but it will also be a DC signal! This is because the Hall voltage polarity is dependent upon *both* the polarity of the magnetic field and the direction of current through the conductor. If both current direction and magnetic field polarity reverses – as it would ever half-cycle of the AC power – the output voltage polarity will stay the same.

If voltage and current in the power circuit are 90^o out of phase (a power factor of zero, meaning *no* real power delivered to the load), the alternate peaks of Hall device current and magnetic field will never coincide with each other : when one is at its peak, the other will be zero. At those points in time, the Hall output voltage will likewise be zero, being the product (multiplication) of current and magnetic field strength. Between those points in time, the Hall output voltage will fluctuate equally between positive and negative, generating a signal corresponding to the instantaneous absorption and release of power through the reactive load. The net DC output voltage will be zero, indicating zero true power in the circuit.

Any phase shift between voltage and current in the power circuit less than 90^o will result in a Hall output voltage that oscillates between positive and negative, but spends more time positive than negative. Consequently there will be a net DC output voltage. Conditioned through a low-pass filter circuit, this net DC voltage can be separated from the AC mixed with it, the final output signal registered on a sensitive DC meter movement.

Often it is useful to have a meter to totalize power usage over a period of time rather than instantaneously. The output of such a meter can be set in units of Joules, or total energy consumed, since *power* is a measure of work being done *per* unit time. Or, more commonly, the output of the meter can be set in units of Watt-Hours.

Mechanical means for measuring Watt-Hours are usually centered around the concept of the motor : build an AC motor that spins at a rate of speed proportional to the instantaneous power in a circuit, then have that motor turn an "odometer" style counting mechanism to keep a running total of energy consumed. The "motor" used in these meters has a rotor made of a thin aluminum disk, with the rotating magnetic field established by sets of coils energized by line voltage and load current so that the rotational speed of the disk is dependent on both voltage and current.

12.4 Power quality measurement

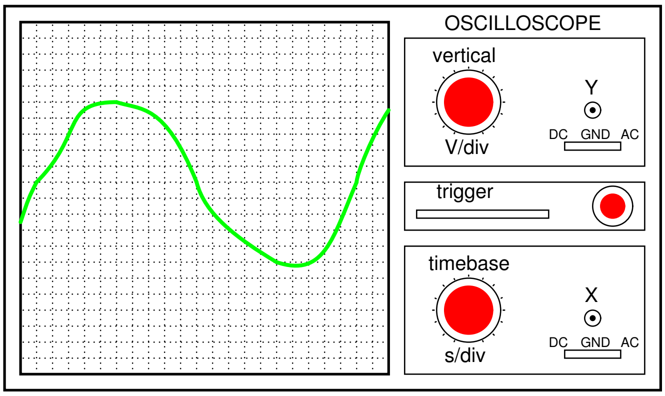
It used to be with large AC power systems that "power quality" was an unheard-of concept, aside from power factor. Almost all loads were of the "linear" variety, meaning that they did not distort the shape of the voltage sine wave, or cause non-sinusoidal currents to flow in the circuit. This is not true anymore. Loads controlled by "nonlinear" electronic components are becoming more prevalent in both home and industry, meaning that the voltages and currents in the power system(s) feeding these loads are rich in harmonics : what should be nice, clean sine-wave voltages and currents are becoming highly distorted, which is equivalent to the presence of an infinite series of high-frequency sine waves at multiples of the fundamental power line frequency.

Excessive harmonics in an AC power system can overheat transformers, cause exceedingly high neutral conductor currents in three-phase systems, create electromagnetic "noise" in the form of radio emissions that can interfere with sensitive electronic equipment, reduce electric motor horsepower output, and can be difficult to pinpoint. With problems like these plaguing power systems, engineers and technicians require ways to precisely detect and measure these conditions.

Power Quality is the general term given to represent an AC power system's freedom from harmonic content. A "power quality" meter is one that gives some form of harmonic content indication.

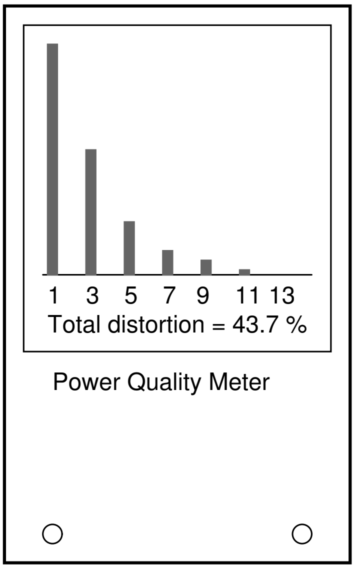
A simple way for a technician to determine power quality in their system without sophisticated equipment is to compare voltage readings between two accurate voltmeters measuring the same system voltage : one meter being an "averaging" type of unit (such as an electromechanical movement meter) and the other being a "true-RMS" type of unit (such as a high-quality digital meter). Remember that "averaging" type meters are calibrated so that their scales indicate volts RMS, *based on the assumption that the AC voltage being measured is sinusoidal*. If the voltage is anything but sinewave-shaped, the averaging meter will *not* register the proper value, whereas the true-RMS meter always will, regardless of waveshape. The rule of thumb here is this : the greater the disparity between the two meters, the worse the power quality is, and the greater its harmonic content. A power system with good quality power should generate equal voltage readings between the two meters, to within the rated error tolerance of the two instruments.

Another qualitative measurement of power quality is the oscilloscope test : connect an oscilloscope (CRT) to the AC voltage and observe the shape of the wave. Anything other than a clean sine wave could be an indication of trouble :



This is a moderately ugly "sine" wave. Definite harmonic content here!

Still, if quantitative analysis (definite, numerical figures) is necessary, there is no substitute for an instrument specifically designed for that purpose. Such an instrument is called a *power quality meter* and is sometimes better known in electronic circles as a low-frequency *spectrum analyzer*. What this instrument does is provide a graphical representation on a CRT or digital display screen of the AC voltage's frequency "spectrum." Just as a prism splits a beam of white light into its constituent color components (how much red, orange, yellow, green, and blue is in that light), the spectrum analyzer splits a mixed-frequency signal into its constituent frequencies, and displays the result in the form of a histogram :



Each number on the horizontal scale of this meter represents a harmonic of the fundamental frequency. For American power systems, the "1" represents 60 Hz (the 1st harmonic, or *fundamental*), the "3" for 180 Hz (the 3rd harmonic), the "5" for 300 Hz (the 5th harmonic), and so on. The black rectangles represent the relative magnitudes of each of these harmonic components in the measured

AC voltage. A pure, 60 Hz sine wave would show only a tall black bar over the "1" with no black bars showing at all over the other frequency markers on the scale, because a pure sine wave has no harmonic content.

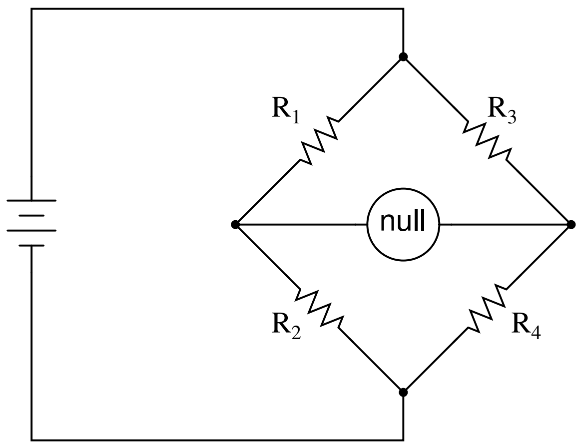
Power quality meters such as this might be better referred to as *overtone* meters, because they are designed to display only those frequencies known to be generated by the power system. In three-phase AC power systems (predominant for large power applications), even-numbered harmonics tend to be canceled out, and so only harmonics existing in significant measure are the odd-numbered.

Meters like these are very useful in the hands of a skilled technician, because different types of nonlinear loads tend to generate different spectrum "signatures" which can clue the troubleshooter to the source of the problem. These meters work by very quickly sampling the AC voltage at many different points along the waveform shape, digitizing those points of information, and using a microprocessor (small computer) to perform numerical Fourier analysis (the *Fast Fourier Transform* or "*FFT*" algorithm) on those data points to arrive at harmonic frequency magnitudes. The process is not much unlike what the SPICE program tells a computer to do when performing a Fourier analysis on a simulated circuit voltage or current waveform.

12.5 AC bridge circuits

As we saw with DC measurement circuits, the circuit configuration known as a *bridge* can be a very useful way to measure unknown values of resistance. This is true with AC as well, and we can apply the very same principle to the accurate measurement of unknown impedances.

To review, the bridge circuit works as a pair of two-component voltage dividers connected across the same source voltage, with a *null-detector* meter movement connected between them to indicate a condition of "balance" at zero volts :



Any one of the four resistors in the above bridge can be the resistor of unknown value, and its value can be determined by a ratio of the other three, which are "calibrated," or whose resistances are known to a precise degree. When the bridge is in a balanced condition (zero voltage as indicated by the null detector), the ratio works out to be this :

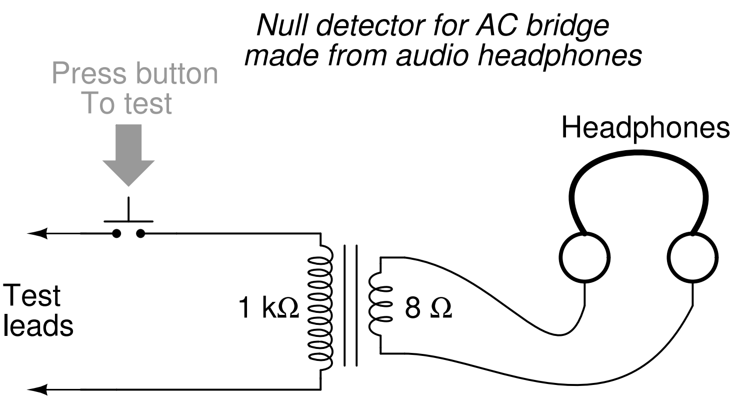
In a condition of ***balance****:*

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

One of the advantages of using a bridge circuit to measure resistance is that the voltage of the power source is irrelevant. Practically speaking, the higher the supply voltage, the easier it is to detect a condition of imbalance between the four resistors with the null detector, and thus the more sensitive it will be. A greater supply voltage leads to the possibility of increased measurement precision. However, there will be no fundamental error introduced as a result of a lesser or greater power supply voltage unlike other types of resistance measurement schemes.

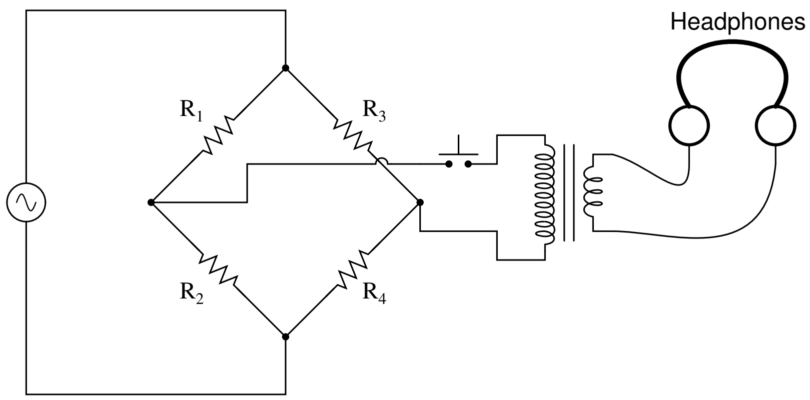
Impedance bridges work the same, only the balance equation is with *complex* quantities, as both magnitude and phase across the components of the two dividers must be equal in order for the null detector to indicate "zero." The null detector, of course, must be a device capable of detecting very small AC voltages. An oscilloscope is often used for this, although very sensitive electromechanical meter movements and even headphones (small speakers) may be used if the source frequency is within audio range.

One way to maximize the effectiveness of audio headphones as a null detector is to connect them to the signal source through an impedance-matching transformer. Headphone speakers are typically low-impedance units (8 Ω), requiring substantial current to drive, and so a step-down transformer helps "match" low-current signals to the impedance of the headphone speakers. An audio output transformer works well for this purpose :



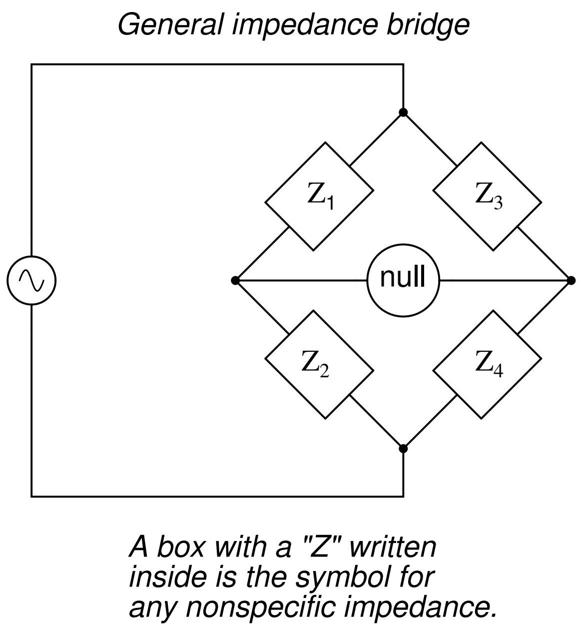
Using a pair of headphones that completely surround the ears (the "closed-cup" type), I've been able to detect currents of less than 0.1 μA with this simple detector circuit. Roughly equal performance was obtained using two different step-down transformers : a small power transformer (120/6 volt ratio), and an audio output transformer (1000 :8 ohm impedance ratio). With the pushbutton switch in place to interrupt current, this circuit is usable for detecting signals from DC to over 2 MHz : even if the frequency is far above or below the audio range, a "click" will be heard from the headphones each time the switch is pressed and released.

Connected to a resistive bridge, the whole circuit looks like this :



Listening to the headphones as one or more of the resistor "arms" of the bridge is adjusted, a condition of balance will be realized when the headphones fail to produce "clicks" (or tones, if the bridge's power source frequency is within audio range) as the switch is actuated.

When describing general AC bridges, where *impedances* and not just resistances must be in proper ratio for balance, it is sometimes helpful to draw the respective bridge legs in the form of box-shaped components, each one with a certain impedance :



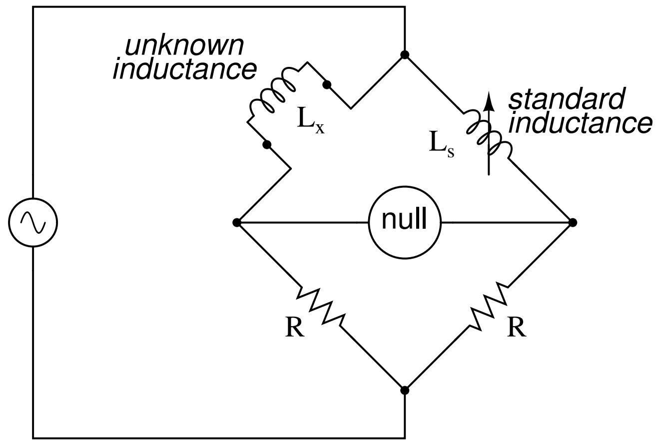
For this general form of AC bridge to balance, the impedance ratios of each branch must be equal :

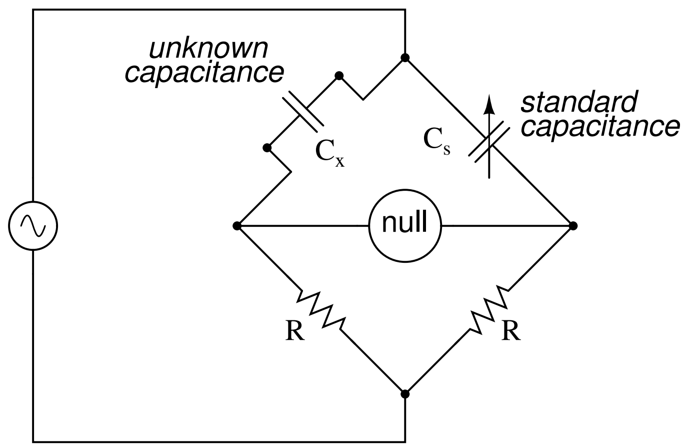
$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

Again, it must be stressed that the impedance quantities in the above equation *must* be complex, accounting for both magnitude and phase angle. It is insufficient that the impedance magnitudes alone be balanced; without phase angles in balance as well, there will still be voltage across the terminals of the null detector and the bridge will not be balanced.

Bridge circuits can be constructed to measure just about any device value desired, be it capacitance, inductance, resistance, or even "Q." As always in bridge measurement circuits, the unknown quantity is always "balanced" against a known standard, obtained from a high-quality, calibrated component that can be adjusted in value until the null detector device indicates a condition of balance. Depending on how the bridge is set up, the unknown component's value may be determined directly from the setting of the calibrated standard, or derived from that standard through a mathematical formula.

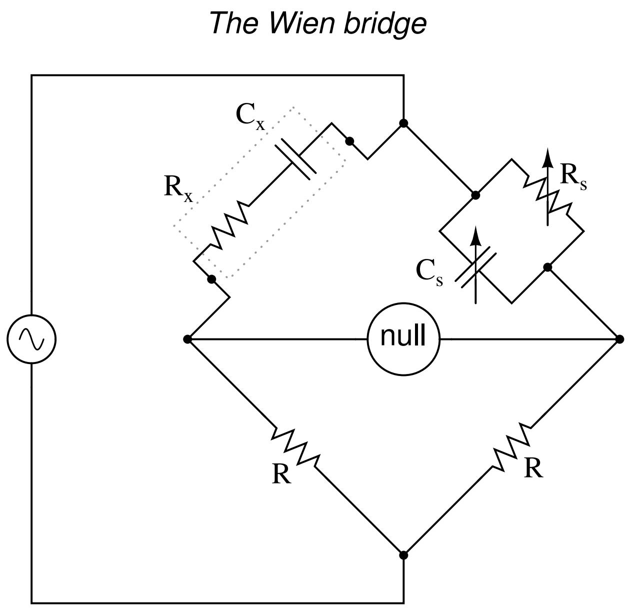
A couple of simple bridge circuits are shown below, one for inductance and one for capacitance :





Simple "symmetrical" bridges such as these are so named because they exhibit symmetry (mirror-image similarity) from left to right. The two bridge circuits shown above are balanced by adjusting the calibrated reactive component (L_s or C_s). They are a bit simplified from their real-life counterparts, as practical symmetrical bridge circuits often have a calibrated, variable resistor in series or parallel with the reactive component to balance out stray resistance in the unknown component. But, in the hypothetical world of perfect components, these simple bridge circuits do just fine to illustrate the basic concept.

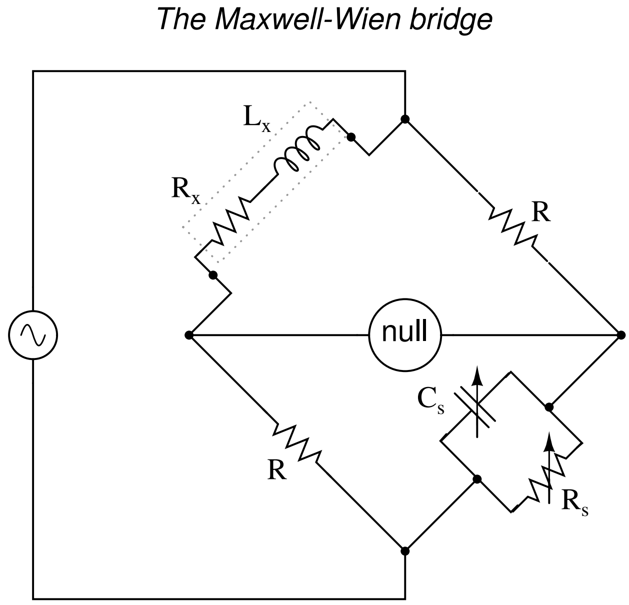
An example of a little extra complexity added to compensate for real-world effects can be found in the so-called *Wien bridge*, which uses a parallel capacitor-resistor standard impedance to balance out an unknown series capacitor-resistor combination. All capacitors have some amount of internal resistance, be it literal or equivalent (in the form of dielectric heating losses) which tend to spoil their otherwise perfectly reactive natures. This internal resistance may be of interest to measure, and so the Wien bridge attempts to do so by providing a balancing impedance that isn't "pure" either :



Being that there are two standard components to be adjusted (a resistor and a capacitor) this bridge will take a little more time to balance than the others we've seen so far. The combined effect of R_s and C_s is to alter the magnitude and phase angle until the bridge achieves a condition of balance. Once that balance is achieved, the settings of R_s and C_s can be read from their calibrated knobs, the parallel impedance of the two determined mathematically, and the unknown capacitance and resistance determined mathematically from the balance equation ($Z_1/Z_2 = Z_3/Z_4$).

It is assumed in the operation of the Wien bridge that the standard capacitor has negligible internal resistance, or at least that resistance is already known so that it can be factored into the balance equation. Wien bridges are useful for determining the values of "lossy" capacitor designs like electrolytics, where the internal resistance is relatively high. They are also used as frequency meters, because the balance of the bridge is frequency-dependent. When used in this fashion, the capacitors are made fixed (and usually of equal value) and the top two resistors are made variable and are adjusted by means of the same knob.

An interesting variation on this theme is found in the next bridge circuit, used to precisely measure inductances.



This ingenious bridge circuit is known as the *Maxwell-Wien bridge* (sometimes known plainly as the *Maxwell bridge*), and is used to measure unknown inductances in terms of calibrated resistance and capacitance. Calibration-grade inductors are more difficult to manufacture than capacitors of similar precision, and so the use of a simple "symmetrical" inductance bridge is not always practical. Because the phase shifts of inductors and capacitors are exactly opposite each other, a capacitive impedance can balance out an inductive impedance if they are located in opposite legs of a bridge, as they are here.

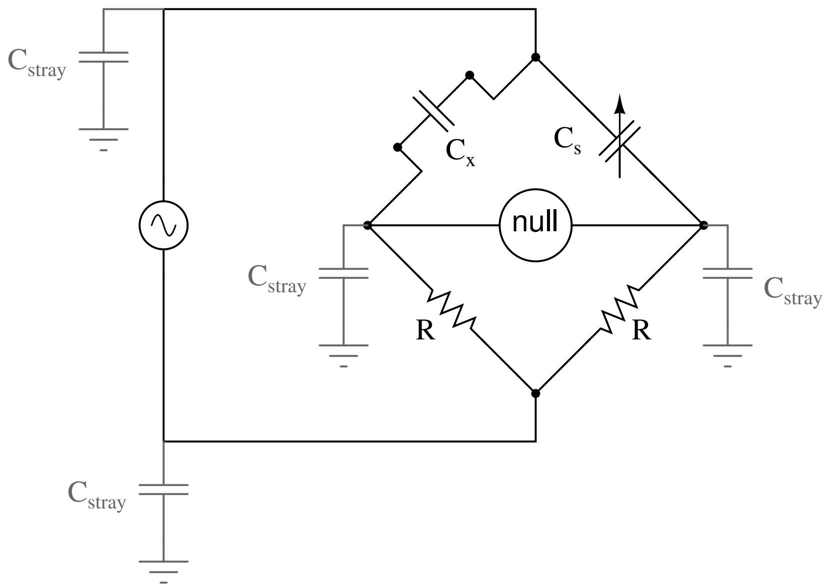
Another advantage of using a Maxwell bridge to measure inductance rather than a symmetrical inductance bridge is the elimination of measurement error due to mutual inductance between two inductors. Magnetic fields can be difficult to shield, and even a small amount of coupling between coils in a bridge can introduce substantial errors in certain conditions. With no second inductor to react with in the Maxwell bridge, this problem is eliminated.

For easiest operation, the standard capacitor (C_s) and the resistor in parallel with it (R_s) are made variable, and both must be adjusted to achieve balance. However, the bridge can be made to work if the capacitor is fixed (non-variable) and more than one resistor made variable (at least the resistor in parallel with the capacitor, and one of the other two). However, in the latter configuration it takes more trial-and-error adjustment to achieve balance, as the different variable resistors interact in balancing magnitude and phase.

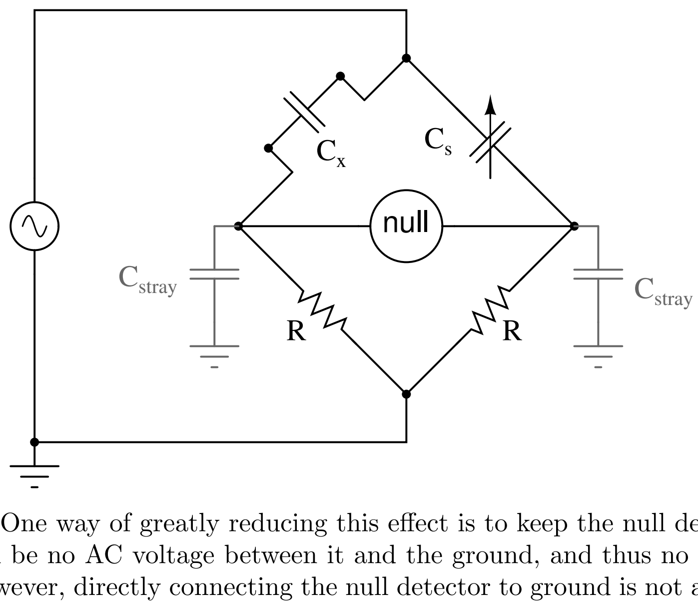
Unlike the plain Wien bridge, the balance of the Maxwell-Wien bridge is independent of source frequency, and in some cases this bridge can be made to balance in the presence of mixed frequencies from the AC voltage source, the limiting factor being the inductor's stability over a wide frequency range.

There are more variations beyond these designs, but a full discussion is not warranted here. General-purpose impedance bridge circuits are manufactured which can be switched into more than one configuration for maximum flexibility of use.

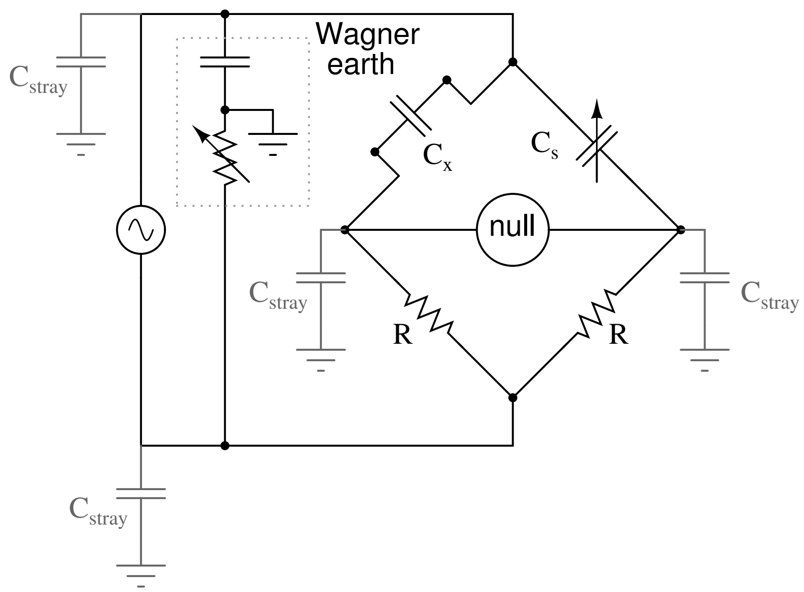
A potential problem in sensitive AC bridge circuits is that of stray capacitance between either end of the null detector unit and ground (earth) potential. Because capacitances can "conduct" alternating current by charging and discharging, they form stray current paths to the AC voltage source which may affect bridge balance :



The problem is worsened if the AC voltage source is firmly grounded at one end, the total stray impedance for leakage currents made far less and any leakage currents through these stray capacitances made greater as a result :

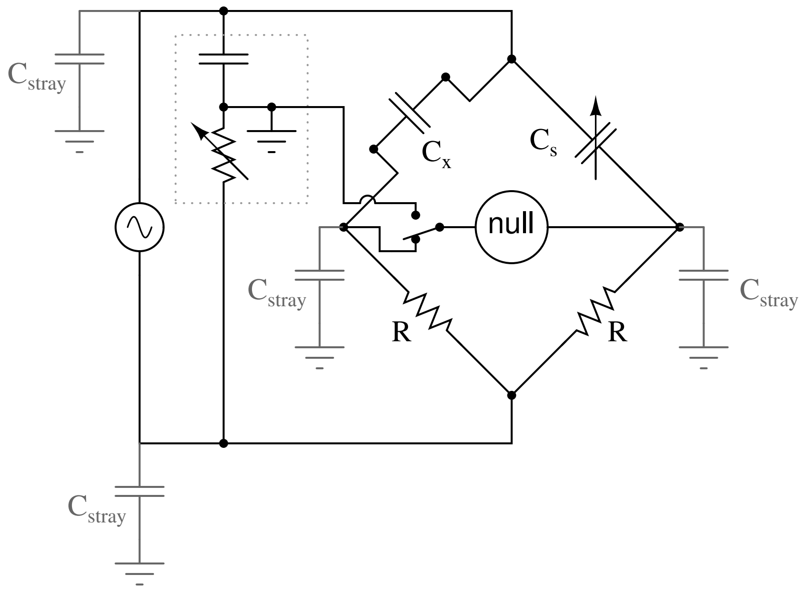


One way of greatly reducing this effect is to keep the null detector at ground potential, so there will be no AC voltage between it and the ground, and thus no current through stray capacitances. However, directly connecting the null detector to ground is not an option, as it would create a *direct* current path for stray currents, which would be worse than any capacitive path. Instead, a special voltage divider circuit called a *Wagner ground* or *Wagner earth* may be used to maintain the null detector at ground potential without the need for a direct connection to the null detector.



The Wagner earth circuit is nothing more than a voltage divider, designed to have the voltage ratio and phase shift as each side of the bridge. Because the midpoint of the Wagner divider is directly grounded, any other divider circuit (including either side of the bridge) having the same voltage proportions and phases as the Wagner divider, and powered by the same AC voltage source, will be at ground potential as well. Thus, the Wagner earth divider forces the null detector to be at ground potential, without a direct connection between the detector and ground.

There is often a provision made in the null detector connection to confirm proper setting of the Wagner earth divider circuit : a two-position switch, so that one end of the null detector may be connected to either the bridge or the Wagner earth. When the null detector registers zero signal in both switch positions, the bridge is not only guaranteed to be balanced, but the null detector is also guaranteed to be at zero potential with respect to ground, thus eliminating any errors due to leakage currents through stray detector-to-ground capacitances :



- **REVIEW :**

- AC bridge circuits work on the same basic principle as DC bridge circuits : that a balanced ratio of impedances (rather than resistances) will result in a "balanced" condition as indicated by the null-detector device.

- Null detectors for AC bridges may be sensitive electromechanical meter movements, oscilloscopes (CRT's), headphones (amplified or unamplified), or any other device capable of registering very small AC voltage levels. Like DC null detectors, its only required point of calibration accuracy is at zero.

- AC bridge circuits can be of the "symmetrical" type where an unknown impedance is balanced by a standard impedance of similar type on the same side (top or bottom) of the bridge. Or, they can be "nonsymmetrical," using parallel impedances to balance series impedances, or even capacitances balancing out inductances.

- AC bridge circuits often have more than one adjustment, since both impedance magnitude *and* phase angle must be properly matched to balance.

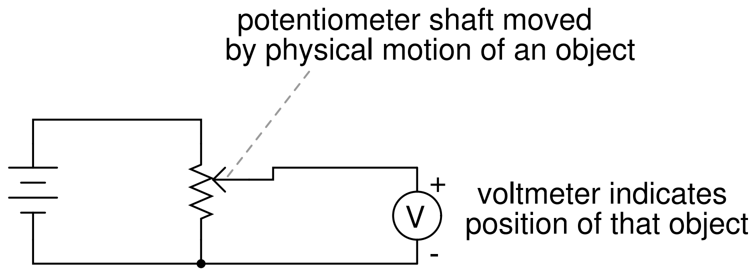
- Some impedance bridge circuits are frequency-sensitive while others are not. The frequency-sensitive types may be used as frequency measurement devices if all component values are accurately known.

- A *Wagner earth* or *Wagner ground* is a voltage divider circuit added to AC bridges to help reduce errors due to stray capacitance coupling the null detector to ground.

12.6 AC instrumentation transducers

Just as devices have been made to measure certain physical quantities and repeat that information in the form of DC electrical signals (thermocouples, strain gauges, pH probes, etc.), special devices have been made that do the same with AC.

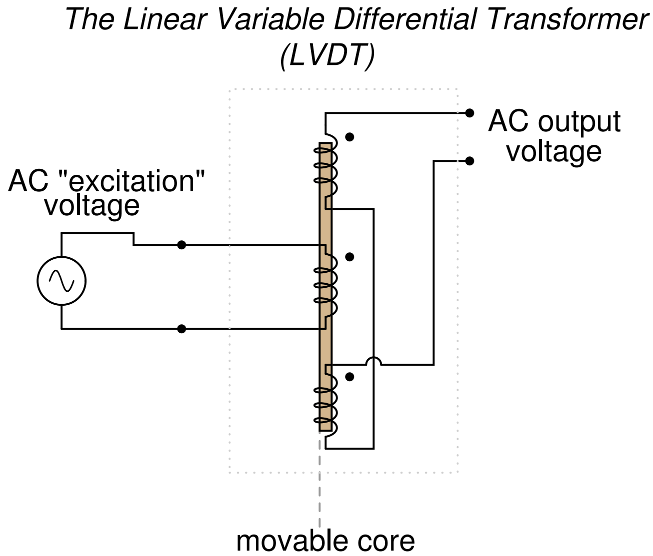
It is often necessary to be able to detect and transmit the physical position of mechanical parts via electrical signals. This is especially true in the fields of automated machine tool control and robotics. A simple and easy way to do this is with a potentiometer :



However, potentiometers have their own unique problems. For one, they rely on physical contact between the "wiper" and the resistance strip, which means they suffer the effects of physical wear over time. As potentiometers wear, their proportional output versus shaft position becomes less and less certain. You might have already experienced this effect when adjusting the volume control on an old radio : when twisting the knob, you might hear "scratching" sounds coming out of the speakers. Those noises are the result of poor wiper contact in the volume control potentiometer.

Also, this physical contact between wiper and strip creates the possibility of arcing (sparking) between the two as the wiper is moved. With most potentiometer circuits, the current is so low that wiper arcing is negligible, but it is a possibility to be considered. If the potentiometer is to be operated in an environment where combustible vapor or dust is present, this potential for arcing translates into a potential for an explosion!

Using AC instead of DC, we are able to completely avoid sliding contact between parts if we use a *variable transformer* instead of a potentiometer. Devices made for this purpose are called LVDT's, which stands for **L**inear **V**ariable **D**ifferential **T**ransformers. The design of an LVDT looks like this :



Obviously, this device is a *transformer* : it has a single primary winding powered by an external source of AC voltage, and two secondary windings connected in series-bucking fashion. It is *variable* because the core is free to move between the windings. It is *differential* because of the way the two secondary windings are connected. Being arranged to oppose each other (180^o out of phase) means that the output of this device will be the *difference* between the voltage output of the two secondary windings. When the core is centered and both windings are outputting the same voltage, the net result at the output terminals will be zero volts. It is called *linear* because the core's freedom of motion is straight-line.

The AC voltage output by an LVDT indicates the position of the movable core. Zero volts means that the core is centered. The further away the core is from center position, the greater percentage of input ("excitation") voltage will be seen at the output. The phase of the output voltage relative to the excitation voltage indicates which direction from center the core is offset.

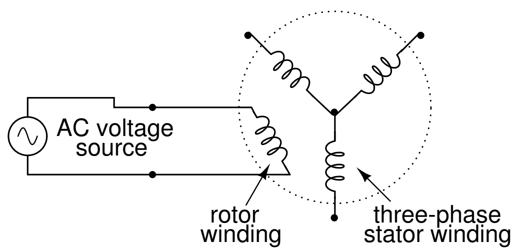
The primary advantage of an LVDT over a potentiometer for position sensing is the absence of physical contact between the moving and stationary parts. The core does not contact the wire windings, but slides in and out within a nonconducting tube. Thus, the LVDT does not "wear" like a potentiometer, nor is there the possibility of creating an arc.

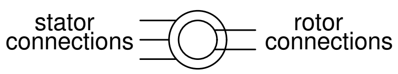
Excitation of the LVDT is typically 10 volts RMS or less, at frequencies ranging from power line to the high audio (20 kHz) range. One potential disadvantage of the LVDT is its response time, which is mostly dependent on the frequency of the AC voltage source. If very quick response times are desired, the frequency must be higher to allow whatever voltage-sensing circuits enough cycles of AC to determine voltage level as the core is moved. To illustrate the potential problem here, imagine this exaggerated scenario : an LVDT powered by a 60 Hz voltage source, with the core being moved in and out hundreds of times per second. The output of this LVDT wouldn't even look like a sine wave because the core would be moved throughout its range of motion before the AC source voltage could complete a single cycle! It would be almost impossible to determine instantaneous core position if it moves faster than the instantaneous source voltage does.

A variation on the LVDT is the RVDT, or **R**otary **V**ariable **D**ifferential **T**ransformer. This device works on almost the same principle, except that the core revolves on a shaft instead of moving in a straight line. RVDT's can be constructed for limited motion of 360^o (full-circle) motion.

Continuing with this principle, we have what is known as a *Synchro* or *Selsyn*, which is a device constructed a lot like a wound-rotor polyphase AC motor or generator. The rotor is free to revolve a full 360^o, just like a motor. On the rotor is a single winding connected to a source of AC voltage, much like the primary winding of an LVDT. The stator windings are usually in the form of a three-phase Y, although synchros with more than three phases have been built :

Synchro (a.k.a "Selsyn")

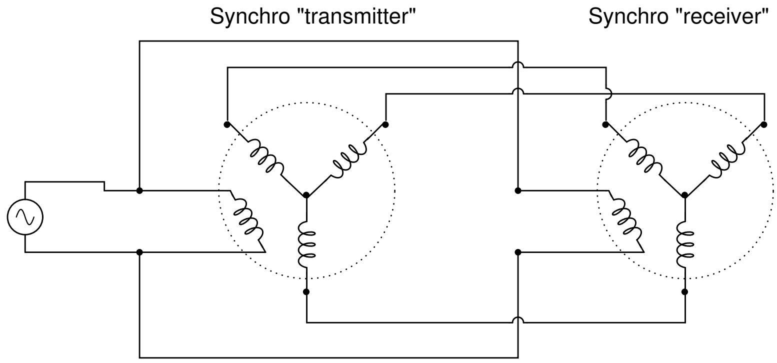




modern schematic symbol

Voltages induced in the stator windings from the rotor's AC excitation are *not* phase-shifted by 120^o as in a real three-phase generator. If the rotor were energized with DC current rather than AC and the shaft spun continuously, then the voltages would be true three-phase. But this is not how a synchro is designed to be operated. Rather, this is a *position-sensing* device much like an RVDT, except that its output signal is much more definite. With the rotor energized by AC, the stator winding voltages will be proportional in magnitude to the angular position of the rotor, phase either 0^o or 180^o shifted, like a regular LVDT or RVDT. You could think of it as a transformer with one primary winding and three secondary windings, each secondary winding oriented at a unique angle. As the rotor is slowly turned, each winding in turn will line up directly with the rotor, producing full voltage, while the other windings will produce something less than full voltage.

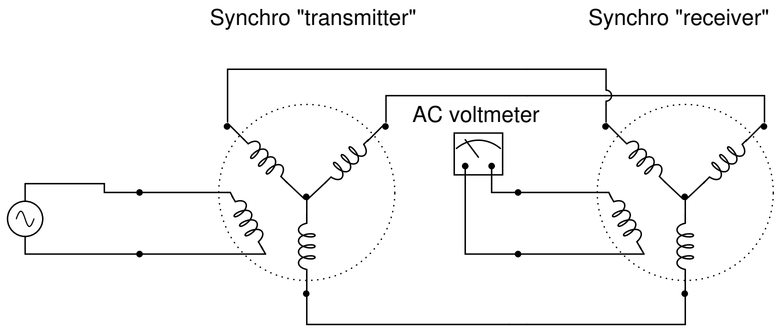
Synchros are often used in pairs. With their rotors connected in parallel and energized by the same AC voltage source, their shafts will match position to a high degree of accuracy :



The receiver rotor will turn to match position with the transmitter rotor so long as the two rotors remain energized.

Such "transmitter/receiver" pairs have been used on ships to relay rudder position, or to relay navigational gyro position over fairly long distances. The only difference between the "transmitter" and the "receiver" is which one gets turned by an outside force. The "receiver" can just as easily be used as the "transmitter" by forcing its shaft to turn and letting the synchro on the left match position.

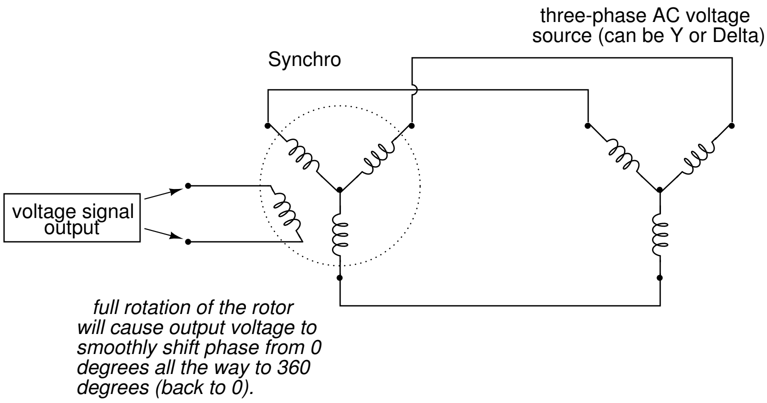
If the receiver's rotor is left unpowered, it will act as a position-error detector, generating an AC voltage at the rotor if the shaft is anything other than 90^o or 270^o shifted from the shaft position of the transmitter. The receiver rotor will no longer generate any torque and consequently will no longer automatically match position with the transmitter's :



AC voltmeter registers a voltage if receiver rotor position isn't exactly 90 or 270 degrees shifted from the position of the transmitter rotor.

This can be thought of almost as a sort of bridge circuit that achieves balance only if the receiver shaft is brought to one of two (matching) positions with the transmitter shaft.

One rather ingenious application of the synchro is in the creation of a phase-shifting device, provided that the stator is energized by three-phase AC :



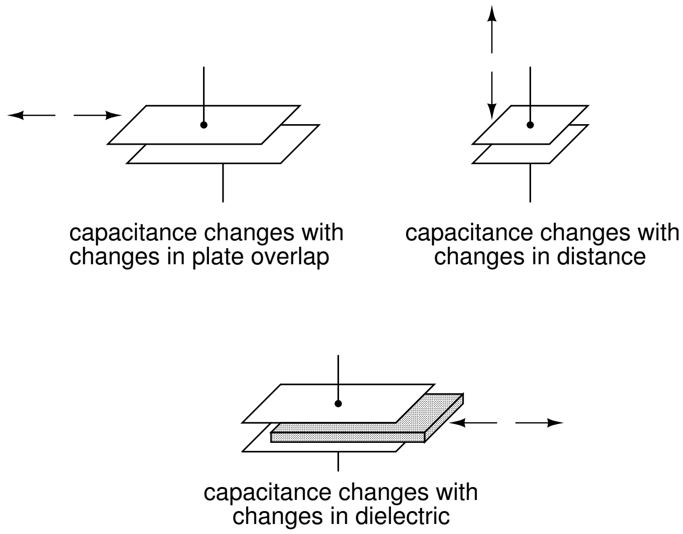
As the synchro's rotor is turned, the rotor coil will progressively align with each stator coil, their respective magnetic fields being 120^o phase-shifted from one another. In between those positions, these phase-shifted fields will mix to produce a rotor voltage somewhere between 0^o, 120^o, or 240^o shift. The practical result is a device capable of providing an infinitely variable-phase AC voltage with the twist of a knob (attached to the rotor shaft).

So far the transducers discussed have all been of the inductive variety. However, it is possible to make transducers which operate on variable capacitance as well, AC being used to sense the change in capacitance and generate a variable output voltage.

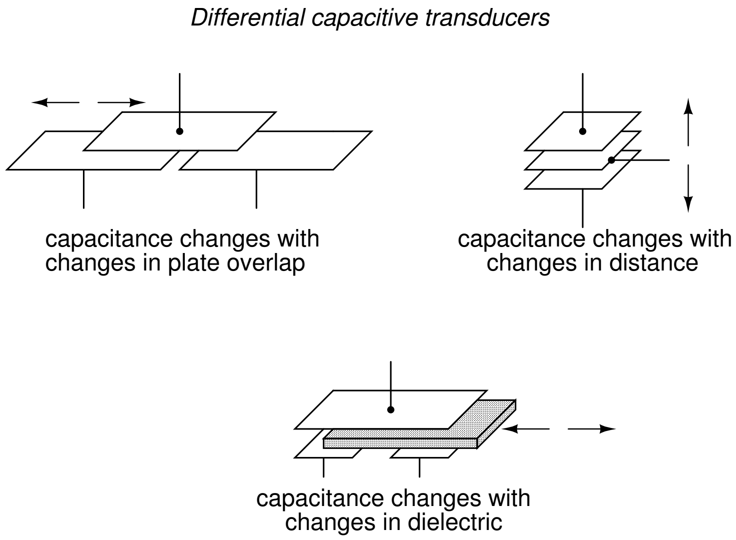
Remember that the capacitance between two conductive surfaces varies with three major factors : the overlapping area of those two surfaces, the distance between them, and the dielectric constant of the material in between the surfaces. If two out of three of these variables can be fixed (stabilized) and the third allowed to vary, then any measurement of capacitance between the surfaces will be solely indicative of changes in that third variable.

Medical researchers have long made use of capacitive sensing to detect physiological changes in living bodies. As early as 1907, a German researcher named H. Cremer placed two metal plates on either side of a beating frog heart and measured the capacitance changes resulting from the heart alternately filling and emptying itself of blood. Similar measurements have been performed on human beings with metal plates placed on the chest and back, recording respiratory and cardiac action by means of capacitance changes. For more precise capacitive measurements of organ activity, metal probes have been inserted into organs (especially the heart) on the tips of catheter tubes, capacitance being measured between the metal probe and the body of the subject. With a sufficiently high AC excitation frequency and sensitive enough voltage detector, not just the pumping action but also the *sounds* of the active heart may be readily interpreted.

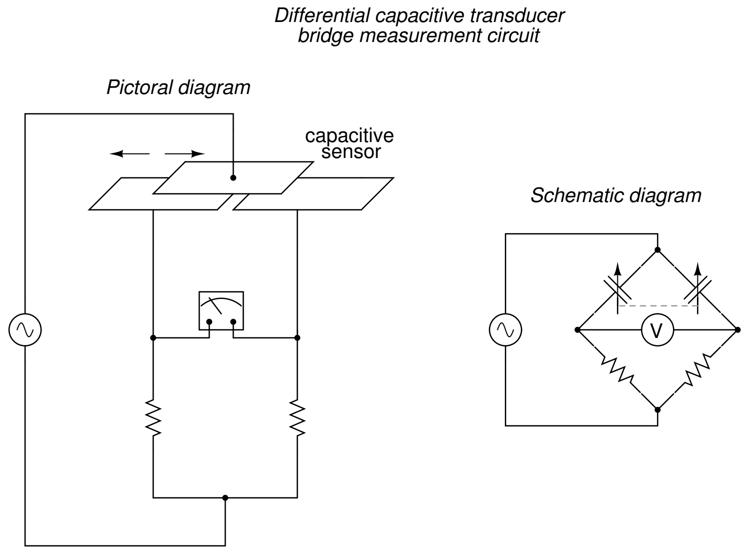
Like inductive transducers, capacitive transducers can also be made to be self-contained units, unlike the direct physiological examples described above. Some transducers work by making one of the capacitor plates movable, either in such a way as to vary the overlapping area or the distance between the plates. Other transducers work by moving a dielectric material in and out between two fixed plates :



Transducers with greater sensitivity and immunity to changes in other variables can be obtained by way of differential design, much like the concept behind the LVDT (Linear Variable *Differential* Transformer). Here are a few examples of differential capacitive transducers :



As you can see, all of the differential devices shown in the above illustration have *three* wire connections rather than two : one wire for each of the "end" plates and one for the "common" plate. As the capacitance between one of the "end" plates and the "common" plate changes, the capacitance between the other "end" plate and the "common" plate is such to change in the opposite direction. This kind of transducer lends itself very well to implementation in a bridge circuit :

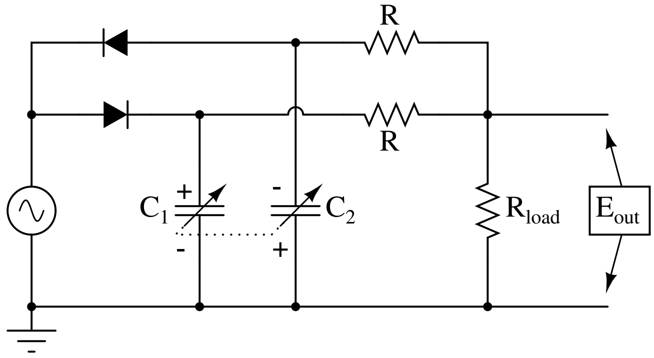


Capacitive transducers provide relatively small capacitances for a measurement circuit to operate with, typically in the *pico*farad range. Because of this, high power supply frequencies (in the megahertz range!) are usually required to reduce these capacitive reactances to reasonable levels. Given the small capacitances provided by typical capacitive transducers, stray capacitances have the potential of being major sources of measurement error. Good conductor shielding is *essential* for reliable and accurate capacitive transducer circuitry!

The bridge circuit is not the only way to effectively interpret the differential capacitance output of such a transducer, but it is one of the simplest to implement and understand. As with the LVDT, the voltage output of the bridge is proportional to the displacement of the transducer action from its center position, and the direction of offset will be indicated by phase shift. This kind of bridge circuit is similar in function to the kind used with strain gauges : it is not intended to be in a "balanced" condition all the time, but rather the degree of imbalance represents the magnitude of the quantity being measured.

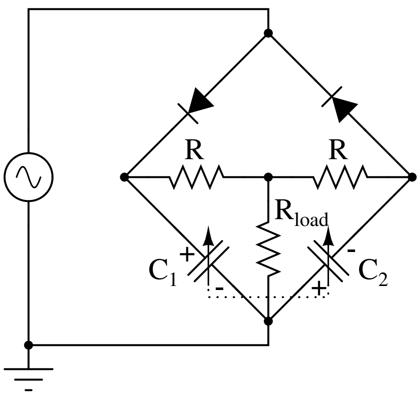
An interesting alternative to the bridge circuit for interpreting differential capacitance is the *twin-T*. It requires the use of diodes, those "one-way valves" for electric current mentioned earlier in the chapter :

Differential capacitive transducer
twin-T measurement circuit



This circuit might be better understood if re-drawn to resemble more of a bridge configuration :

Differential capacitive transducer
twin-T measurement circuit



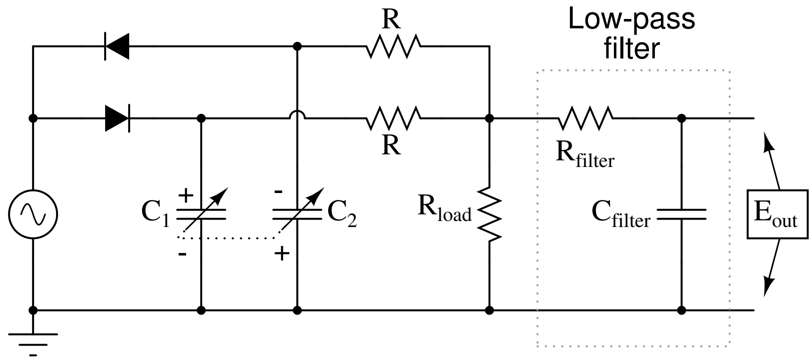
Output voltage measured across R_{load}

Capacitor C_1 is charged by the AC voltage source during every positive half-cycle (positive as measured in reference to the ground point), while C_2 is charged during every negative half-cycle. While one capacitor is being charged, the other capacitor discharges (at a slower rate than it was charged) through the three-resistor network. As a consequence, C_1 maintains a positive DC voltage with respect to ground, and C_2 a negative DC voltage with respect to ground.

If the capacitive transducer is displaced from center position, one capacitor will increase in capacitance while the other will decrease. This has little effect on the peak voltage charge of each capacitor, as there is negligible resistance in the charging current path from source to capacitor, resulting in a very short time constant (τ). However, when it comes time to discharge through the resistors, the capacitor with the greater capacitance value will hold its charge longer, resulting in a greater average DC voltage over time than the lesser-value capacitor.

The load resistor (R_{load}), connected at one end to the point between the two equal-value resistors (R) and at the other end to ground, will drop no DC voltage if the two capacitors' DC voltage charges are equal in magnitude. If, on the other hand, one capacitor maintains a greater DC voltage charge than the other due to a difference in capacitance, the load resistor will drop a voltage proportional to the difference between these voltages. Thus, differential capacitance is translated into a DC voltage across the load resistor.

Across the load resistor, there is both AC and DC voltage present, with only the DC voltage being significant to the difference in capacitance. If desired, a low-pass filter may be added to the output of this circuit to block the AC, leaving only a DC signal to be interpreted by measurement circuitry :



As a measurement circuit for differential capacitive sensors, the twin-T configuration enjoys many advantages over the standard bridge configuration. First and foremost, transducer displacement is indicated by a simple DC voltage, not an AC voltage whose magnitude *and* phase must be interpreted to tell which capacitance is greater. Furthermore, given the proper component values and power supply output, this DC output signal may be strong enough to directly drive an electromechanical meter movement, eliminating the need for an amplifier circuit. Another important advantage is that all important circuit elements have one terminal directly connected to ground : the source, the load resistor, and both capacitors are all ground-referenced. This helps minimize the ill effects of stray capacitance commonly plaguing bridge measurement circuits, likewise eliminating the need for compensatory measures such as the Wagner earth.

This circuit is also easy to specify parts for. Normally, a measurement circuit incorporating complementary diodes requires the selection of "matched" diodes for good accuracy. Not so with this circuit! So long as the power supply voltage is significantly greater than the deviation in voltage drop between the two diodes, the effects of mismatch are minimal and contribute little to measurement error. Furthermore, supply frequency variations have a relatively low impact on gain (how much output voltage is developed for a given amount of transducer displacement), and square-wave supply voltage works as well as sine-wave, assuming a 50% duty cycle (equal positive and negative half-cycles), of course.

Personal experience with using this circuit has confirmed its impressive performance. Not only is it easy to prototype and test, but its relative insensitivity to stray capacitance and its high output voltage as compared to traditional bridge circuits makes it a very robust alternative.

12.7 Contributors

Les contributeurs de ce chapitre sont listés dans l'ordre chronologique de leurs contributions, depuis le plus récent jusqu'au premier. Voyez l'Annexe 2 (Liste des contributeur) pour les dates et les informations de contact.

Jason Starck (June 2000) : HTML document formatting, which led to a much better-looking second edition.

Chapter 13

AC MOTORS

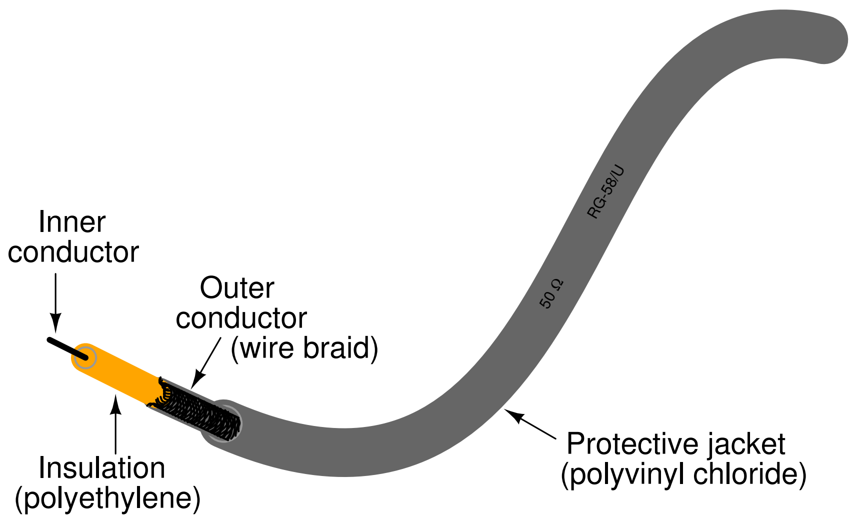
*** PENDING ***

Chapter 14

TRANSMISSION LINES

14.1 A 50-ohm cable?

Early in my explorations of electricity, I came across a length of *coaxial cable* with the label "50 ohms" printed along its outer sheath. Now, coaxial cable is a two-conductor cable made of a single conductor surrounded by a braided wire jacket, with a plastic insulating material separating the two. As such, the outer (braided) conductor completely surrounds the inner (single wire) conductor, the two conductors insulated from each other for the entire length of the cable. This type of cabling is often used to conduct weak (low-amplitude) voltage signals, due to its excellent ability to shield such signals from external interference.



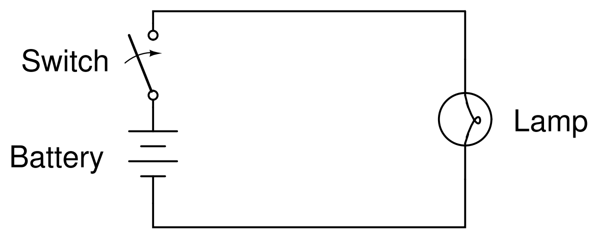
I was mystified by the "50 ohms" label on this coaxial cable. How could two conductors, insulated from each other by a relatively thick layer of plastic, have 50 ohms of resistance between them? Measuring resistance between the outer and inner conductors with my ohmmeter, I found it to be infinite (open-circuit), just as I would have expected from two insulated conductors. Measuring each of the two conductors' resistances from one end of the cable to the other indicated nearly zero ohms of resistance: again, exactly what I would have expected from continuous, unbroken lengths of wire. Nowhere was I able to measure 50 Ω of resistance on this cable, regardless of which points I connected my ohmmeter between.

What I didn't understand at the time was the cable's response to short-duration voltage "pulses" and high-frequency AC signals. Continuous direct current (DC) – such as that used by my ohmmeter to check the cable's resistance – shows the two conductors to be completely insulated from each other, with nearly infinite resistance between the two. However, due to the effects of capacitance and inductance distributed along the length of the cable, the cable's response to rapidly-changing voltages is such that it acts as a *finite* impedance, drawing current proportional to an applied voltage. What we would normally dismiss as being just a pair of wires becomes an important circuit element in the presence of transient and high-frequency AC signals, with characteristic properties all its own. When expressing such properties, we refer to the wire pair as a *transmission line.*

This chapter explores transmission line behavior. Many transmission line effects do not appear in significant measure in AC circuits of powerline frequency (50 or 60 Hz), or in continuous DC circuits, and so we haven't had to concern ourselves with them in our study of electric circuits thus far. However, in circuits involving high frequencies and/or extremely long cable lengths, the effects are very significant. Practical applications of transmission line effects abound in radio-frequency ("RF") communication circuitry, including computer networks, and in low-frequency circuits subject to voltage transients ("surges") such as lightning strikes on power lines.

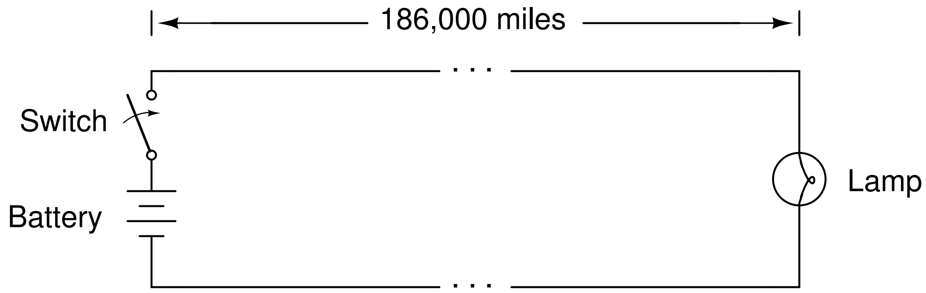
14.2 Circuits and the speed of light

Suppose we had a simple one-battery, one-lamp circuit controlled by a switch. When the switch is closed, the lamp immediately lights. When the switch is opened, the lamp immediately darkens:



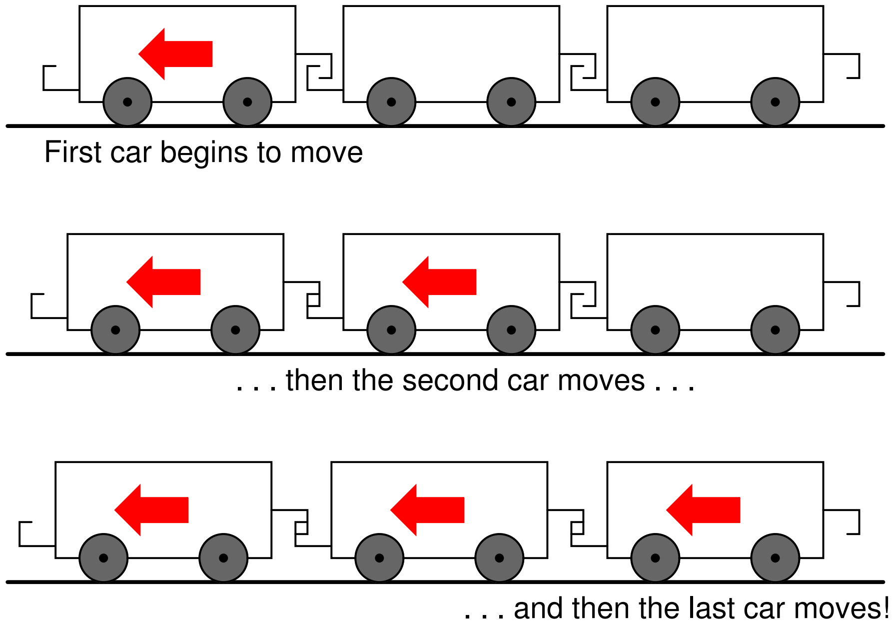
Actually, an incandescent lamp takes a short time for its filament to warm up and emit light after receiving an electric current of sufficient magnitude to power it, so the effect is not instant. However, what I'd like to focus on is the immediacy of the electric current itself, not the response time of the lamp filament. For all practical purposes, the effect of switch action is instant at the lamp's location. Although electrons move through wires very slowly, the overall effect of electrons pushing against each other happens at the speed of light (approximately 186,000 miles per *second*!).

What would happen, though, if the wires carrying power to the lamp were 186,000 miles long? Since we know the effects of electricity do have a finite speed (albeit very fast), a set of very long wires should introduce a time delay into the circuit, delaying the switch's action on the lamp:

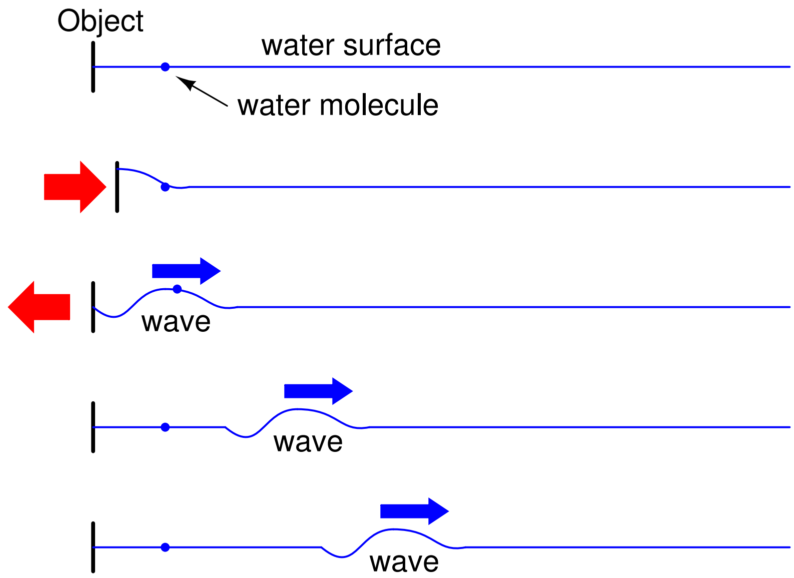


Assuming no warm-up time for the lamp filament, and no resistance along the 372,000 mile length of both wires, the lamp would light up approximately one second after the switch closure. Although the construction and operation of superconducting wires 372,000 miles in length would pose enormous practical problems, it is theoretically possible, and so this "thought experiment" is valid. When the switch is opened again, the lamp will continue to receive power for one second of time after the switch opens, then it will de-energize.

One way of envisioning this is to imagine the electrons within a conductor as rail cars in a train: linked together with a small amount of "slack" or "play" in the couplings. When one rail car (electron) begins to move, it pushes on the one ahead of it and pulls on the one behind it, but not before the slack is relieved from the couplings. Thus, motion is transferred from car to car (from electron to electron) at a maximum velocity limited by the coupling slack, resulting in a much faster transfer of motion from the left end of the train (circuit) to the right end than the actual speed of the cars (electrons):



Another analogy, perhaps more fitting for the subject of transmission lines, is that of waves in water. Suppose a flat, wall-shaped object is suddenly moved horizontally along the surface of water, so as to produce a wave ahead of it. The wave will travel as water molecules bump into each other, transferring wave motion along the water's surface far faster than the water molecules themselves are actually traveling:

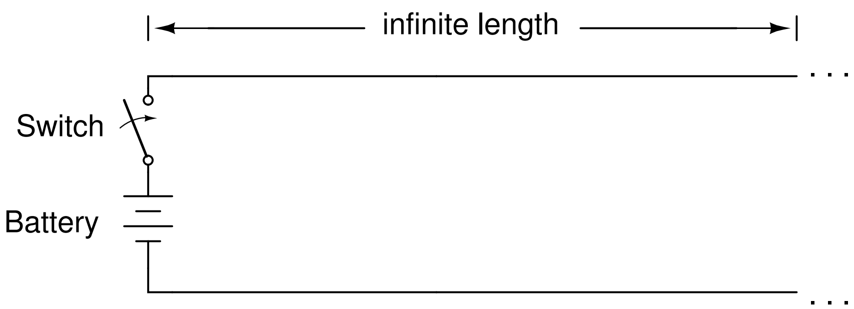


Likewise, electron motion "coupling" travels approximately at the speed of light, although the electrons themselves don't move that quickly. In a very long circuit, this "coupling" speed would become noticeable to a human observer in the form of a short time delay between switch action and lamp action.

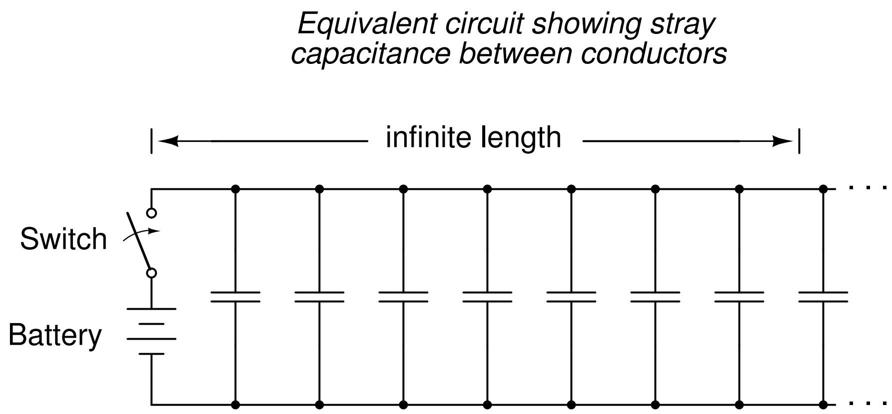
- **REVIEW:**
- In an electric circuit, the effects of electron motion travel approximately at the speed of light, although electrons within the conductors do not travel anywhere near that velocity.

14.3 Characteristic impedance

Suppose, though, that we had a set of parallel wires of *infinite* length, with no lamp at the end. What would happen when we close the switch? Being that there is no longer a load at the end of the wires, this circuit is open. Would there be no current at all?

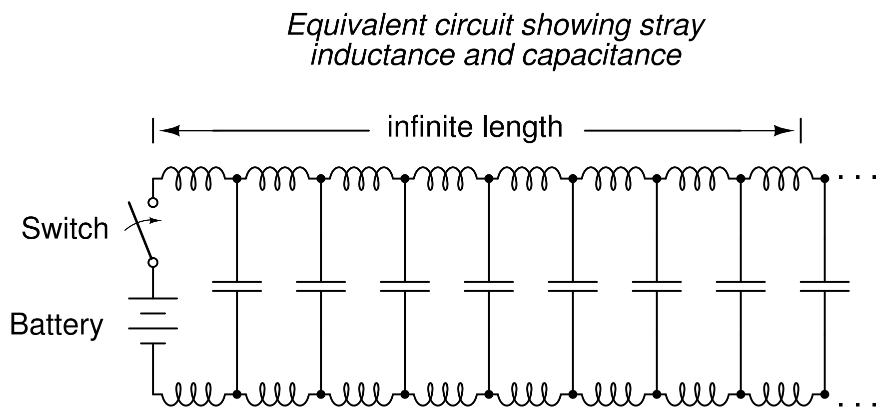


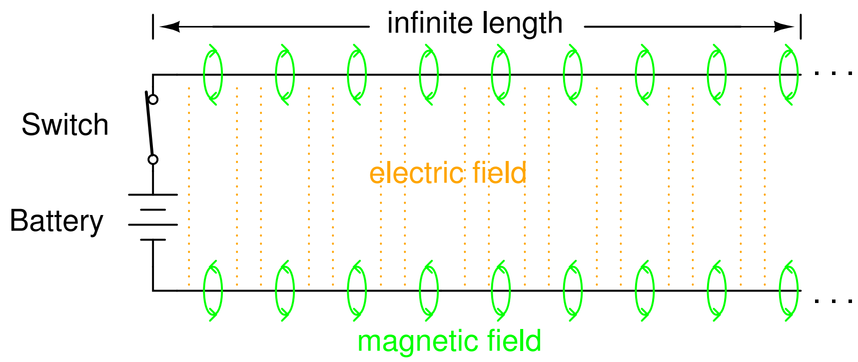
Despite being able to avoid wire resistance through the use of superconductors in this "thought experiment," we cannot eliminate capacitance along the wires' lengths. *Any* pair of conductors separated by an insulating medium creates capacitance between those conductors:



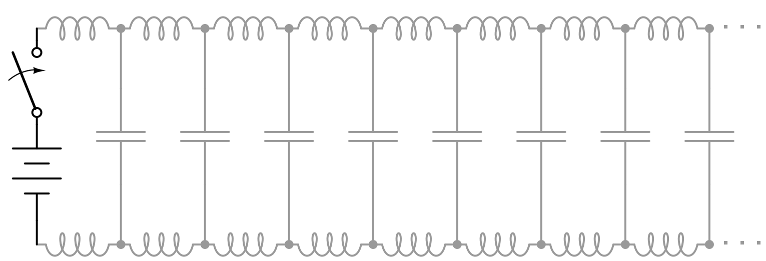
Voltage applied between two conductors creates an electric field between those conductors. Energy is stored in this electric field, and this storage of energy results in an opposition to change in voltage. The reaction of a capacitance against changes in voltage is described by the equation i = C(de/dt), which tells us that current will be drawn proportional to the voltage's rate of change over time. Thus, when the switch is closed, the capacitance between conductors will react against the sudden voltage increase by charging up and drawing current from the source. According to the equation, an instant rise in applied voltage (as produced by perfect switch closure) gives rise to an infinite charging current.

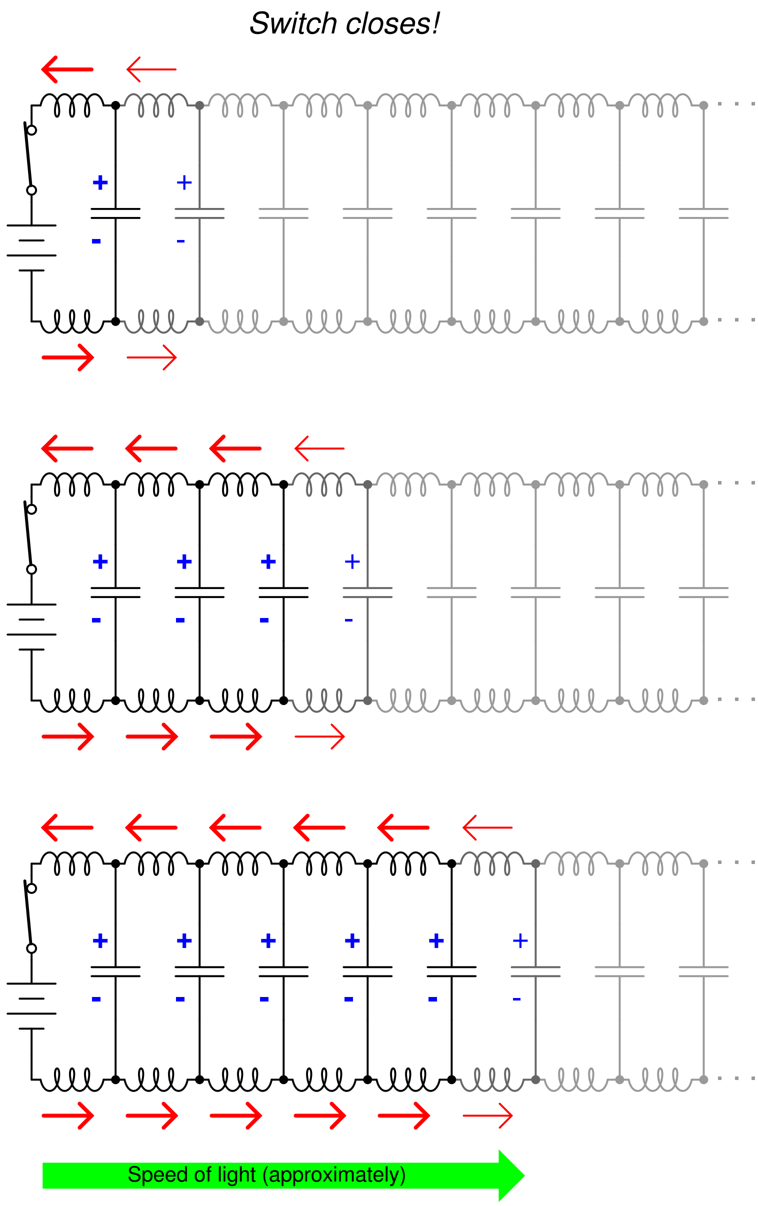
However, the current drawn by a pair of parallel wires will not be infinite, because there exists series impedance along the wires due to inductance. Remember that current through *any* conductor develops a magnetic field of proportional magnitude. Energy is stored in this magnetic field, and this storage of energy results in an opposition to change in current. Each wire develops a magnetic field as it carries charging current for the capacitance between the wires, and in so doing drops voltage according to the inductance equation e = L(di/dt). This voltage drop limits the voltage rate-of-change across the distributed capacitance, preventing the current from ever reaching an infinite magnitude:





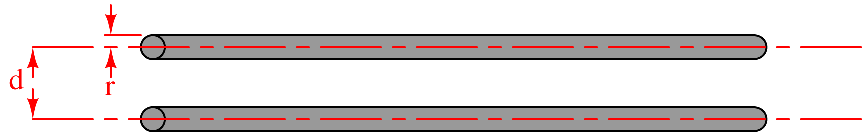
Because the electrons in the two wires transfer motion to and from each other at nearly the speed of light, the "wave front" of voltage and current change will propagate down the length of the wires at that same velocity, resulting in the distributed capacitance and inductance progressively charging to full voltage and current, respectively, like this:





The end result of these interactions is a constant current of limited magnitude through the battery source. Since the wires are infinitely long, their distributed capacitance will never fully charge to the source voltage, and their distributed inductance will never allow unlimited charging current. In other words, this pair of wires will draw current from the source so long as the switch is closed, behaving as a constant load. No longer are the wires merely conductors of electrical current and carriers of voltage, but now constitute a circuit component in themselves, with unique characteristics. No longer are the two wires merely *a pair of conductors*, but rather a *transmission line*.

As a constant load, the transmission line's response to applied voltage is resistive rather than reactive, despite being comprised purely of inductance and capacitance (assuming superconducting wires with zero resistance). We can say this because there is no difference from the battery's perspective between a resistor eternally dissipating energy and an infinite transmission line eternally absorbing energy. The impedance (resistance) of this line in ohms is called the *characteristic impedance*, and it is fixed by the geometry of the two conductors. For a parallel-wire line with air insulation, the characteristic impedance may be calculated as such:



$$Z_0 = \frac{276}{\sqrt{k}} \log \frac{d}{r}$$

Where,

Z_0 = Characteristic impedance of line

d = Distance between conductor centers

r = Conductor radius

k = Relative permittivity of insulation between conductors

If the transmission line is coaxial in construction, the characteristic impedance follows a different equation:

$$Z_0 = \frac{138}{\sqrt{k}} \log \frac{d_1}{d_2}$$

Where,

Z_0 = Characteristic impedance of line

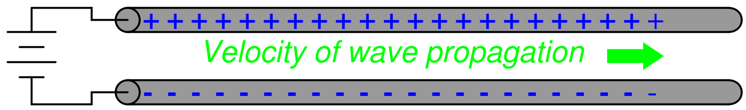
d_1 = Inside diameter of outer conductor

d_2 = Outside diameter of inner conductor

k = Relative permittivity of insulation between conductors

In both equations, identical units of measurement must be used in both terms of the fraction. If the insulating material is other than air (or a vacuum), both the characteristic impedance and the propagation velocity will be affected. The ratio of a transmission line's true propagation velocity and the speed of light in a vacuum is called the *velocity factor* of that line.

Velocity factor is purely a factor of the insulating material's relative permittivity (otherwise known as its *dielectric constant*), defined as the ratio of a material's electric field permittivity to that of a pure vacuum. The velocity factor of any cable type – coaxial or otherwise – may be calculated quite simply by the following formula:



$$\boxed{\text{Velocity factor} = \frac{v}{c} = \frac{1}{\sqrt{k}}}$$

Where,

v = Velocity of wave propagation

c = Velocity of light in a vacuum

k = Relative permittivity of insulation between conductors

Characteristic impedance is also known as *natural impedance*, and it refers to the equivalent resistance of a transmission line if it were infinitely long, owing to distributed capacitance and inductance as the voltage and current "waves" propagate along its length at a propagation velocity equal to some large fraction of light speed.

It can be seen in either of the first two equations that a transmission line's characteristic impedance (Z_0) increases as the conductor spacing increases. If the conductors are moved away from each other, the distributed capacitance will decrease (greater spacing between capacitor "plates"), and the distributed inductance will increase (less cancellation of the two opposing magnetic fields). Less parallel capacitance and more series inductance results in a smaller current drawn by the line for any given amount of applied voltage, which by definition is a greater impedance. Conversely, bringing the two conductors closer together increases the parallel capacitance and decreases the series inductance. Both changes result in a larger current drawn for a given applied voltage, equating to a lesser impedance.

Barring any dissipative effects such as dielectric "leakage" and conductor resistance, the characteristic impedance of a transmission line is equal to the square root of the ratio of the line's inductance per unit length divided by the line's capacitance per unit length:

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

Where,

Z_0 = Characteristic impedance of line

L = Inductance per unit length of line

C = Capacitance per unit length of line

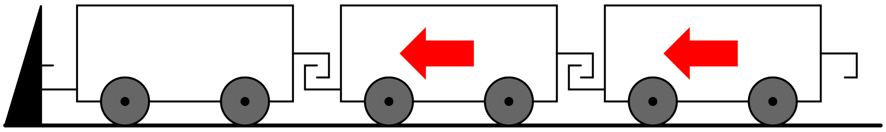
- **REVIEW:**
- A *transmission line* is a pair of parallel conductors exhibiting certain characteristics due to distributed capacitance and inductance along its length.
- When a voltage is suddenly applied to one end of a transmission line, both a voltage "wave" and a current "wave" propagate along the line at nearly light speed.
- If a DC voltage is applied to one end of an infinitely long transmission line, the line will draw current from the DC source as though it were a constant resistance.
- The *characteristic impedance* (Z_0) of a transmission line is the resistance it would exhibit if it were infinite in length. This is entirely different from leakage resistance of the dielectric separating the two conductors, and the metallic resistance of the wires themselves. Characteristic impedance is purely a function of the capacitance and inductance distributed along the line's length, and would exist even if the dielectric were perfect (infinite parallel resistance) and the wires superconducting (zero series resistance).
- *Velocity factor* is a fractional value relating a transmission line's propagation speed to the speed of light in a vacuum. Values range between 0.66 and 0.80 for typical two-wire lines and coaxial cables. For any cable type, it is equal to the reciprocal (1/x) of the square root of the relative permittivity of the cable's insulation.

14.4 Finite-length transmission lines

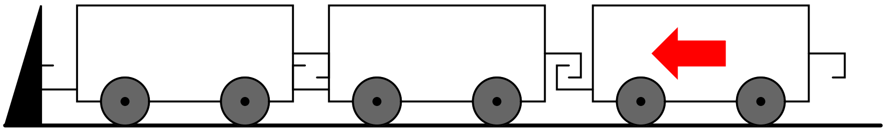
A transmission line of infinite length is an interesting abstraction, but physically impossible. All transmission lines have some finite length, and as such do not behave precisely the same as an infinite line. If that piece of 50 Ω "RG-58/U" cable I measured with an ohmmeter years ago had been infinitely long, I actually would have been able to measure 50 Ω worth of resistance between the inner and outer conductors. But it was not infinite in length, and so it measured as "open" (infinite resistance).

Nonetheless, the characteristic impedance rating of a transmission line is important even when dealing with limited lengths. An older term for characteristic impedance, which I like for its descriptive value, is *surge impedance*. If a transient voltage (a "surge") is applied to the end of a transmission line, the line will draw a current proportional to the surge voltage magnitude divided by the line's surge impedance (I=E/Z). This simple, Ohm's Law relationship between current and voltage will hold true for a limited period of time, but not indefinitely.

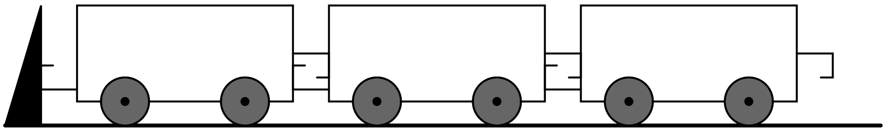
If the end of a transmission line is open-circuited – that is, left unconnected – the current "wave" propagating down the line's length will have to stop at the end, since electrons cannot flow where there is no continuing path. This abrupt cessation of current at the line's end causes a "pile-up" to occur along the length of the transmission line, as the electrons successively find no place to go. Imagine a train traveling down the track with slack between the rail car couplings: if the lead car suddenly crashes into an immovable barricade, it will come to a stop, causing the one behind it to come to a stop as soon as the first coupling slack is taken up, which causes the next rail car to stop as soon as the next coupling's slack is taken up, and so on until the last rail car stops. The train does not come to a halt together, but rather in sequence from first car to last:



First car stops



. . . then the second car stops . . .



. . . and then the last car stops!

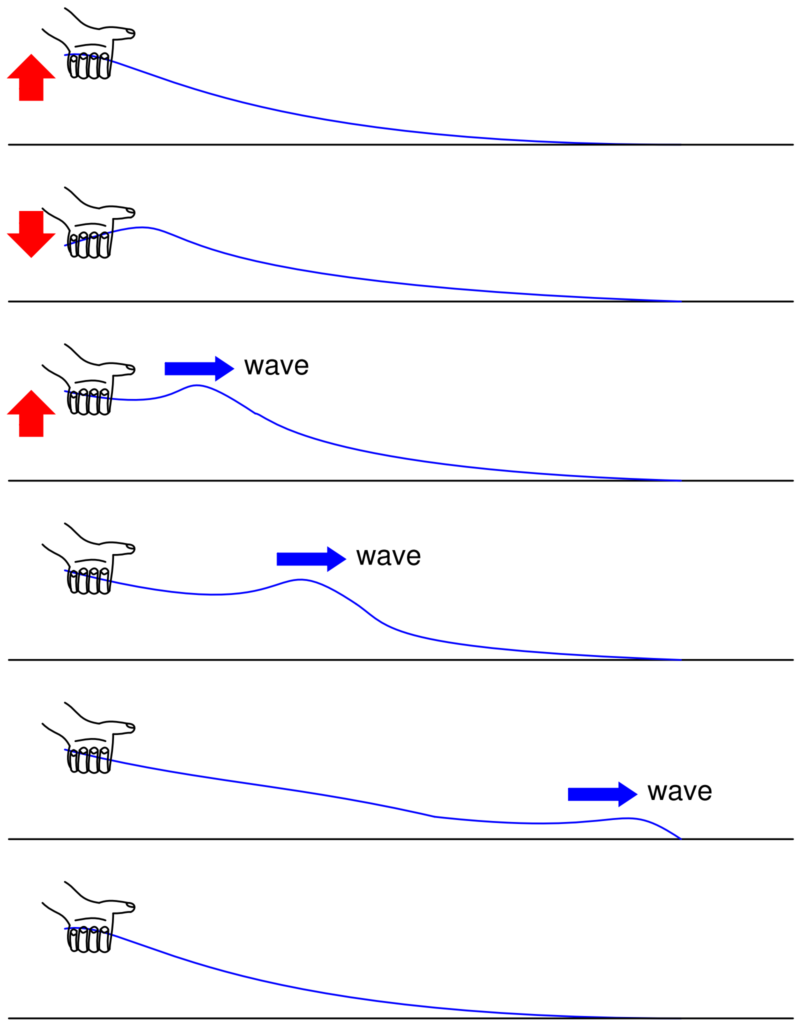
A signal propagating from the source-end of a transmission line to the load-end is called an *incident wave*. The propagation of a signal from load-end to source-end (such as what happened in this example with current encountering the end of an open-circuited transmission line) is called a *reflected wave*.

When this electron "pile-up" propagates back to the battery, current at the battery ceases, and the line acts as a simple open circuit. All this happens very quickly for transmission lines of reasonable length, and so an ohmmeter measurement of the line never reveals the brief time period where the line actually behaves as a resistor. For a mile-long cable with a velocity factor of 0.66 (signal propagation velocity is 66% of light speed, or 122,760 miles per second), it takes only 1/122,760 of a second (8.146 microseconds) for a signal to travel from one end to the other. For the current signal to reach the line's end and "reflect" back to the source, the round-trip time is twice this figure, or 16.292 μs.

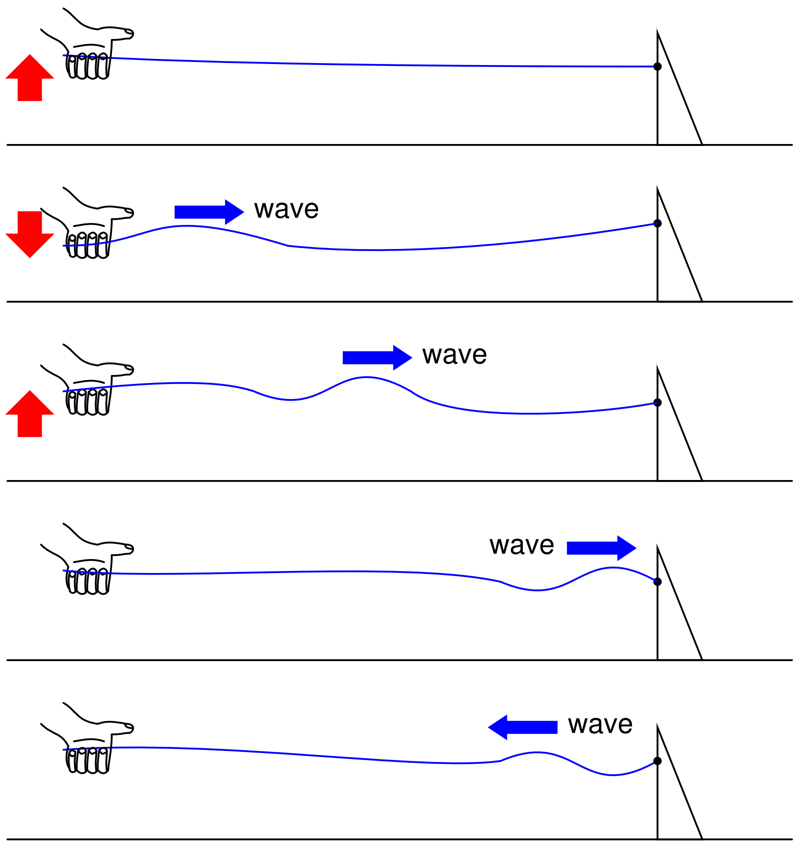
High-speed measurement instruments are able to detect this transit time from source to line-end and back to source again, and may be used for the purpose of determining a cable's length. This technique may also be used for determining the presence *and* location of a break in one or both of the cable's conductors, since a current will "reflect" off the wire break just as it will off the end of an open-circuited cable. Instruments designed for such purposes are called *time-domain reflectometers* (TDRs). The basic principle is identical to that of sonar range-finding: generating a sound pulse and measuring the time it takes for the echo to return.

A similar phenomenon takes place if the end of a transmission line is short-circuited: when the voltage wave-front reaches the end of the line, it is reflected back to the source, because voltage cannot exist between two electrically common points. When this reflected wave reaches the source, the source sees the entire transmission line as a short-circuit. Again, this happens as quickly as the signal can propagate round-trip down and up the transmission line at whatever velocity allowed by the dielectric material between the line's conductors.

A simple experiment illustrates the phenomenon of wave reflection in transmission lines. Take a length of rope by one end and "whip" it with a rapid up-and-down motion of the wrist. A wave may be seen traveling down the rope's length until it dissipates entirely due to friction:



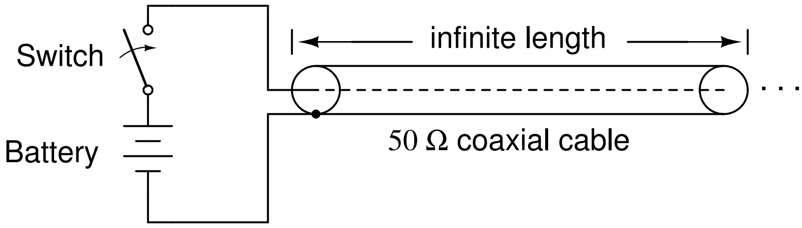
This is analogous to a long transmission line with internal loss: the signal steadily grows weaker as it propagates down the line's length, never reflecting back to the source. However, if the far end of the rope is secured to a solid object at a point prior to the incident wave's total dissipation, a second wave will be reflected back to your hand:



Usually, the purpose of a transmission line is to convey electrical energy from one point to another. Even if the signals are intended for information only, and not to power some significant load device, the ideal situation would be for all of the original signal energy to travel from the source to the load, and then be completely absorbed or dissipated by the load for maximum signal-to-noise ratio. Thus, "loss" along the length of a transmission line is undesirable, as are reflected waves, since reflected energy is energy not delivered to the end device.

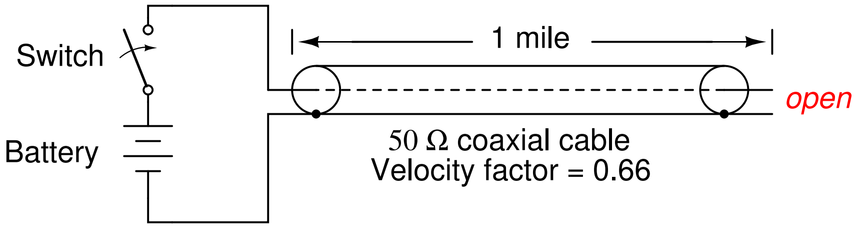
Reflections may be eliminated from the transmission line if the load's impedance exactly equals the characteristic ("surge") impedance of the line. For example, a 50 Ω coaxial cable that is either open-circuited or short-circuited will reflect all of the incident energy back to the source. However, if a 50 Ω resistor is connected at the end of the cable, there will be no reflected energy, all signal energy being dissipated by the resistor.

This makes perfect sense if we return to our hypothetical, infinite-length transmission line example. A transmission line of 50 Ω characteristic impedance and infinite length behaves exactly like a 50 Ω resistance as measured from one end. If we cut this line to some finite length, it will behave as a 50 Ω resistor to a constant source of DC voltage for a brief time, but then behave like an open- or a short-circuit, depending on what condition we leave the cut end of the line: open or shorted. However, if we *terminate* the line with a 50 Ω resistor, the line will once again behave as a 50 Ω resistor, indefinitely: the same as if it were of infinite length again:



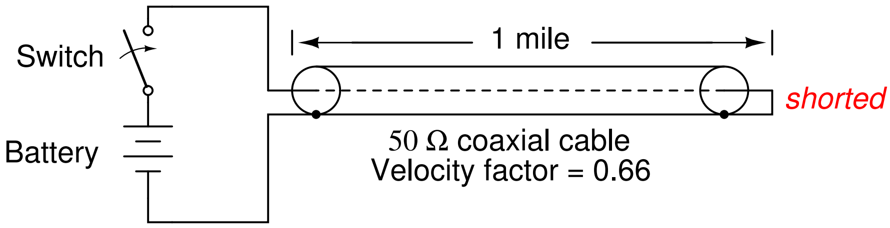
Cable's behavior from perspective of battery:

Exactly like a 50 Ω resistor



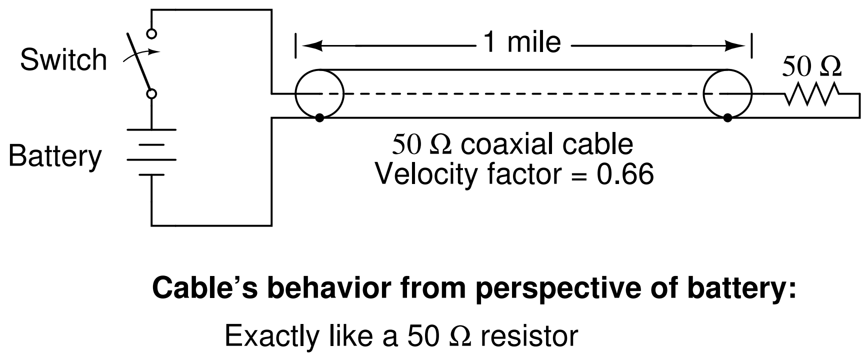
Cable's behavior from perspective of battery:

Like a 50 Ω resistor for 16.292 µs,
then like an open (infinite resistance)



Cable's behavior from perspective of battery:

Like a 50 Ω resistor for 16.292 µs,
then like a short (zero resistance)



In essence, a terminating resistor matching the natural impedance of the transmission line makes the line "appear" infinitely long from the perspective of the source, because a resistor has the ability to eternally dissipate energy in the same way a transmission line of infinite length is able to eternally absorb energy.

Reflected waves will also manifest if the terminating resistance isn't precisely equal to the characteristic impedance of the transmission line, not just if the line is left unconnected (open) or jumpered (shorted). Though the energy reflection will not be total with a terminating impedance of slight mismatch, it will be partial. This happens whether or not the terminating resistance is *greater* or *less* than the line's characteristic impedance.

Re-reflections of a reflected wave may also occur at the *source end* of a transmission line, if the source's internal impedance (Thevenin equivalent impedance) is not exactly equal to the line's characteristic impedance. A reflected wave returning back to the source will be dissipated entirely if the source impedance matches the line's, but will be reflected back toward the line end like another incident wave, at least partially, if the source impedance does not match the line. This type of reflection may be particularly troublesome, as it makes it appear that the source has transmitted another pulse.

- **REVIEW:**
- Characteristic impedance is also known as *surge impedance*, due to the temporarily resistive behavior of any length transmission line.
- A finite-length transmission line will appear to a DC voltage source as a constant resistance for some short time, then as whatever impedance the line is terminated with. Therefore, an open-ended cable simply reads "open" when measured with an ohmmeter, and "shorted" when its end is short-circuited.
- A transient ("surge") signal applied to one end of an open-ended or short-circuited transmission line will "reflect" off the far end of the line as a secondary wave. A signal traveling on a transmission line from source to load is called an *incident wave*; a signal "bounced" off the end of a transmission line, traveling from load to source, is called a *reflected wave.*
- Reflected waves will also appear in transmission lines terminated by resistors not precisely matching the characteristic impedance.

- A finite-length transmission line may be made to appear infinite in length if terminated by a resistor of equal value to the line's characteristic impedance. This eliminates all signal reflections.

- A reflected wave may become re-reflected off the source-end of a transmission line if the source's internal impedance does not match the line's characteristic impedance. This re-reflected wave will appear, of course, like another pulse signal transmitted from the source.

14.5 "Long" and "short" transmission lines

In DC and low-frequency AC circuits, the characteristic impedance of parallel wires is usually ignored. This includes the use of coaxial cables in instrument circuits, often employed to protect weak voltage signals from being corrupted by induced "noise" caused by stray electric and magnetic fields. This is due to the relatively short timespans in which reflections take place in the line, as compared to the period of the waveforms or pulses of the significant signals in the circuit. As we saw in the last section, if a transmission line is connected to a DC voltage source, it will behave as a resistor equal in value to the line's characteristic impedance only for as long as it takes the incident pulse to reach the end of the line and return as a reflected pulse, back to the source. After that time (a brief 16.292 μs for the mile-long coaxial cable of the last example), the source "sees" only the terminating impedance, whatever that may be.

If the circuit in question handles low-frequency AC power, such short time delays introduced by a transmission line between when the AC source outputs a voltage peak and when the source "sees" that peak loaded by the terminating impedance (round-trip time for the incident wave to reach the line's end and reflect back to the source) are of little consequence. Even though we know that signal magnitudes along the line's length are not equal at any given time due to signal propagation at (nearly) the speed of light, the actual phase difference between start-of-line and end-of-line signals is negligible, because line-length propagations occur within a very small fraction of the AC waveform's period. For all practical purposes, we can say that voltage along all respective points on a low-frequency, two-conductor line are equal and in-phase with each other at any given point in time.

In these cases, we can say that the transmission lines in question are *electrically short*, because their propagation effects are much quicker than the periods of the conducted signals. By contrast, an *electrically long* line is one where the propagation time is a large fraction or even a multiple of the signal period. A "long" line is generally considered to be one where the source's signal waveform completes at least a quarter-cycle (90^o of "rotation") before the incident signal reaches line's end. Up until this chapter in the *Lessons In Electric Circuits* book series, all connecting lines were assumed to be electrically short.

To put this into perspective, we need to express the distance traveled by a voltage or current signal along a transmission line in relation to its source frequency. An AC waveform with a frequency of 60 Hz completes one cycle in 16.66 ms. At light speed (186,000 m/s), this equates to a distance of 3100 miles that a voltage or current signal will propagate in that time. If the velocity factor of the transmission line is less than 1, the propagation velocity will be less than 186,000 miles per second, and the distance less by the same factor. But even if we used the coaxial cable's velocity factor from the last example (0.66), the distance is still a very long 2046 miles! Whatever distance we calculate for a given frequency is called the *wavelength* of the signal.

A simple formula for calculating wavelength is as follows:

$$\lambda = \frac{v}{f}$$

Where,

λ = Wavelength

v = Velocity of propagation

f = Frequency of signal

The lower-case Greek letter "lambda" (λ) represents wavelength, in whatever unit of length used in the velocity figure (if miles per second, then wavelength in miles; if meters per second, then wavelength in meters). Velocity of propagation is usually the speed of light when calculating signal wavelength in open air or in a vacuum, but will be less if the transmission line has a velocity factor less than 1.

If a "long" line is considered to be one at least 1/4 wavelength in length, you can see why all connecting lines in the circuits discussed thusfar have been assumed "short." For a 60 Hz AC power system, power lines would have to exceed 775 miles in length before the effects of propagation time became significant. Cables connecting an audio amplifier to speakers would have to be over 4.65 miles in length before line reflections would significantly impact a 10 kHz audio signal!

When dealing with radio-frequency systems, though, transmission line length is far from trivial. Consider a 100 MHz radio signal: its wavelength is a mere 9.8202 feet, even at the full propagation velocity of light (186,000 m/s). A transmission line carrying this signal would not have to be more than about 2-1/2 feet in length to be considered "long!" With a cable velocity factor of 0.66, this critical length shrinks to 1.62 feet.

When an electrical source is connected to a load via a "short" transmission line, the load's impedance dominates the circuit. This is to say, when the line is short, its own characteristic impedance is of little consequence to the circuit's behavior. We see this when testing a coaxial cable with an ohmmeter: the cable reads "open" from center conductor to outer conductor if the cable end is left unterminated. Though the line acts as a resistor for a very brief period of time after the meter is connected (about 50 Ω for an RG-58/U cable), it immediately thereafter behaves as a simple "open circuit:" the impedance of the line's open end. Since the combined response time of an ohmmeter and the human being using it *greatly exceeds* the round-trip propagation time up and down the cable, it is "electrically short" for this application, and we only register the terminating (load) impedance. It is the extreme speed of the propagated signal that makes us unable to detect the cable's 50 Ω transient impedance with an ohmmeter.

If we use a coaxial cable to conduct a DC voltage or current to a load, and no component in the circuit is capable of measuring or responding quickly enough to "notice" a reflected wave, the cable is considered "electrically short" and its impedance is irrelevant to circuit function. Note how the electrical "shortness" of a cable is relative to the application: in a DC circuit where voltage and current values change slowly, nearly any physical length of cable would be considered "short" from the standpoint of characteristic impedance and reflected waves. Taking the same length of cable, though, and using it to conduct a high-frequency AC signal could result in a vastly different assessment of that cable's "shortness!"

When a source is connected to a load via a "long" transmission line, the line's own characteristic impedance dominates over load impedance in determining circuit behavior. In other words, an electrically "long" line acts as the principal component in the circuit, its own characteristics overshadowing the load's. With a source connected to one end of the cable and a load to the other, current drawn from the source is a function primarily of the line and not the load. This is increasingly true the longer the transmission line is. Consider our hypothetical 50 Ω cable of infinite length, surely the ultimate example of a "long" transmission line: no matter what kind of load we connect to one end of this line, the source (connected to the other end) will only see 50 Ω of impedance, because the line's infinite length prevents the signal from *ever reaching* the end where the load is connected. In this scenario, line impedance exclusively defines circuit behavior, rendering the load completely irrelevant.

The most effective way to minimize the impact of transmission line length on circuit behavior is to match the line's characteristic impedance to the load impedance. If the load impedance is equal to the line impedance, then *any* signal source connected to the other end of the line will "see" the exact same impedance, and will have the exact same amount of current drawn from it, regardless of line length. In this condition of perfect impedance matching, line length only affects the amount of time delay from signal departure at the source to signal arrival at the load. However, perfect matching of line and load impedances is not always practical or possible.

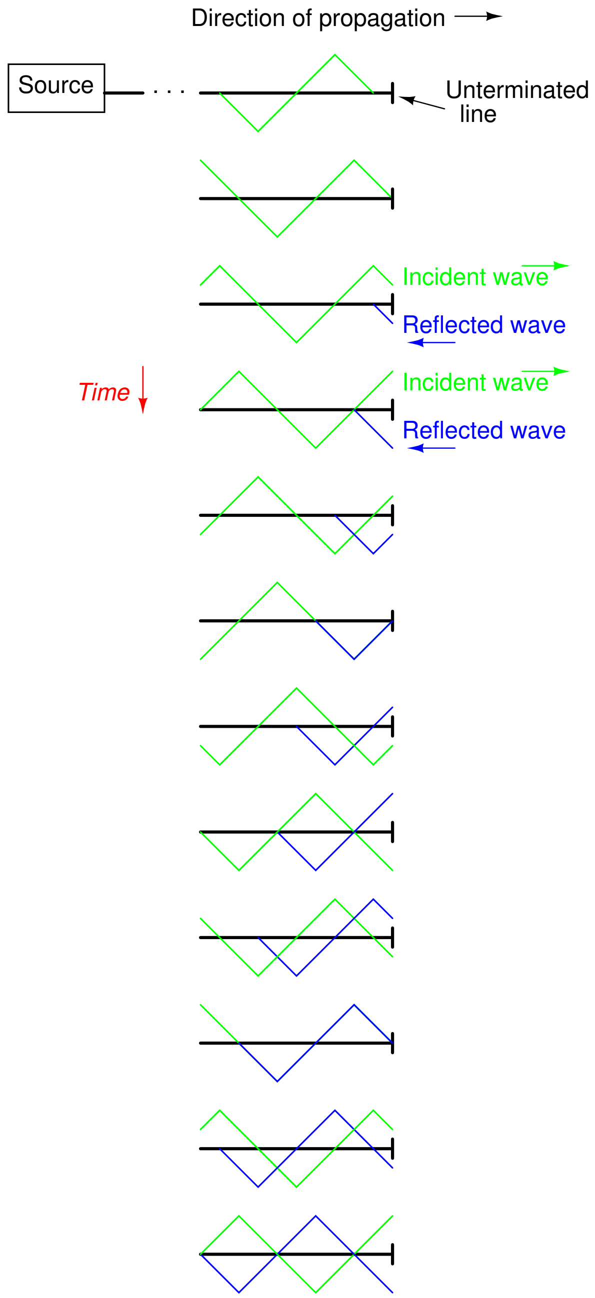
The next section discusses the effects of "long" transmission lines, especially when line length happens to match specific fractions or multiples of signal wavelength.

- **REVIEW:**
- Coaxial cabling is sometimes used in DC and low-frequency AC circuits as well as in high-frequency circuits, for the excellent immunity to induced "noise" that it provides for signals.
- When the period of a transmitted voltage or current signal greatly exceeds the propagation time for a transmission line, the line is considered *electrically short*. Conversely, when the propagation time is a large fraction or multiple of the signal's period, the line is considered *electrically long*.
- A signal's *wavelength* is the physical distance it will propagate in the timespan of one period. Wavelength is calculated by the formula λ=v/f, where "λ" is the wavelength, "v" is the propagation velocity, and "f" is the signal frequency.
- A rule-of-thumb for transmission line "shortness" is that the line must be at least 1/4 wavelength before it is considered "long."
- In a circuit with a "short" line, the terminating (load) impedance dominates circuit behavior. The source effectively sees nothing but the load's impedance, barring any resistive losses in the transmission line.
- In a circuit with a "long" line, the line's own characteristic impedance dominates circuit behavior. The ultimate example of this is a transmission line of infinite length: since the signal will *never* reach the load impedance, the source only "sees" the cable's characteristic impedance.
- When a transmission line is terminated by a load precisely matching its impedance, there are no reflected waves and thus no problems with line length.

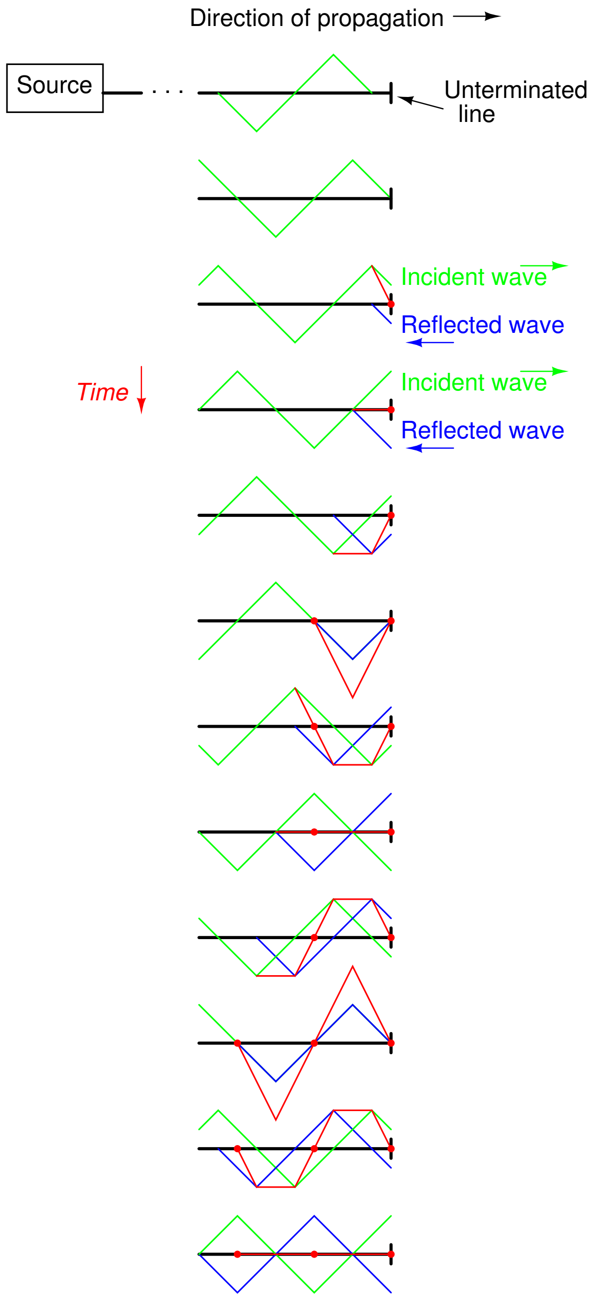
14.6 Standing waves and resonance

Whenever there is a mismatch of impedance between transmission line and load, reflections will occur. If the incident signal is a continuous AC waveform, these reflections will mix with more of the oncoming incident waveform to produce stationary waveforms called *standing waves.*

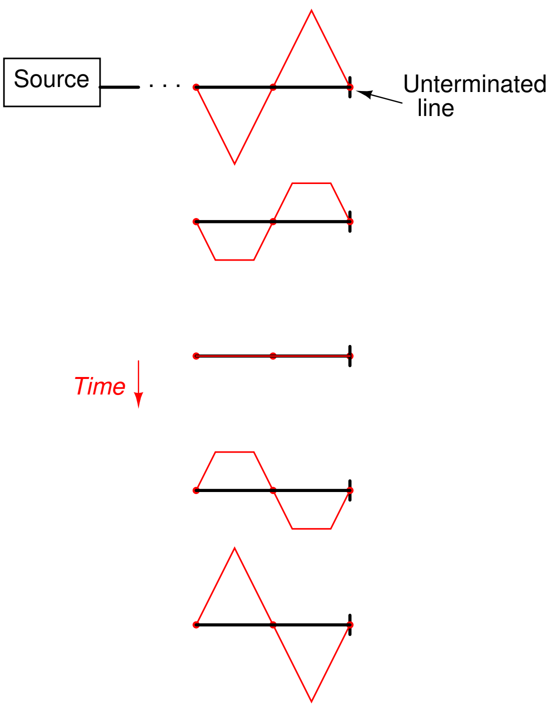
The following illustration shows how a triangle-shaped incident waveform turns into a mirror-image reflection upon reaching the line's unterminated end. The transmission line in this illustrative sequence is shown as a single, thick line rather than a pair of wires, for simplicity's sake. The incident wave is shown traveling from left to right, while the reflected wave travels from right to left:



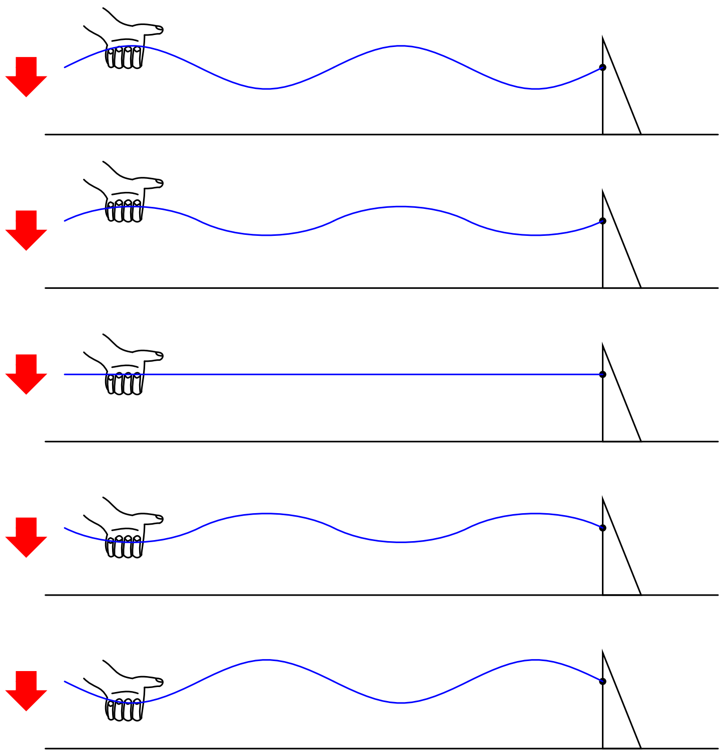
If we add the two waveforms together, we find that a third, stationary waveform is created along the line's length:



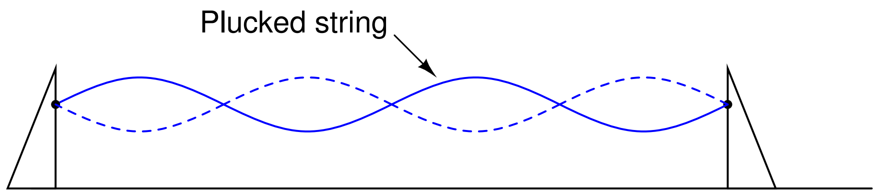
This third, "standing" wave, in fact, represents the only voltage along the line, being the representative sum of incident and reflected voltage waves. It oscillates in instantaneous magnitude, but does not propagate down the cable's length like the incident or reflected waveforms causing it. Note the dots along the line length marking the "zero" points of the standing wave (where the incident and reflected waves cancel each other), and how those points never change position:



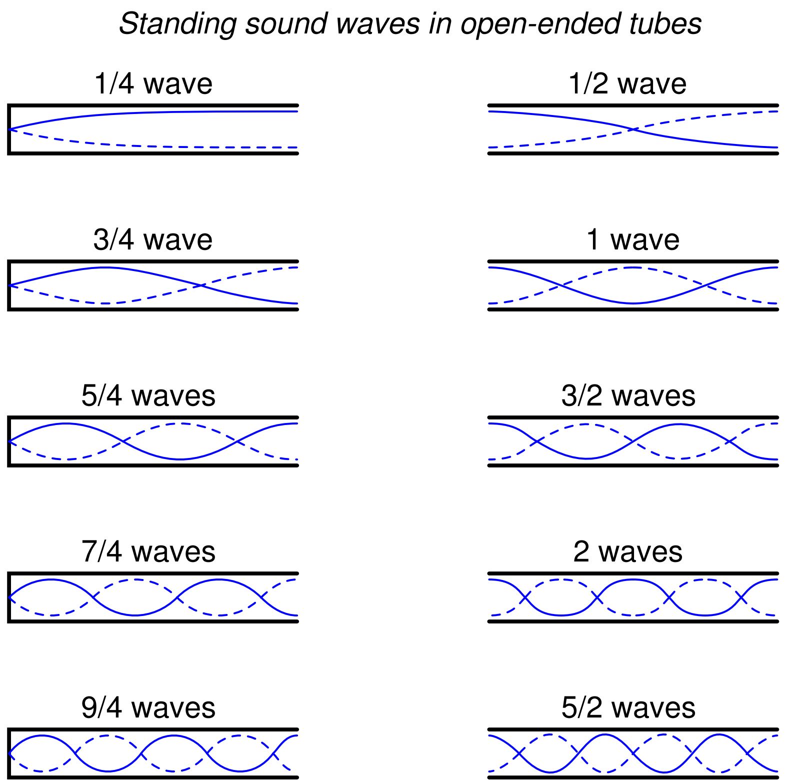
Standing waves are quite abundant in the physical world. Consider a string or rope, shaken at one end, and tied down at the other (only one half-cycle of hand motion shown, moving downward):



Both the nodes (points of little or no vibration) and the antinodes (points of maximum vibration) remain fixed along the length of the string or rope. The effect is most pronounced when the free end is shaken at just the right frequency. Plucked strings exhibit the same "standing wave" behavior, with "nodes" of maximum and minimum vibration along their length. The major difference between a plucked string and a shaken string is that the plucked string supplies its own "correct" frequency of vibration to maximize the standing-wave effect:



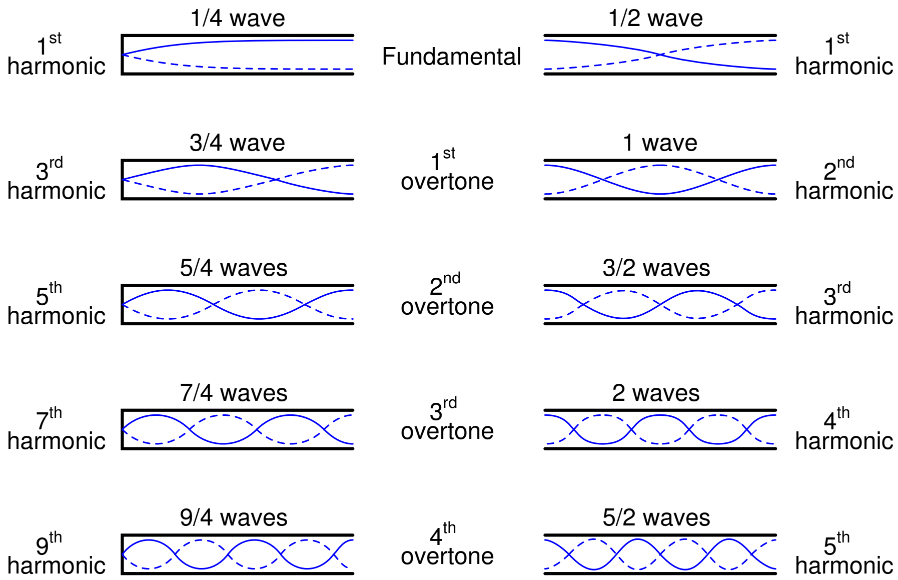
Wind blowing across an open-ended tube also produces standing waves; this time, the waves are vibrations of air molecules (sound) within the tube rather than vibrations of a solid object. Whether the standing wave terminates in a node (minimum amplitude) or an antinode (maximum amplitude) depends on whether the other end of the tube is open or closed:



A closed tube end must be a wave node, while an open tube end must be an antinode. By analogy, the anchored end of a vibrating string must be a node, while the free end (if there is any) must be an antinode.

Note how there is more than one wavelength suitable for producing standing waves of vibrating air within a tube that precisely match the tube's end points. This is true for all standing-wave systems: standing waves will resonate with the system for any frequency (wavelength) correlating to the node/antinode points of the system. Another way of saying this is that there are multiple resonant frequencies for any system supporting standing waves.

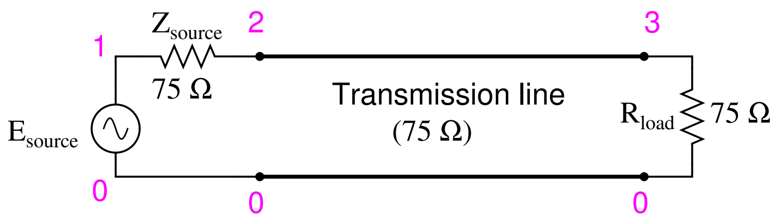
All higher frequencies are integer-multiples of the lowest (fundamental) frequency for the system. The sequential progression of harmonics from one resonant frequency to the next defines the *overtone* frequencies for the system:



The actual frequencies (measured in Hertz) for any of these harmonics or overtones depends on the physical length of the tube and the waves' propagation velocity, which is the speed of sound in air.

Because transmission lines support standing waves, and force these waves to possess nodes and antinodes according to the type of termination impedance at the load end, they also exhibit resonance at frequencies determined by physical length and propagation velocity. Transmission line resonance, though, is a bit more complex than resonance of strings or of air in tubes, because we must consider both voltage waves and current waves.

This complexity is made easier to understand by way of computer simulation. To begin, let's examine a perfectly matched source, transmission line, and load. All components have an impedance of 75 Ω:

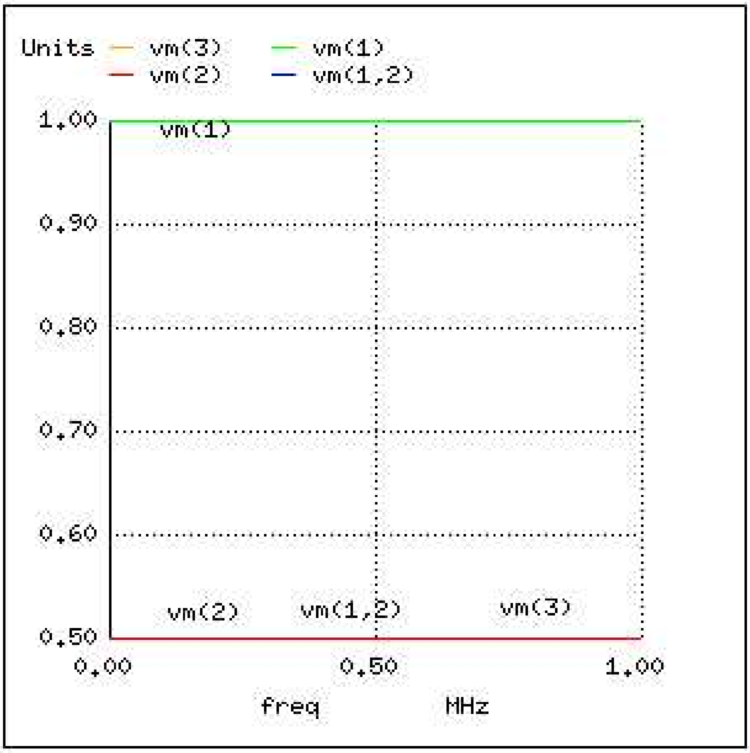


Using SPICE to simulate the circuit, we'll specify the transmission line (`t1`) with a 75 Ω characteristic impedance (`z0=75`) and a propagation delay of 1 microsecond (`td=1u`). This is a convenient method for expressing the physical length of a transmission line: the amount of time it takes a wave to propagate down its entire length. If this were a real 75 Ω cable – perhaps a type "RG-59B/U" coaxial cable, the type commonly used for cable television distribution – with a velocity factor of 0.66, it would be about 648 feet long. Since 1 μs is the period of a 1 MHz signal, I'll choose to sweep the frequency of the AC source from (nearly) zero to that figure, to see how the system reacts when exposed to signals ranging from DC to 1 wavelength.

Here is the SPICE netlist for the circuit shown above:

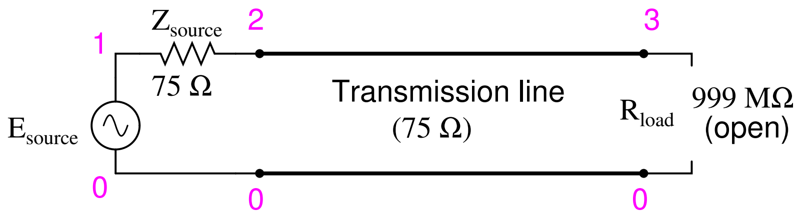
```
Transmission line
v1 1 0 ac 1 sin
rsource 1 2 75
t1 2 0 3 0 z0=75 td=1u
rload 3 0 75
.ac lin 101 1m 1meg
* Using "Nutmeg" program to plot analysis
.end
```

Running this simulation and plotting the source impedance drop (as an indication of current), the source voltage, the line's source-end voltage, and the load voltage, we see that the source voltage – shown as `vm(1)` (voltage magnitude between node 1 and the implied ground point of node 0) on the graphic plot – registers a steady 1 volt, while every other voltage registers a steady 0.5 volts:

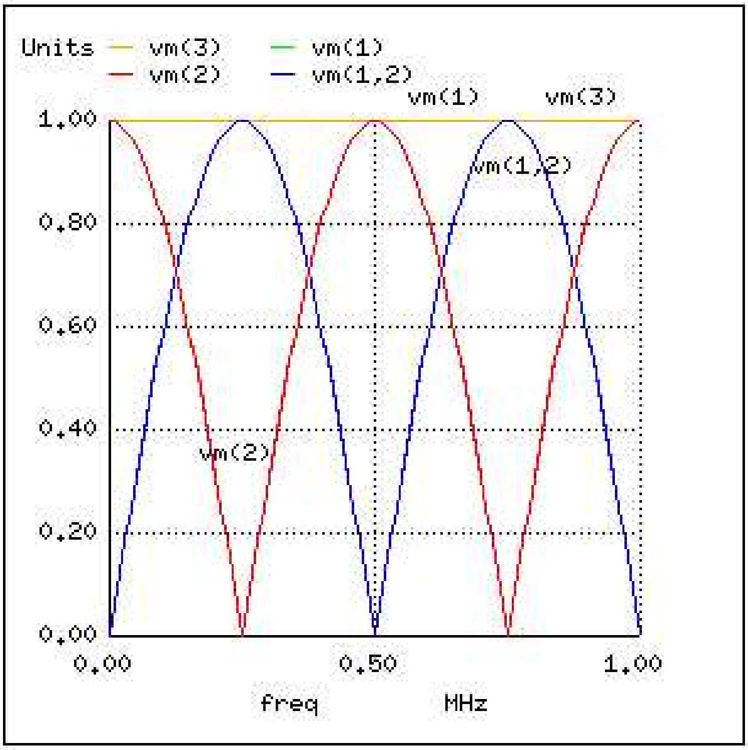


In a system where all impedances are perfectly matched, there can be no standing waves, and therefore no resonant "peaks" or "valleys" in the Bode plot.

Now, let's change the load impedance to 999 MΩ, to simulate an open-ended transmission line. We should definitely see some reflections on the line now as the frequency is swept from 1 mHz to 1 MHz:

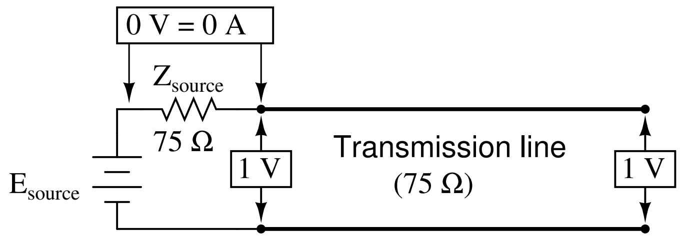



```
Transmission line
v1 1 0 ac 1 sin
rsource 1 2 75
t1 2 0 3 0 z0=75 td=1u
rload 3 0 999meg
.ac lin 101 1m 1meg
* Using "Nutmeg" program to plot analysis
.end
```

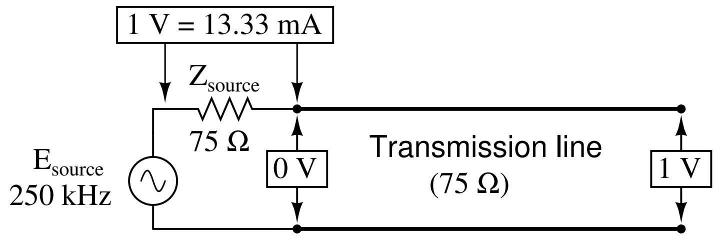


Here, both the supply voltage `vm(1)` and the line's load-end voltage `vm(3)` remain steady at 1 volt. The other voltages dip and peak at different frequencies along the sweep range of 1 mHz to 1 MHz. There are five points of interest along the horizontal axis of the analysis: 0 Hz, 250 kHz, 500 kHz, 750 kHz, and 1 MHz. We will investigate each one with regard to voltage and current at different points of the circuit.

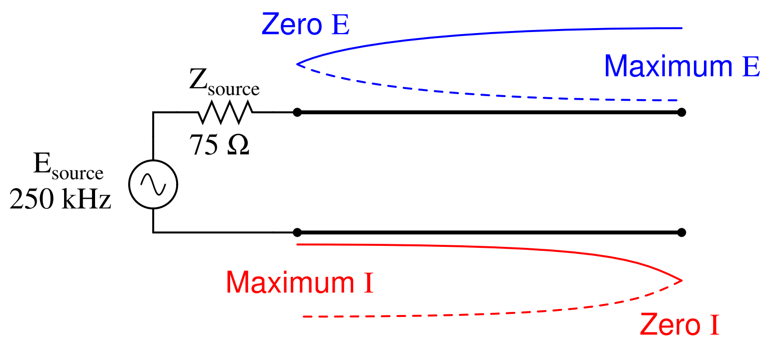
At 0 Hz (actually 1 mHz), the signal is practically DC, and the circuit behaves much as it would given a 1-volt DC battery source. There is no circuit current, as indicated by zero voltage drop across the source impedance (Z_{source}: `vm(1,2)`), and full source voltage present at the source-end of the transmission line (voltage measured between node 2 and node 0: `vm(2)`).



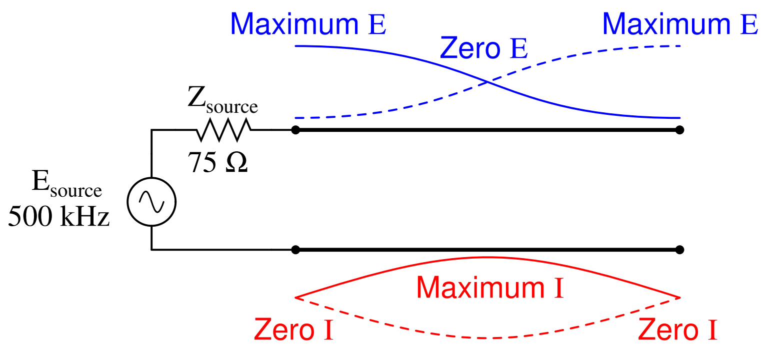
At 250 kHz, we see zero voltage and maximum current at the source-end of the transmission line, yet still full voltage at the load-end:



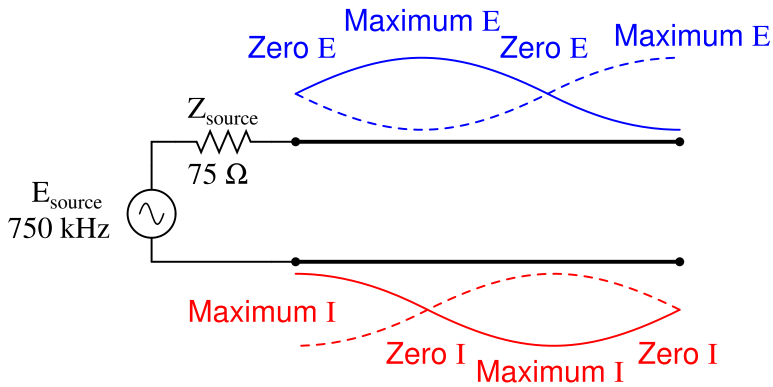
You might be wondering, how can this be? How can we get full source voltage at the line's open end while there is zero voltage at its entrance? The answer is found in the paradox of the standing wave. With a source frequency of 250 kHz, the line's length is precisely right for 1/4 wavelength to fit from end to end. With the line's load end open-circuited, there can be no current, but there will be voltage. Therefore, the load-end of an open-circuited transmission line is a current node (zero point) and a voltage antinode (maximum amplitude):



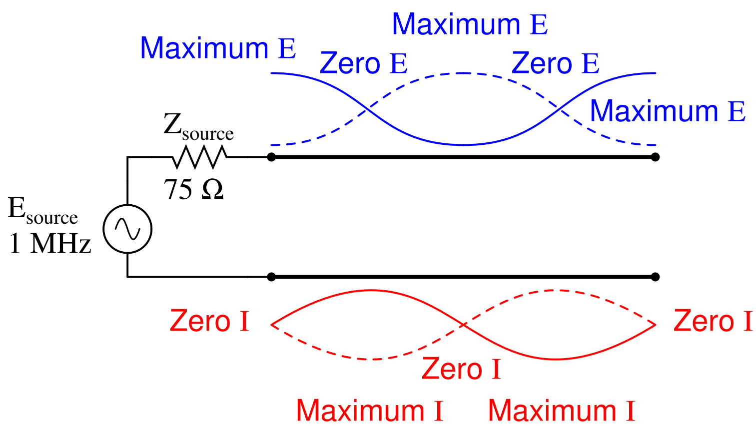
At 500 kHz, exactly one-half of a standing wave rests on the transmission line, and here we see another point in the analysis where the source current drops off to nothing and the source-end voltage of the transmission line rises again to full voltage:



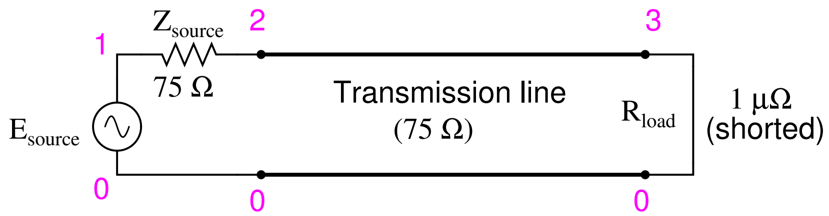
At 750 kHz, the plot looks a lot like it was at 250 kHz: zero source-end voltage (`vm(2)`) and maximum current (`vm(1,2)`). This is due to 3/4 of a wave poised along the transmission line, resulting in the source "seeing" a short-circuit where it connects to the transmission line, even though the other end of the line is open-circuited:



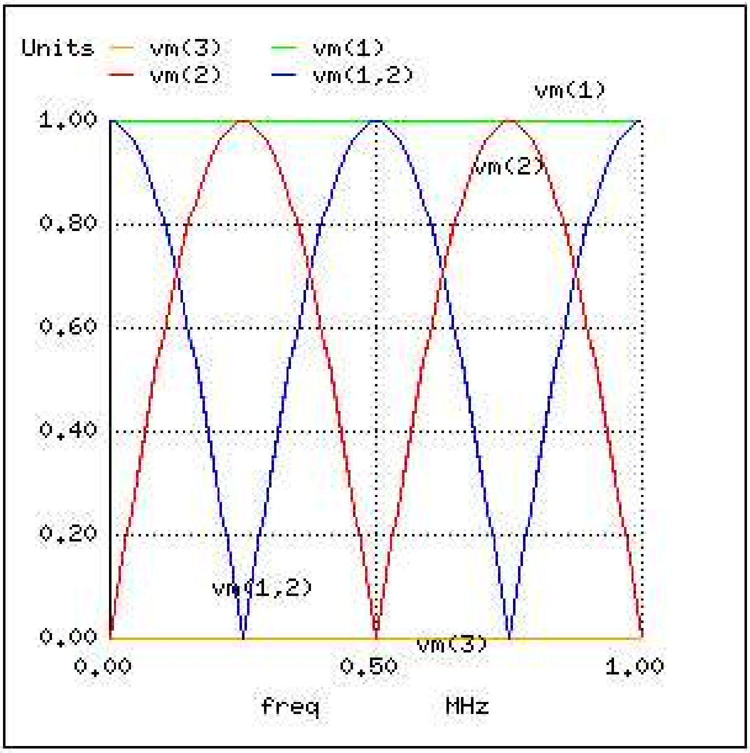
When the supply frequency sweeps up to 1 MHz, a full standing wave exists on the transmission line. At this point, the source-end of the line experiences the same voltage and current amplitudes as the load-end: full voltage and zero current. In essence, the source "sees" an open circuit at the point where it connects to the transmission line.



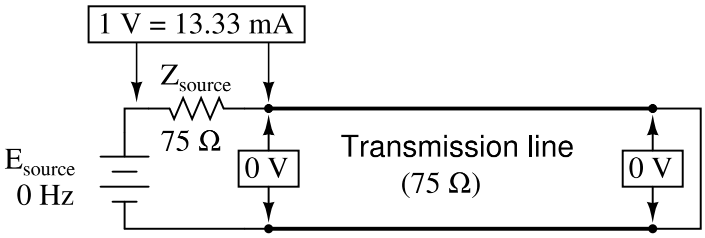
In a similar fashion, a short-circuited transmission line generates standing waves, although the node and antinode assignments for voltage and current are reversed: at the shorted end of the line, there will be zero voltage (node) and maximum current (antinode). What follows is the SPICE simulation and illustrations of what happens at all the interesting frequencies: 0 Hz, 250 kHz, 500 kHz, 750 kHz, and 1 MHz. The short-circuit jumper is simulated by a 1 $\mu\Omega$ load impedance:



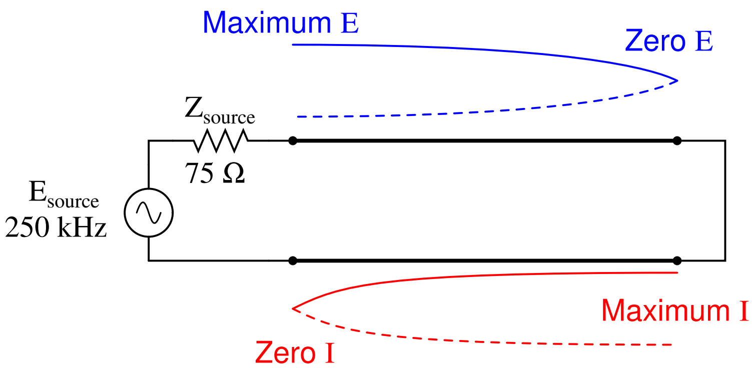

```
Transmission line
v1 1 0 ac 1 sin
rsource 1 2 75
t1 2 0 3 0 z0=75 td=1u
rload 3 0 1u
.ac lin 101 1m 1meg
* Using "Nutmeg" program to plot analysis
.end
```



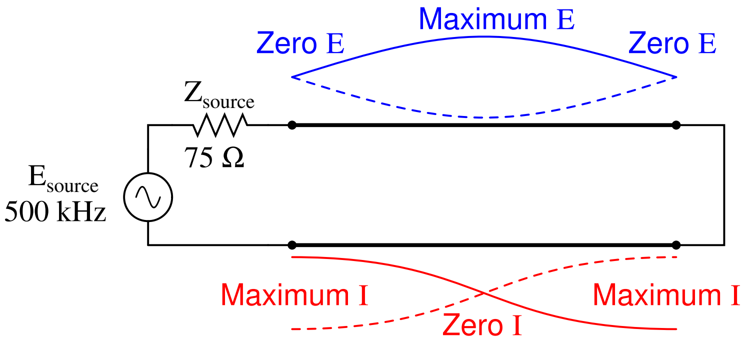
Units
vm(3)
vm(2)
vm(1)
vm(1,2)
vm(1)
vm(2)
vm(1,2)
vm(3)
1.00
0.80
0.60
0.40
0.20
0.00
0.00
0.50
1.00
freq
MHz

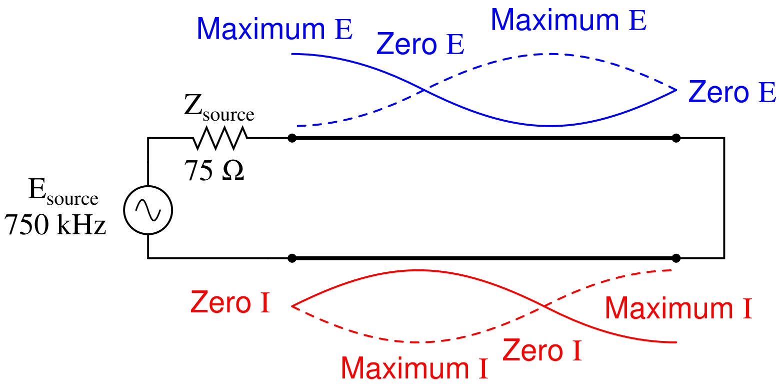


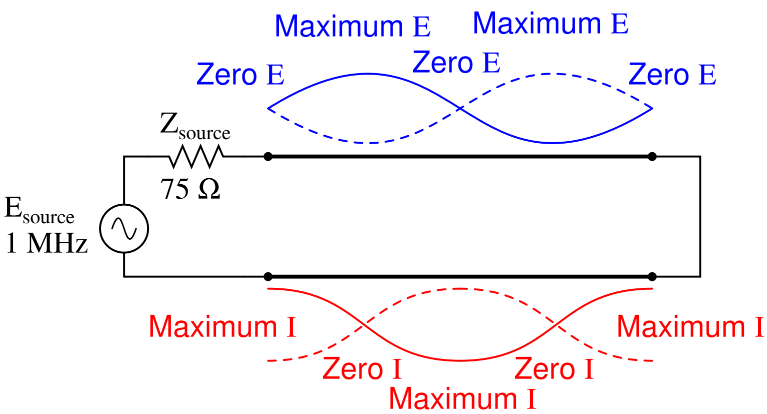
1 V = 13.33 mA
Z source
75 Ω
E source
0 Hz
0 V
Transmission line
(75 Ω)
0 V



Maximum E
Zero E
Z source
75 Ω
E source
250 kHz
Maximum I
Zero I



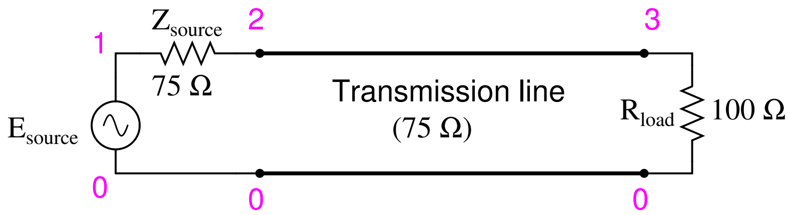




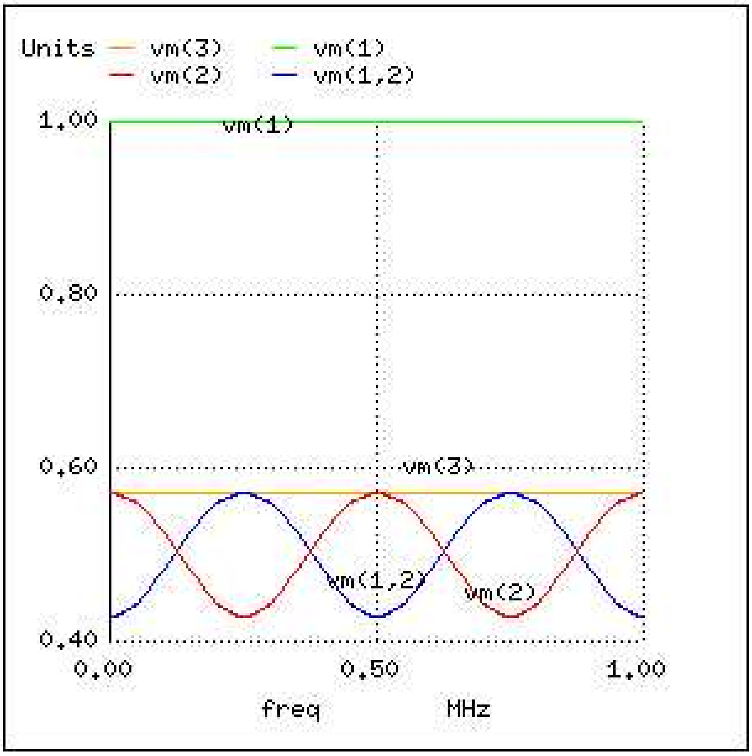
In both these circuit examples, an open-circuited line and a short-circuited line, the energy reflection is total: 100% of the incident wave reaching the line's end gets reflected back toward the source. If, however, the transmission line is terminated in some impedance other than an open or a short, the reflections will be less intense, as will be the difference between minimum and maximum values of voltage and current along the line.

Suppose we were to terminate our example line with a 100 Ω resistor instead of a 75 Ω resistor.

Examine the results of the corresponding SPICE analysis to see the effects of impedance mismatch at different source frequencies:




```
Transmission line
v1 1 0 ac 1 sin
rsource 1 2 75
t1 2 0 3 0 z0=75 td=1u
rload 3 0 100
.ac lin 101 1m 1meg
* Using "Nutmeg" program to plot analysis
.end
```

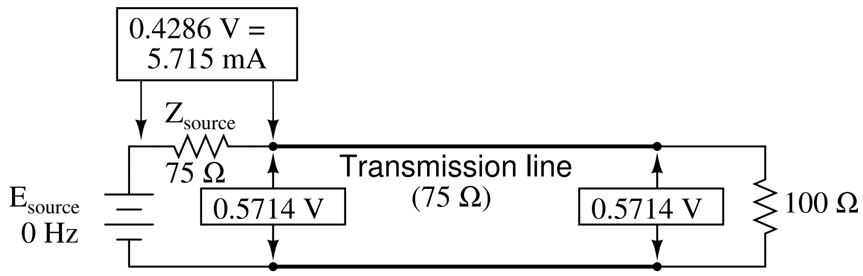


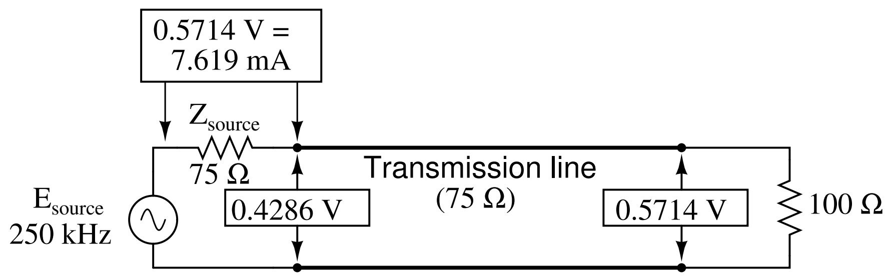
If we run another SPICE analysis, this time printing numerical results rather than plotting them, we can discover exactly what is happening at all the interesting frequencies (DC, 250 kHz, 500 kHz, 750 kHz, and 1 MHz):

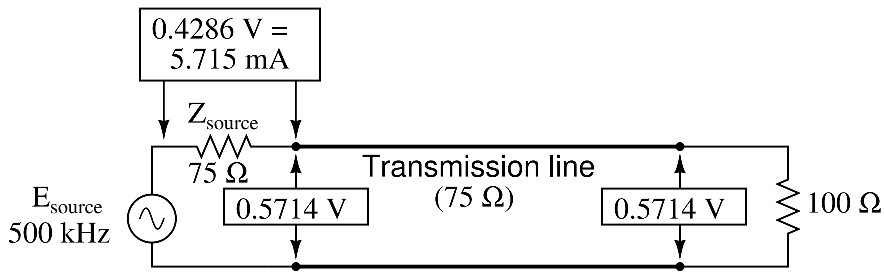
```
Transmission line
v1 1 0 ac 1 sin
rsource 1 2 75
t1 2 0 3 0 z0=75 td=1u
rload 3 0 100
.ac lin 5 1m 1meg
.print ac v(1,2) v(1) v(2) v(3)
.end
```

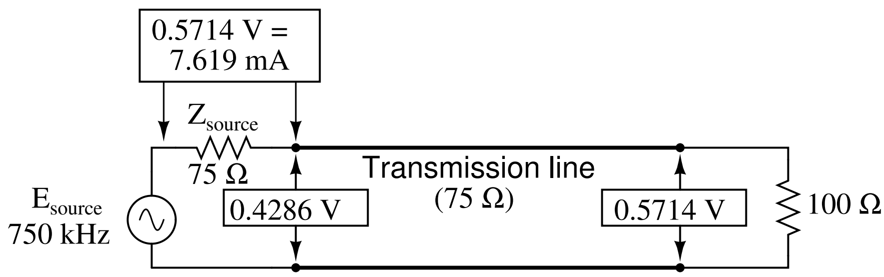
```
freq           v(1,2)       v(1)         v(2)         v(3)
1.000E-03      4.286E-01    1.000E+00    5.714E-01    5.714E-01
2.500E+05      5.714E-01    1.000E+00    4.286E-01    5.714E-01
5.000E+05      4.286E-01    1.000E+00    5.714E-01    5.714E-01
7.500E+05      5.714E-01    1.000E+00    4.286E-01    5.714E-01
1.000E+06      4.286E-01    1.000E+00    5.714E-01    5.714E-01
```

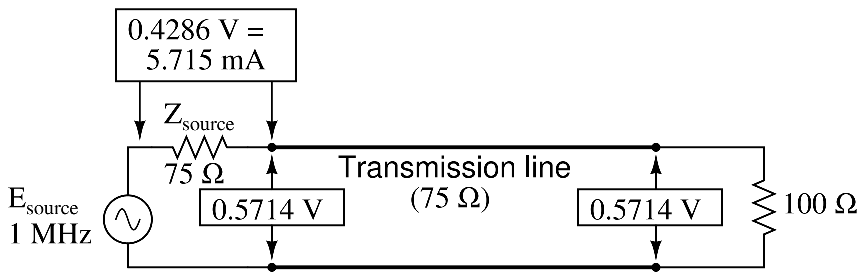
At all frequencies, the source voltage, `v(1)`, remains steady at 1 volt, as it should. The load voltage, `v(3)`, also remains steady, but at a lesser voltage: 0.5714 volts. However, both the line input voltage (`v(2)`) and the voltage dropped across the source's 75 Ω impedance (`v(1,2)`, indicating current drawn from the source) vary with frequency.











At odd harmonics of the fundamental frequency (250 kHz and 750 kHz), we see differing levels of voltage at each end of the transmission line, because at those frequencies the standing waves terminate at one end in a node and at the other end in an antinode. Unlike the open-circuited and short-circuited transmission line examples, the maximum and minimum voltage levels along this transmission line do not reach the same extreme values of 0% and 100% source voltage, but we still have points of "minimum" and "maximum" voltage. The same holds true for current: if the line's terminating impedance is mismatched to the line's characteristic impedance, we will have points of minimum and maximum current at certain fixed locations on the line, corresponding to the standing current wave's nodes and antinodes, respectively.

One way of expressing the severity of standing waves is as a ratio of maximum amplitude (antinode) to minimum amplitude (node), for voltage or for current. When a line is terminated by an open or a short, this *standing wave ratio*, or *SWR* is valued at infinity, since the minimum amplitude will be zero, and any finite value divided by zero results in an infinite (actually, "undefined") quotient. In this example, with a 75 Ω line terminated by a 100 Ω impedance, the SWR will be finite: 1.333, calculated by taking the maximum line voltage at either 250 kHz or 750 kHz (0.5714 volts) and dividing by the minimum line voltage (0.4286 volts).

Standing wave ratio may also be calculated by taking the line's terminating impedance and the line's characteristic impedance, and dividing the larger of the two values by the smaller. In this example, the terminating impedance of 100 Ω divided by the characteristic impedance of 75 Ω yields a quotient of exactly 1.333, matching the previous calculation very closely.

$$\boxed{SWR = \frac{E_{maximum}}{E_{minimum}} = \frac{I_{maximum}}{I_{minimum}}}$$

$$\boxed{\begin{array}{c} SWR = \dfrac{Z_{load}}{Z_0} \quad \textit{or} \quad \dfrac{Z_0}{Z_{load}} \\ \textit{which ever is greater} \end{array}}$$

A perfectly terminated transmission line will have an SWR of 1, since voltage at any location along the line's length will be the same, and likewise for current. Again, this is usually considered ideal, not only because reflected waves constitute energy not delivered to the load, but because the high values of voltage and current created by the antinodes of standing waves may over-stress the transmission line's insulation (high voltage) and conductors (high current), respectively.

Also, a transmission line with a high SWR tends to act as an antenna, radiating electromagnetic energy away from the line, rather than channeling all of it to the load. This is usually undesirable, as the radiated energy may "couple" with nearby conductors, producing signal interference. An interesting footnote to this point is that antenna structures – which typically resemble open- or short-circuited transmission lines – are often designed to operate at *high* standing wave ratios, for the very reason of maximizing signal radiation and reception.

The following photograph shows a set of transmission lines at a junction point in a radio transmitter system. The large, copper tubes with ceramic insulator caps at the ends are rigid coaxial transmission lines of 50 Ω characteristic impedance. These lines carry RF power from the radio transmitter circuit to a small, wooden shelter at the base of an antenna structure, and from that shelter on to other shelters with other antenna structures:

Flexible coaxial cable connected to the rigid lines (also of 50 Ω characteristic impedance) conduct the RF power to capacitive and inductive "phasing" networks inside the shelter. The white, plastic tube joining two of the rigid lines together carries "filling" gas from one sealed line to the other. The lines are gas-filled to avoid collecting moisture inside them, which would be a definite problem for a coaxial line. Note the flat, copper "straps" used as jumper wires to connect the conductors of the flexible coaxial cables to the conductors of the rigid lines. Why flat straps of copper and not round wires? Because of the skin effect, which renders most of the cross-sectional area of a round conductor useless at radio frequencies.

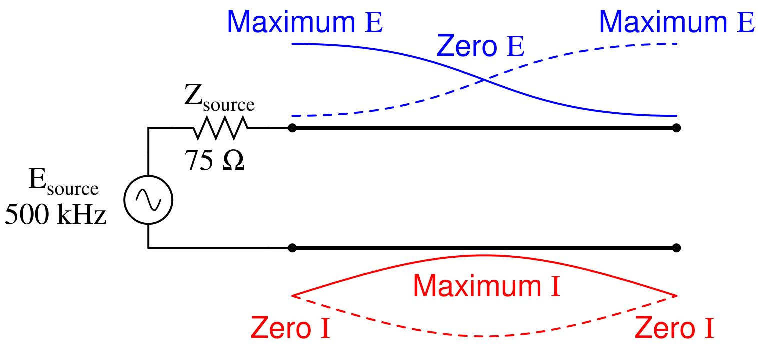
Like many transmission lines, these are operated at low SWR conditions. As we will see in the next section, though, the phenomenon of standing waves in transmission lines is not always undesirable, as it may be exploited to perform a useful function: impedance transformation.

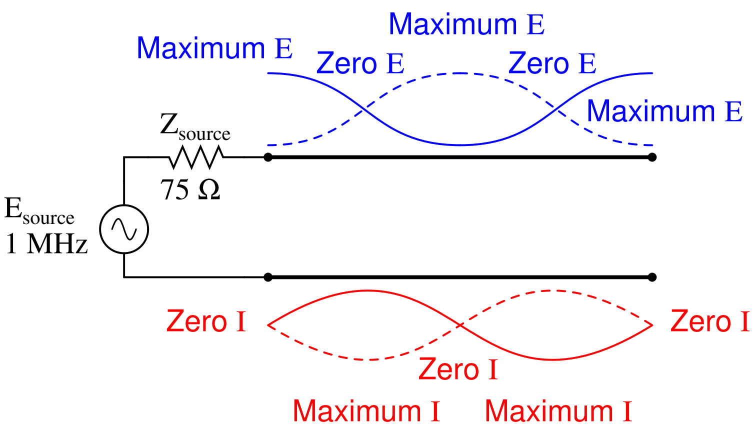
- **REVIEW:**
- *Standing waves* are waves of voltage and current which do not propagate (i.e. they are stationary), but are the result of interference between incident and reflected waves along a transmission line.
- A *node* is a point on a standing wave of *minimum* amplitude.
- An *antinode* is a point on a standing wave of *maximum* amplitude.
- Standing waves can only exist in a transmission line when the terminating impedance does not match the line's characteristic impedance. In a perfectly terminated line, there are no reflected waves, and therefore no standing waves at all.
- At certain frequencies, the nodes and antinodes of standing waves will correlate with the ends of a transmission line, resulting in *resonance*.

- The lowest-frequency resonant point on a transmission line is where the line is one quarter-wavelength long. Resonant points exist at every harmonic (integer-multiple) frequency of the fundamental (quarter-wavelength).
- *Standing wave ratio*, or *SWR*, is the ratio of maximum standing wave amplitude to minimum standing wave amplitude. It may also be calculated by dividing termination impedance by characteristic impedance, or visa-versa, which ever yields the greatest quotient. A line with no standing waves (perfectly matched: Z_{load} to Z_0) has an SWR equal to 1.
- Transmission lines may be damaged by the high maximum amplitudes of standing waves. Voltage antinodes may break down insulation between conductors, and current antinodes may overheat conductors.

14.7 Impedance transformation

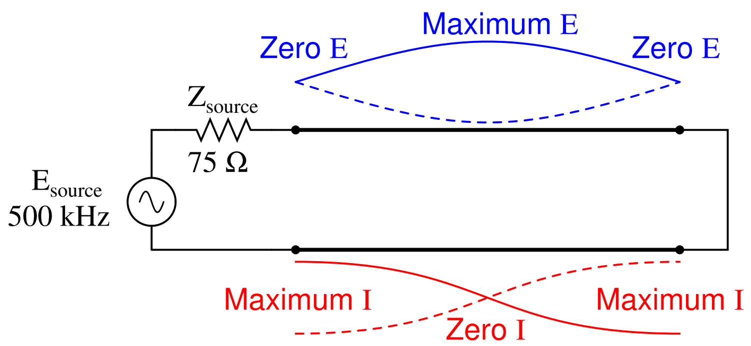
Standing waves at the resonant frequency points of an open- or short-circuited transmission line produce unusual effects. When the signal frequency is such that exactly 1/2 wave or some multiple thereof matches the line's length, the source "sees" the load impedance as it is. The following pair of illustrations shows an open-circuited line operating at 1/2 and 1 wavelength frequencies:

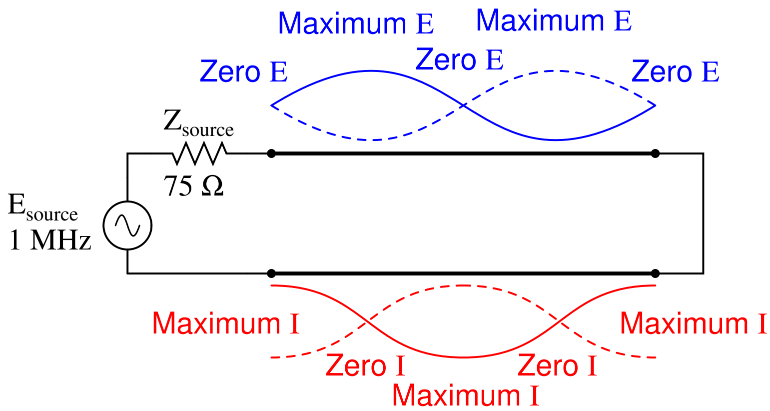




In either case, the line has voltage antinodes at both ends, and current nodes at both ends. That is to say, there is maximum voltage and minimum current at either end of the line, which corresponds to the condition of an open circuit. The fact that this condition exists at *both* ends of the line tells us that the line faithfully reproduces its terminating impedance at the source end, so that the source "sees" an open circuit where it connects to the transmission line, just as if it were directly open-circuited.

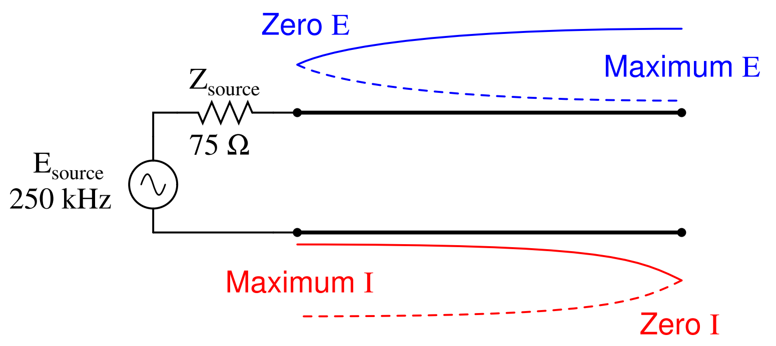
The same is true if the transmission line is terminated by a short: at signal frequencies corresponding to 1/2 wavelength or some multiple thereof, the source "sees" a short circuit, with minimum voltage and maximum current present at the connection points between source and transmission line:

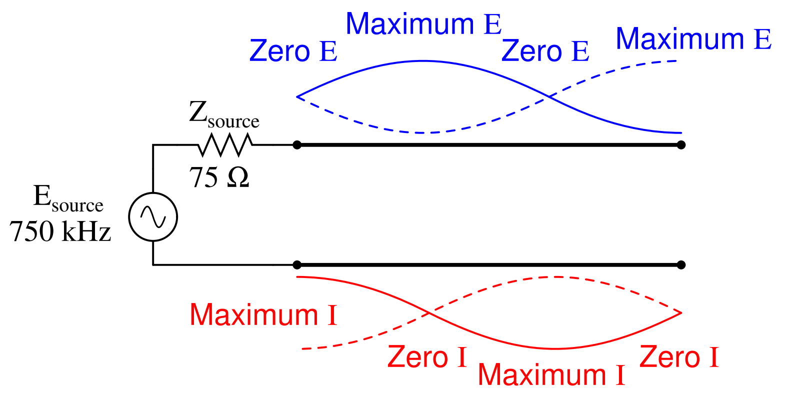




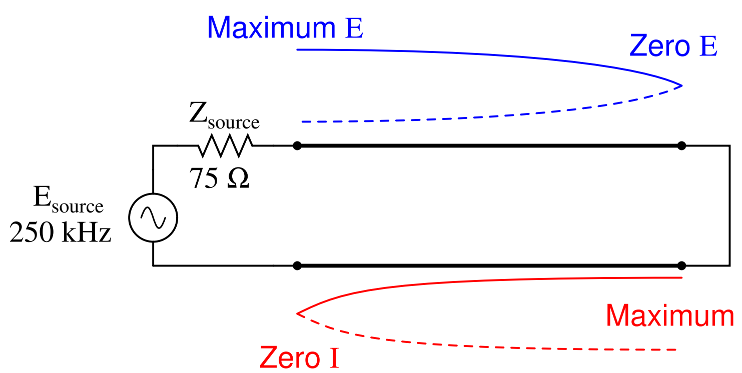
However, if the signal frequency is such that the line resonates at *1/4* wavelength or some multiple thereof, the source will "see" the exact opposite of the termination impedance. That is, if the line is open-circuited, the source will "see" a short-circuit at the point where it connects to the line; and if the line is short-circuited, the source will "see" an open circuit:

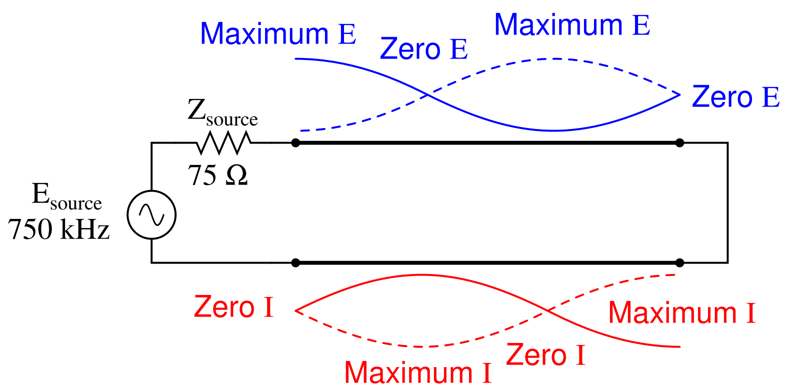
Line open-circuited; source "sees" a short circuit:





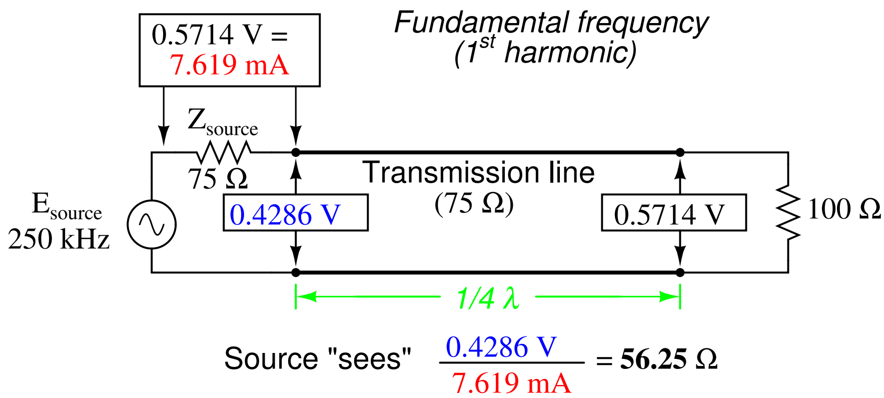
Line short-circuited; source "sees" an open circuit:

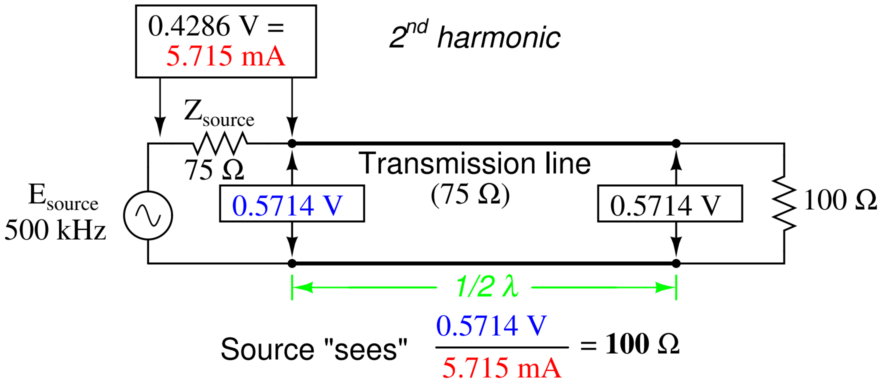


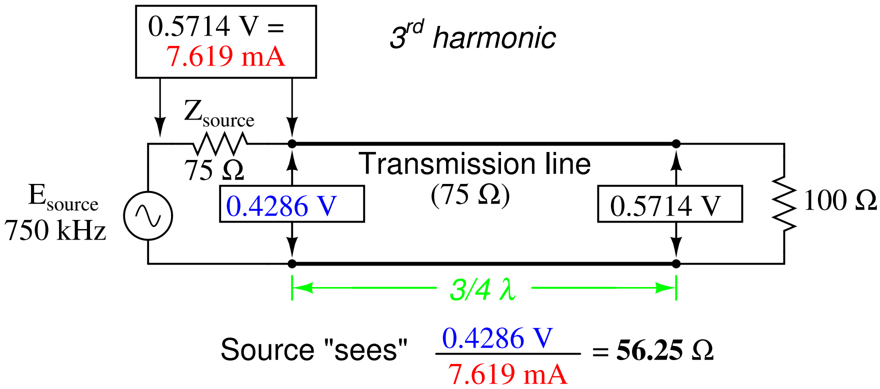


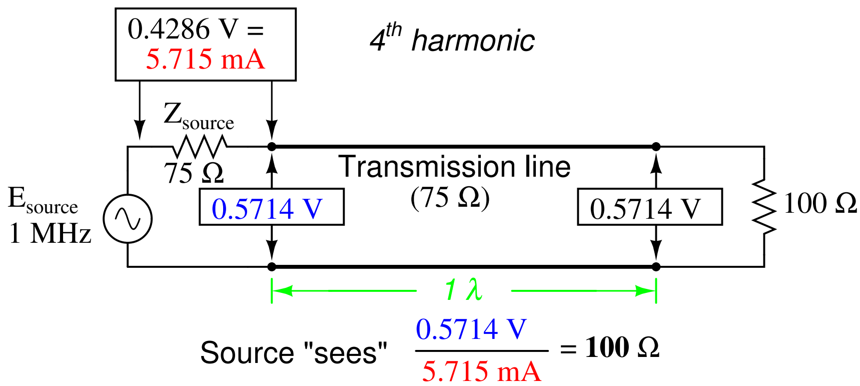
At these frequencies, the transmission line is actually functioning as an *impedance transformer*, transforming an infinite impedance into zero impedance, or visa-versa. Of course, this only occurs at resonant points resulting in a standing wave of 1/4 cycle (the line's fundamental, resonant frequency) or some odd multiple (3/4, 5/4, 7/4, 9/4 . . .), but if the signal frequency is known and unchanging, this phenomenon may be used to match otherwise unmatched impedances to each other.

Take for instance the example circuit from the last section where a 75 Ω source connects to a 75 Ω transmission line, terminating in a 100 Ω load impedance. From the numerical figures obtained via SPICE, let's determine what impedance the source "sees" at its end of the transmission line at the line's resonant frequencies:




0.4286 V =
5.715 mA
2nd harmonic
Zsource
75 Ω
Esource
500 kHz
Transmission line
(75 Ω)
0.5714 V
0.5714 V
100 Ω
1/2 λ
Source "sees"
0.5714 V / 5.715 mA = 100 Ω


0.5714 V =
7.619 mA
3rd harmonic
Zsource
75 Ω
Esource
750 kHz
Transmission line
(75 Ω)
0.4286 V
0.5714 V
100 Ω
3/4 λ
Source "sees"
0.4286 V / 7.619 mA = 56.25 Ω



A simple equation relates line impedance (Z_0), load impedance (Z_{load}), and input impedance (Z_{input}) for an unmatched transmission line operating at an odd harmonic of its fundamental frequency:

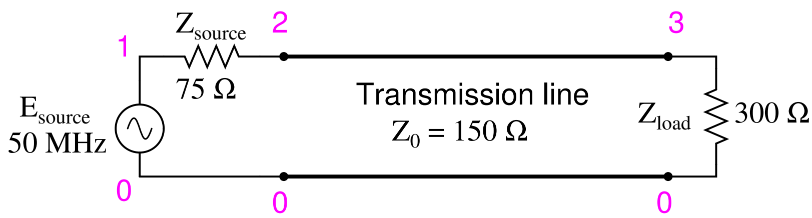
$$Z_0 = \sqrt{Z_{input} Z_{load}}$$

One practical application of this principle would be to match a 300 Ω load to a 75 Ω signal source at a frequency of 50 MHz. All we need to do is calculate the proper transmission line impedance (Z_0), and length so that exactly 1/4 of a wave will "stand" on the line at a frequency of 50 MHz.

First, calculating the line impedance: taking the 75 Ω we desire the source to "see" at the source-end of the transmission line, and multiplying by the 300 Ω load resistance, we obtain a figure of 22,500. Taking the square root of 22,500 yields 150 Ω for a characteristic line impedance.

Now, to calculate the necessary line length: assuming that our cable has a velocity factor of 0.85, and using a speed-of-light figure of 186,000 miles per second, the velocity of propagation will be 158,100 miles per second. Taking this velocity and dividing by the signal frequency gives us a wavelength of 0.003162 miles, or 16.695 feet. Since we only need one-quarter of this length for the cable to support a quarter-wave, the requisite cable length is 4.1738 feet.

Here is a schematic diagram for the circuit, showing node numbers for the SPICE analysis we're about to run:



$$150 = \sqrt{(75)(300)}$$

We can specify the cable length in SPICE in terms of time delay from beginning to end. Since the frequency is 50 MHz, the signal period will be the reciprocal of that, or 20 nano-seconds (20 ns). One-quarter of that time (5 ns) will be the time delay of a transmission line one-quarter wavelength long:

```
Transmission line
```

```
v1 1 0 ac 1 sin
rsource 1 2 75
t1 2 0 3 0 z0=150 td=5n
rload 3 0 300
.ac lin 1 50meg 50meg
.print ac v(1,2) v(1) v(2) v(3)
.end

freq          v(1,2)      v(1)        v(2)        v(3)
5.000E+07     5.000E-01   1.000E+00   5.000E-01   1.000E+00
```

At a frequency of 50 MHz, our 1-volt signal source drops half of its voltage across the series 75 Ω impedance (`v(1,2)`) and the other half of its voltage across the input terminals of the transmission line (`v(2)`). This means the source "thinks" it is powering a 75 Ω load. The actual load impedance, however, receives a full 1 volt, as indicated by the 1.000 figure at `v(3)`. With 0.5 volt dropped across 75 Ω, the source is dissipating 3.333 mW of power: the same as dissipated by 1 volt across the 300 Ω load, indicating a perfect match of impedance, according to the Maximum Power Transfer Theorem. The 1/4-wavelength, 150 Ω, transmission line segment has successfully matched the 300 Ω load to the 75 Ω source.

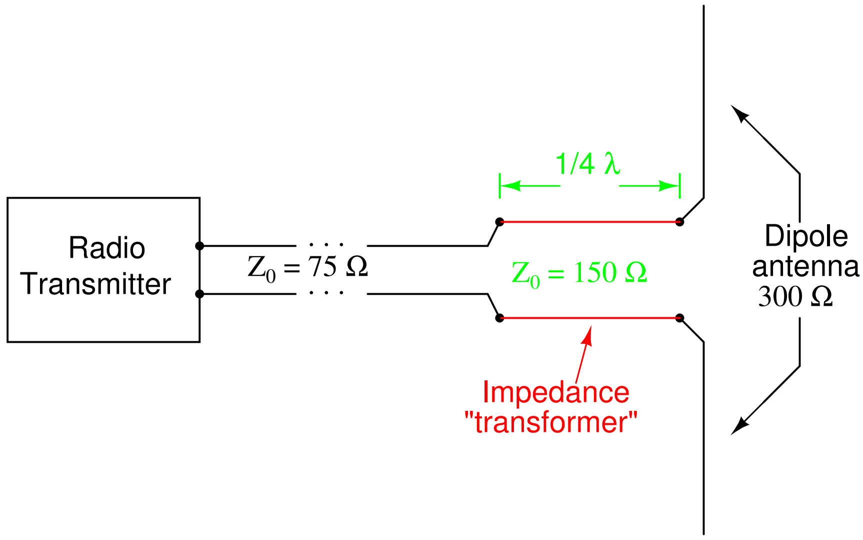
Bear in mind, of course, that this only works for 50 MHz and its odd-numbered harmonics. For any other signal frequency to receive the same benefit of matched impedances, the 150 Ω line would have to lengthened or shortened accordingly so that it was exactly 1/4 wavelength long.

Strangely enough, the exact same line can also match a 75 Ω load to a 300 Ω source, demonstrating how this phenomenon of impedance transformation is fundamentally different in principle from that of a conventional, two-winding transformer:

```
Transmission line
v1 1 0 ac 1 sin
rsource 1 2 300
t1 2 0 3 0 z0=150 td=5n
rload 3 0 75
.ac lin 1 50meg 50meg
.print ac v(1,2) v(1) v(2) v(3)
.end

freq          v(1,2)      v(1)        v(2)        v(3)
5.000E+07     5.000E-01   1.000E+00   5.000E-01   2.500E-01
```

Here, we see the 1-volt source voltage equally split between the 300 Ω source impedance (`v(1,2)`) and the line's input (`v(2)`), indicating that the load "appears" as a 300 Ω impedance from the source's perspective where it connects to the transmission line. This 0.5 volt drop across the source's 300 Ω internal impedance yields a power figure of 833.33 μW, the same as the 0.25 volts across the 75 Ω load, as indicated by voltage figure `v(3)`. Once again, the impedance values of source and load have been matched by the transmission line segment.

This technique of impedance matching is often used to match the differing impedance values of transmission line and antenna in radio transmitter systems, because the transmitter's frequency is generally well-known and unchanging. The use of an impedance "transformer" 1/4 wavelength in length provides impedance matching using the shortest conductor length possible.



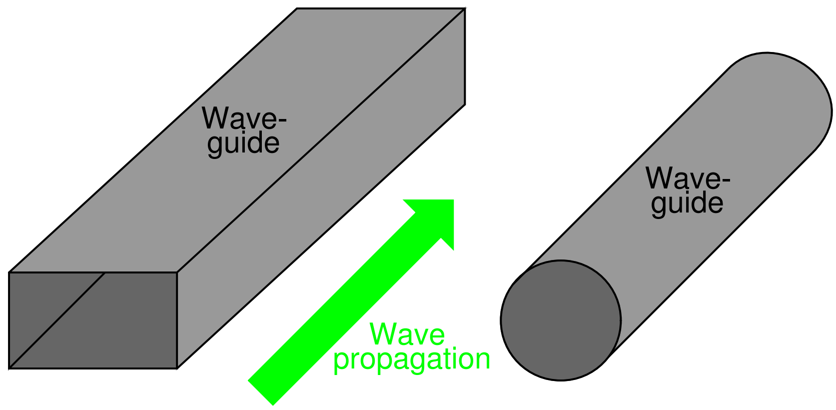
- **REVIEW:**

- A transmission line with standing waves may be used to match different impedance values if operated at the correct frequency(ies).

- When operated at a frequency corresponding to a standing wave of 1/4-wavelength along the transmission line, the line's characteristic impedance necessary for impedance transformation must be equal to the square root of the product of the source's impedance and the load's impedance.

14.8 Waveguides

A *waveguide* is a special form of transmission line consisting of a hollow, metal tube. The tube wall provides distributed inductance, while the empty space between the tube walls provide distributed capacitance:

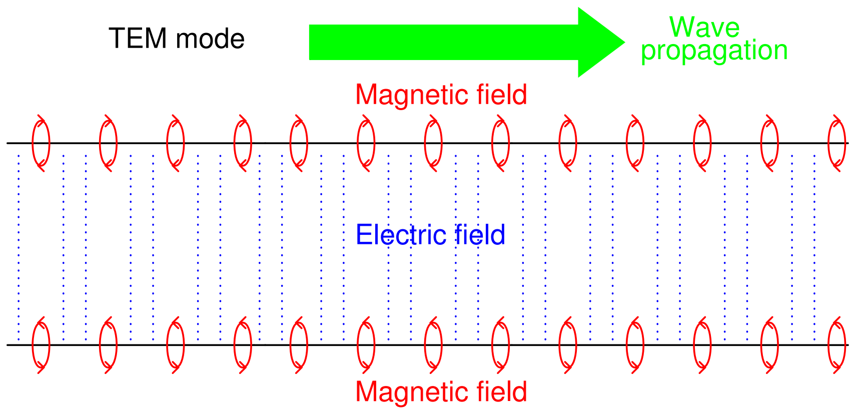


Waveguides are practical only for signals of extremely high frequency, where the wavelength approaches the cross-sectional dimensions of the waveguide. Below such frequencies, waveguides are useless as electrical transmission lines.

When functioning as transmission lines, though, waveguides are considerably simpler than two-conductor cables – especially coaxial cables – in their manufacture and maintenance. With only a single conductor (the waveguide's "shell"), there are no concerns with proper conductor-to-conductor spacing, or of the consistency of the dielectric material, since the only dielectric in a waveguide is air. Moisture is not as severe a problem in waveguides as it is within coaxial cables, either, and so waveguides are often spared the necessity of gas "filling."

Waveguides may be thought of as conduits for electromagnetic energy, the waveguide itself acting as nothing more than a "director" of the energy rather than as a signal conductor in the normal sense of the word. In a sense, all transmission lines function as conduits of electromagnetic energy when transporting pulses or high-frequency waves, directing the waves as the banks of a river direct a tidal wave. However, because waveguides are single-conductor elements, the propagation of electrical energy down a waveguide is of a very different nature than the propagation of electrical energy down a two-conductor transmission line.

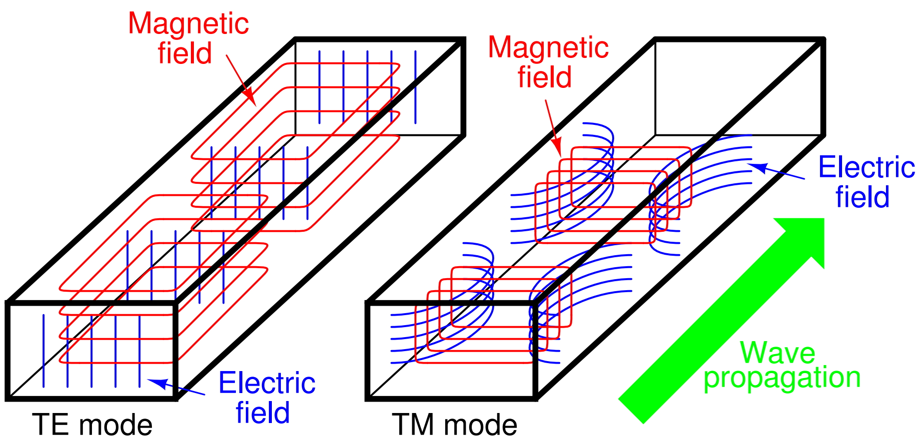
All electromagnetic waves consist of electric and magnetic fields propagating in the same direction of travel, but perpendicular to each other. Along the length of a normal transmission line, both electric and magnetic fields are perpendicular (transverse) to the direction of wave travel. This is known as the *principal mode*, or *TEM* (**T**ransverse **E**lectric and **M**agnetic) mode. This mode of wave propagation can exist only where there are two conductors, and it is the dominant mode of wave propagation where the cross-sectional dimensions of the transmission line are small compared to the wavelength of the signal.



Both field planes perpendicular (transverse) to direction of signal propagation.

At *microwave* signal frequencies (between 100 MHz and 300 GHz), two-conductor transmission lines of any substantial length operating in standard TEM mode become impractical. Lines small enough in cross-sectional dimension to maintain TEM mode signal propagation for microwave signals tend to have low voltage ratings, and suffer from large, parasitic power losses due to conductor "skin" and dielectric effects. Fortunately, though, at these short wavelengths there exist other modes of propagation that are not as "lossy," if a conductive tube is used rather than two parallel conductors. It is at these high frequencies that waveguides become practical.

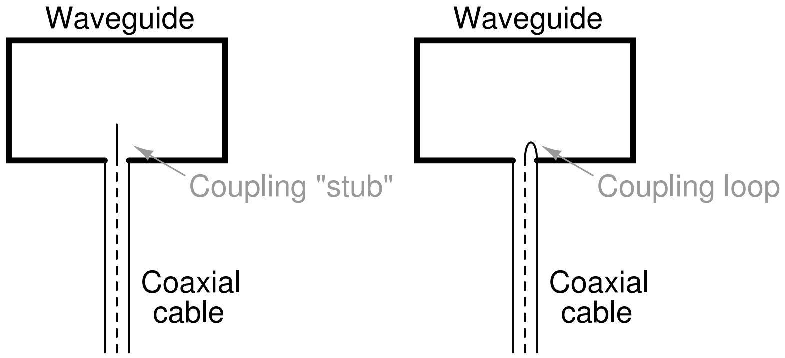
When an electromagnetic wave propagates down a hollow tube, only one of the fields – either electric or magnetic – will actually be transverse to the wave's direction of travel. The other field will "loop" longitudinally to the direction of travel, but still be perpendicular to the other field. Whichever field remains transverse to the direction of travel determines whether the wave propagates in *TE* mode (**T**ransverse **E**lectric) or *TM* (**T**ransverse **M**agnetic) mode.



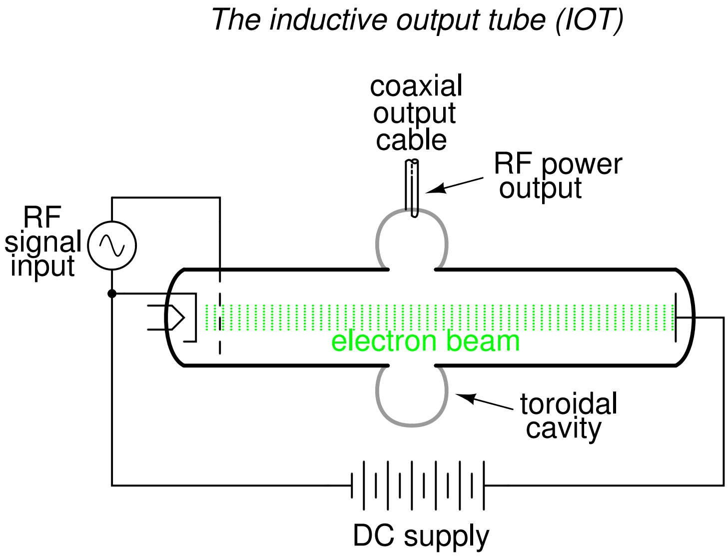
Magnetic flux lines appear as continuous loops
Electric flux lines appear with beginning and end points

Many variations of each mode exist for a given waveguide, and a full discussion of this is subject well beyond the scope of this book.

Signals are typically introduced to and extracted from waveguides by means of small antenna-like coupling devices inserted into the waveguide. Sometimes these coupling elements take the form of a dipole, which is nothing more than two open-ended stub wires of appropriate length. Other times, the coupler is a single stub (a half-dipole, similar in principle to a "whip" antenna, $1/4\lambda$ in physical length), or a short loop of wire terminated on the inside surface of the waveguide:



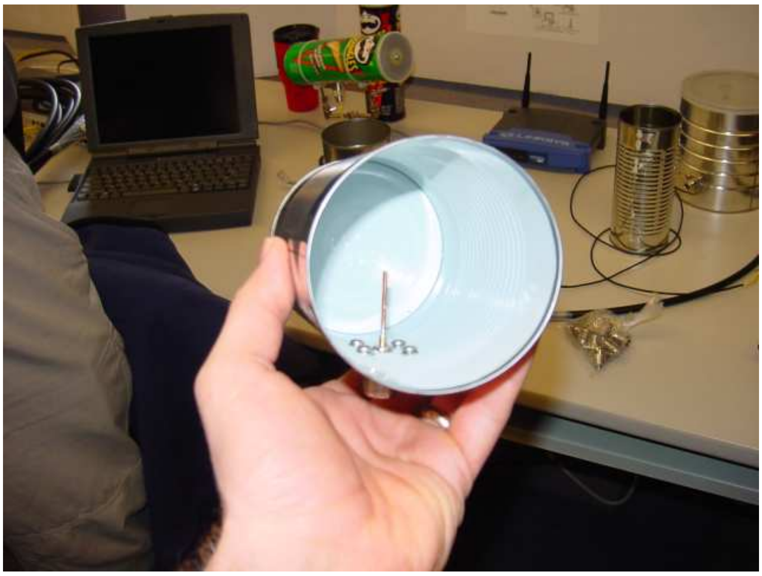
In some cases, such as a class of vacuum tube devices called *inductive output tubes* (the so-called *klystron* tube falls into this category), a "cavity" formed of conductive material may intercept electromagnetic energy from a modulated beam of electrons, having no contact with the beam itself:



Just as transmission lines are able to function as resonant elements in a circuit, especially when terminated by a short-circuit or an open-circuit, a dead-ended waveguide may also resonate at particular frequencies. When used as such, the device is called a *cavity resonator*. Inductive output tubes use toroid-shaped cavity resonators to maximize the power transfer efficiency between the electron beam and the output cable.

A cavity's resonant frequency may be altered by changing its physical dimensions. To this end, cavities with movable plates, screws, and other mechanical elements for tuning are manufactured to provide coarse resonant frequency adjustment.

If a resonant cavity is made open on one end, it functions as a unidirectional antenna. The following photograph shows a home-made waveguide formed from a tin can, used as an antenna for a 2.4 GHz signal in an "802.11b" computer communication network. The coupling element is a quarter-wave stub: nothing more than a piece of solid copper wire about 1-1/4 inches in length extending from the center of a coaxial cable connector penetrating the side of the can:



A few more tin-can antennae may be seen in the background, one of them a "Pringles" potato chip can. Although this can is of cardboard (paper) construction, its metallic inner lining provides the necessary conductivity to function as a waveguide. Some of the cans in the background still have their plastic lids in place. The plastic, being nonconductive, does not interfere with the RF signal, but functions as a physical barrier to prevent rain, snow, dust, and other physical contaminants from entering the waveguide. "Real" waveguide antennae use similar barriers to physically enclose the tube, yet allow electromagnetic energy to pass unimpeded.

- **REVIEW:**

- *Waveguides* are metal tubes functioning as "conduits" for carrying electromagnetic waves. They are practical only for signals of extremely high frequency, where the signal wavelength approaches the cross-sectional dimensions of the waveguide.

- Wave propagation through a waveguide may be classified into two broad categories: *TE* (Transverse Electric), or *TM* (Transverse Magnetic), depending on which field (electric or magnetic) is perpendicular (transverse) to the direction of wave travel. Wave travel along a standard, two-conductor transmission line is of the *TEM* (Transverse Electric and Magnetic) mode, where both fields are oriented perpendicular to the direction of travel. TEM mode is only possible with two conductors and cannot exist in a waveguide.

- A dead-ended waveguide serving as a resonant element in a microwave circuit is called a *cavity resonator*.
- A cavity resonator with an open end functions as a unidirectional antenna, sending or receiving RF energy to/from the direction of the open end.

Chapter 15

À PROPOS DE CE LIVRE

15.1 Objectif

Il se dit que nécessité fait loi. Au moins, dans le cas de ce livre, cet adage est vrai. En tant que formateur en électronique industrielle, j'ai été contraint d'utiliser un sous-produit de littérature électronique pendant ma première année d'enseignement. Mes étudiants étaient quotidiennement frustrés avec les nombreuses erreurs typographiques et les explications obscures de ce livre, passant beaucoup de temps à la maison pour comprendre ce qu'il y a dedans. Le pire était les nombreuses réponses incorrectes à la fin du livre pour les problèmes sélectionnés. Lorsque les insultes se sont ajoutées au préjudice, cela a été le sommet.

Contacter l'éditeur s'est révélé futile. Même si le texte particulier que j'utilisais était imprimé et utilisé depuis quelques années, ils m'ont fait savoir que ma plainte était la première qu'ils avaient reçu jusqu'ici. Ma requête pour relire le brouillon de l'édition suivante de leur livre a reçu leur désintérêt et je me suis décidé de rechercher un texte alternatif.

Trouver une alternative adaptée fut plus difficile que je ne l'avais imaginé. Bien sûr, il y avait beaucoup de textes imprimés mais les livres réellement bons semblent être trop lourd en mathématiques et les livres moins intimidants oubliaient beaucoup d'informations que je trouvais importantes. Quelques uns des meilleurs livres étaient épuisés et ceux qui étaient disponibles étaient assez chers.

J'étais tellement frustré que j'ai compilé *Lessons in Electric Circuits* depuis des notes et des idées que j'avais collecté pendant des années. Mon objectif premier était de mettre dans les mains de mes étudiants, des informations de haute qualité mais le but secondaire était de rendre le livre aussi abordable que possible. Pendant des années, j'ai expérimenté le bénéfice de recevoir de l'instruction gratuite et des encouragements dans ma poursuite de l'apprentissage de l'électronique de la part de plusieurs personnes, incluant plusieurs de mes enseignants de l'école primaire et secondaire. Leur assistance désintéressée a joué un rôle clé dans mes propres études, en pavant le chemin d'une carrière réussie et d'un hobby fascinant. Si seulement je peut étendre le don de leur aide en donnant à d'autres personne ce qui m'a été donné...

J'ai donc décidé de rendre ce livre librement disponible. Plus que ça, j'ai décidé de le rendre "ouvert", en suivant le même modèle de développement utilisé dans la réalisation de logiciels libres (plus précisément les divers utilitaires UNIX fournis par la Free Software Foundation et le système d'exploitation GNU/Linux, dont la popularité croît pendant que j'écris ses lignes). Le but était de copyrighter le texte – donc comme pour protéger mes droits d'auteur – mais permettant expressément à quiconque de distribuer et/ou modifier le texte pour l'adapter à leur propre besoin avec un minimum de gêne légale. Ce rejet intentionnel et formel des limitations de la distribution standard sous copyright est étrangement appelé *copyleft*. Quiconque peut "copylefter" son travail créatif en ajoutant simplement une note pour cet effet sur sa réalisation mais plusieurs Licences existent déjà, couvrant les points légaux avec un grand détail.

La première licence que j'ai appliqué à mon travail était la GPL – General Public License – de la Free Software Foundation (GNU). La GPL, néanmoins, est destinée aux travaux copyleft des réalisations logicielles et bien que le texte d'introduction est suffisamment large pour couvrir les travaux de texte, il n'était pas suffisamment clair pour cette application. Alors que d'autres licences copyleft moins spécifiques commencent à apparaître dans la communauté des logiciels libres, j'ai choisi l'un d'elles (la Design Science License ou DSL) comme la note officielle pour mon projet.

En "copyleftant" ce texte, je garantis qu'aucun formateur ne sera limité par un texte insuffisant pour ses besoins, comme je l'ai été avec des textes remplis d'erreurs des publications importantes. Je suis sûr que ce livre, dans cette forme initiale ne satisfera pas tout le monde mais quiconque à la liberté de la changer, poursuivant mes efforts pour l'adapter aux nécessités variées et individuelles. Pour le démarrage des étudiants en électronique, apprenez ce que vous pouvez de ce livre, éditez-le comme vous le sentez nécessaire si vous tombez sur des éléments d'informations utiles. Alors, si vous le passez à quelqu'un d'autre, vous lui donnerez quelque chose de mieux que ce que vous avez reçu. Les formateurs ou les professionnels de l'électronique peuvent se sentir libres de l'utiliser comme manuel de référence, ajoutant ou éditant du contenu selon son besoin. La seule "contrainte" est celle-ci : si vous avez prévu de distribuer une version modifiée de ce texte, vous devez insérer le crédit où il doit l'être (à moi, l'auteur originel et quiconque d'autre dont les modifications sont contenues dans votre version) et vous devez vous assurer que quiconque recevant le texte soit au courant de sa liberté de partage et d'édition du texte. Le chapitre suivant couvre ce processus plus en détail.

Je dois mentionner que bien que je me sois efforcé de maintenir une exactitude technique dans tout le contenu de ce livre, les sujets sont vastes et les embûches sont multiples. L'ÉLECTRICITÉ BLESSE, TUE SANS AVERTISSEMENT ET NÉCESSITE UNE ATTENTION EXTRÊME. J'encourage fortement les expérimentations comme partie de la lecture mais seulement avec des circuits alimentés par des petites batteries où il n'y a pas de risques de choc électrique, de feu, d'explosions, etc... Les circuits contenant de la haute tension doivent être laissés aux professionnels entraînés! La Design Science License statue clairement que ni moi, ni un des contributeur de ce livre ne porte une responsabilité quelconque avec son contenu.

15.2 L'utilisation de SPICE

Une des meilleurs manières d'apprendre comment les choses fonctionnent est de suivre l'approche inductive : pour observer les instances spécifiques des choses en fonctionnement et pour en dériver des conclusions générales de ses observations. Dans l'éducation scientifique, le travail de laboratoire est une vue traditionnellement acceptée pour ce type d'apprentissage, bien que dans beaucoup de cas, les laboratoires sont conçus par les enseignants pour renforcer les principes précédemment appris lors de cours ou de lecture de livres, plutôt que permettre à leurs étudiants d'apprendre par eux-mêmes avec un processus réellement exploratoire.

Ayant appris par moi-même la grande partie de l'électronique que je connais, je comprend les étudiants frustrés dans leur apprentissage depuis des livres. Bien que les composants soient très bon marché, il n'est pas donné à tout le monde de monter son laboratoire à la maison et lorsque les choses ne tournent pas rond, il n'y a personne à qui demander de l'aide. La plupart des livres ont une approche de l'éducation par la perspective déductive : indiquer aux étudiants comment les choses sont supposées fonctionner puis appliquer ces principes aux instances spécifiques que l'étudiant peut ou ne peut pas explorer de lui-même. L'approche inductive, toute aussi utile qu'elle soit, est difficile à trouver dans les pages d'un livre.

Néanmoins, les livres n'ont pas besoin d'abonder dans ce sens. Je l'ai découvert lorsque j'ai démarré l'apprentissage avec un programme d'ordinateur appelé SPICE. C'est un un programme à commande texte, destiné à simuler les circuits et fournir des analyses de tension, courant, fréquence, etc. Bien que rien ne soit aussi bien que de construire des circuits réels pour acquérir de la connaissance en électronique, la simulation par ordinateur est une excellente alternative. Dans l'apprentissage sur l'utilisation de cet outil puissant, j'ai fait une découverte : SPICE peut être utilisé dans un livre présentant des simulations de circuits pour permettre aux étudiants "d'observer" le phénomène par eux mêmes. De cette manière, les lecteurs peuvent apprendre les concepts d'une manière inductive (en interprétant la sortie de SPICE) de même que déductive (en interprétant mes explications). En outre, en voyant SPICE utilisé et abusé, ils doivent être capable de comprendre comment l'utiliser par eux-mêmes, fournissant des moyens d'expérimentations sans risque sur leur propre ordinateur avec des simulations de circuits de leur propre conception.

Un autre avantage d'inclure des analyses d'ordinateur dans un livre est de permettre les vérifications empiriques sur les concepts présentés. Sans démonstrations, le lecteur doit croire les déclarations de l'auteur sur parole, faisant confiance à l'exactitude de ce qui est écrit. Le problème avec la croyance, bien sûr, est qu'elle ne vaut que dans l'autorité dans laquelle elle est placée et l'exactitude de l'interprétation au travers de laquelle elle est comprise. Les auteurs, comme toute chose humaine, sont sujets aux erreurs et/ou aux problèmes de communication. Avec les démonstrations, néanmoins, le lecteur peut voir immédiatement de lui-même si ce que décrit l'auteur est vrai. Les démonstrations servent aussi à clarifier la signification du texte avec des exemples concrets.

SPICE a été introduit plus tôt dans le volume I (DC) de cette série de livres et, fort heureusement, d'une manière suffisamment douce pour qu'il ne se crée pas de confusion. Pour ceux qui veulent en apprendre plus, un chapitre dans le volume de Reference (volume V) contient un survol de SPICE avec plusieurs circuits d'exemple. Il existe sûrement des simulateur de circuits plus voyants (graphiques) dans l'existence mais SPICE est libre, une vertu complémentant admirablement la philosophie charitable de ce livre.

15.3 Remerciements

D'abord, je veux remercier ma femme, dont la patience pendant ces nombreuses et longues soirées (et weekends!) de saisie a été extraordinaire.

Je souhaite aussi remercier ceux dont les efforts de développement des logiciels open-source ont rendu cette tentative plus abordable et plaisante. Ce qui suit est une liste des divers logiciels libres qui ont été utilisé pour réaliser ce livre et les programmeurs respectifs :

- *GNU/Linux* Système d'exploitation – Linus Torvalds, Richard Stallman et une pléthore d'autres trop nombreux pour être cités.

- *Vim* éditeur de texte – Bram Moolenaar et les autres.
- *Xcircuit* programme de dessin – Tim Edwards.
- *SPICE* programme de simulation de circuits – trop de contributeurs pour les mentionner.
- *Nutmeg* programme post-processeur pour SPICE – Wayne Christopher.
- TEX système de traitement de texte – Donald Knuth et d'autes.
- *Texinfo* système de formatage de documents – Free Software Foundation.
- LATEX système de formatage de documents – Leslie Lamport et d'autres.
- *Gimp* programme de manipulation d'image – trop de contributeurs à mentionner.
- *Winscope* logiciel d'analyse de signal – Dr. Constantin Zeldovich. (Libre pour utilisation personnelle et académique.)

Mon appréciation est aussi étendue à Robert L. Boylestad, dont la première édition du *Introductory Circuit Analysis* m'a plus appris sur les circuits électriques que tout autre livre. Les autres textes importantes dans mes études électroniques incluent l'édition 1939 de *The "Radio" Handbook*, la seconde édition de *Introduction to Electronics I* de Bernard Grob et l'original de Forrest Mims, *Engineer's Notebook*.

Merci à l'encadrement du Bellingham Antique Radio Museum, qui ont été suffisamment généreux pour me laisser terroriser leur établissement avec ma caméra et son flash. Des remerciements similaires à Jim Swartos et KARI radio de Blaine, Washington pour la tournée d'information de leur installations étendue (50 kW) de même que leur équipement de transmission dernier cru.

Je veux spécialement remercier Jeffrey Elkner et ceux, à Yorktown High School pour avoir hébergé mon livre comme partie de leur Open Book Project et pour avoir fait le premier effort de contribution de la forme et du contenu. Merci aussi à David Sweet (site web : `(http ://www.andamooka.org)`) et Ben Crowell (site web : `(http ://www.lightandmatter.com)`) pour avoir donné des encouragements, des critiques constructives et une grande diffusion pour la version en ligne de ce livre.

Merci à Michael Stutz pour la Design Science License et à Richard Stallman pour avoir été dans les premiers à faire du copyleft.

En dernier et mais non des moindres, beaucoup de remerciements à mes parents et les professeurs qui ont vu en moi le souhait d'apprendre l'électricité et qui ont changé cette flamme en passion pour la découverte et l'aventure intellectuelle. Je vous honore en aidant les autres de la même manière dont vous m'avez aidé.

Tony Kuphaldt, Avril 2002

> *"Une bougie ne perd pas de lumière lorsqu'elle en allume une autre"*
> **Kahlil Gibran**

Chapter 16

CONTRIBUTOR LIST

16.1 How to contribute to this book

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16.2.1 Tony R. Kuphaldt

- **Date(s) of contribution(s) :** 1996 to present
- **Nature of contribution :** Original author.
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16.2.2 Jason Starck

- **Date(s) of contribution(s) :** May-June 2000
- **Nature of contribution :** HTML formatting, some error corrections.
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16.2.3 Traduction

- **Date(s) de la contribution :** Mai 2004
- **Nature de la contribution :** Traduction en français.
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16.2.4 Corrections typographiques et autres contributions “mineures”

- *The students of Bellingham Technical College’s Instrumentation program.*
- **Bart Anderson** (January 2004) Corrected conceptual and safety errors regarding Tesla coils.
- **Ed Beroset** (May 2002) Suggested better ways to illustrate the meaning of the prefix ”poly-” in chapter 10.
- **Duane Damiano** (February 2003) Pointed out magnetic polarity error in DC generator illustration.
- **Harvey Lew** (February 2003) Typo correction in Basic AC chapter : word ”circuit” should have been ”circle”.
- **Jim Palmer** (May 2002) Typo correction on complex number math.
- **Don Stalkowski** (June 2002) Technical help with PostScript-to-PDF file format conversion.
- **John Symonds** (March 2002) Suggested an improved explanation of the unit ”Hertz.”
- **Joseph Teichman** (June 2002) Suggestion and technical help regarding use of PNG images instead of JPEG.
- **Mark D. Zarella** (April 2002) Suggested an improved explanation for the ”average” value of a waveform.

Chapter 17

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